Teleparallel Cosmology
From theory to observational constraints

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Outline

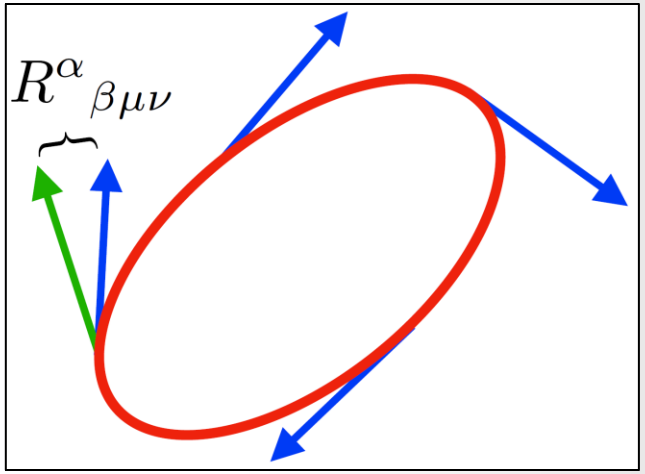
- **Standard gravity** and the motivation for modified gravity
- **Modified Gravity** through other branches of physics
- The **Teleparallel Gravity** (TG) formalism
- $f(T)$ **cosmology** observations
- **Model-independent cosmology**
- **Teleparallel Horndeski Gavity**
- Concluding Remarks

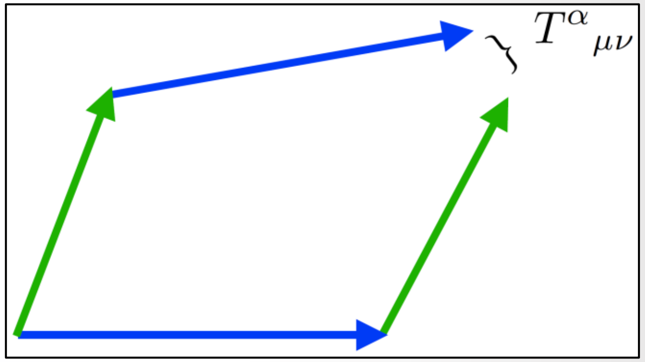
Why do we need modifications to standard cosmology?

Introduction

- Einstein 1915: **General Relativity (GR)**
  Energy-momentum source of curvature
  Levi-Civita connection: Zero Torsion, Metricity

- Einstein 1928: **Teleparallel Equivalent of GR (TEGR)**
  Energy-momentum source of torsion
  Teleparallel connection: Zero Curvature, Metricity

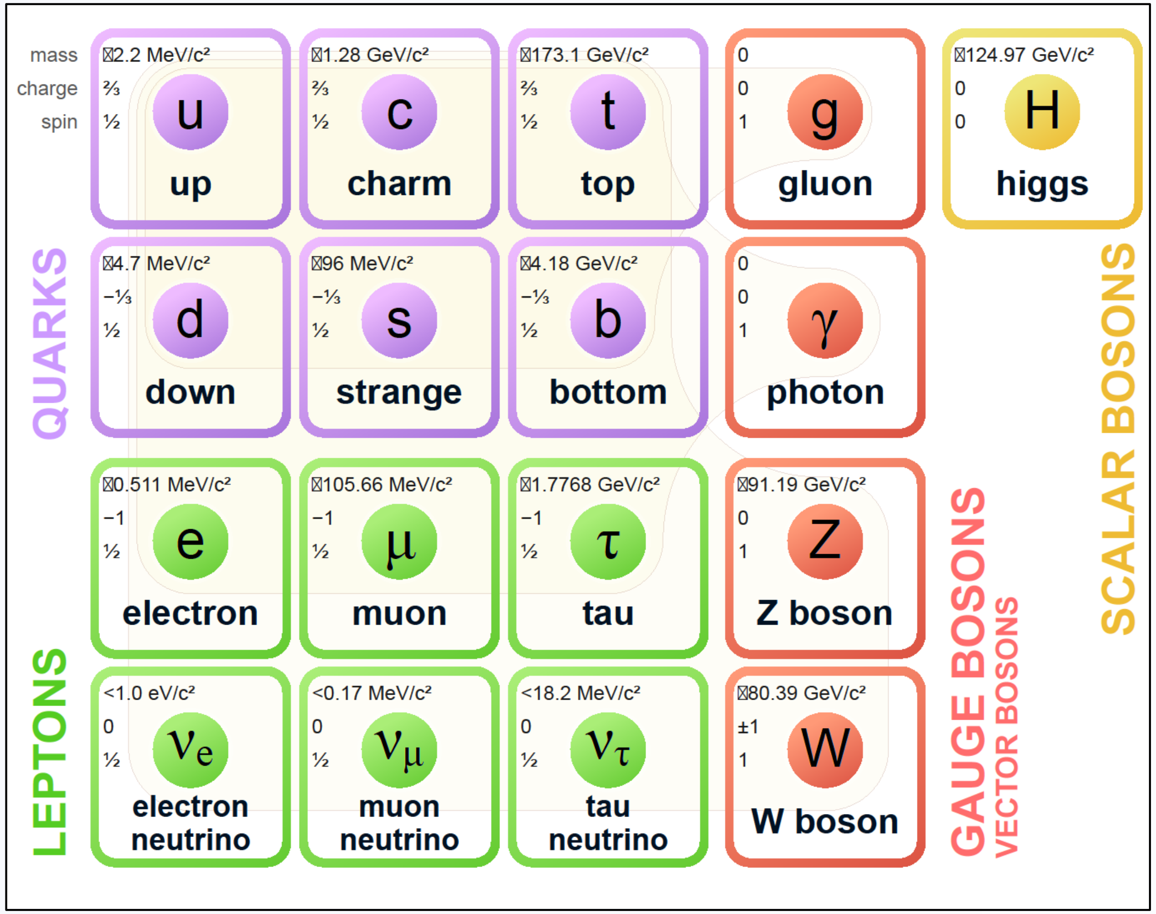




arXiv:1903.06830

Fundamental Physics

Standard Model of Particle physics + GR



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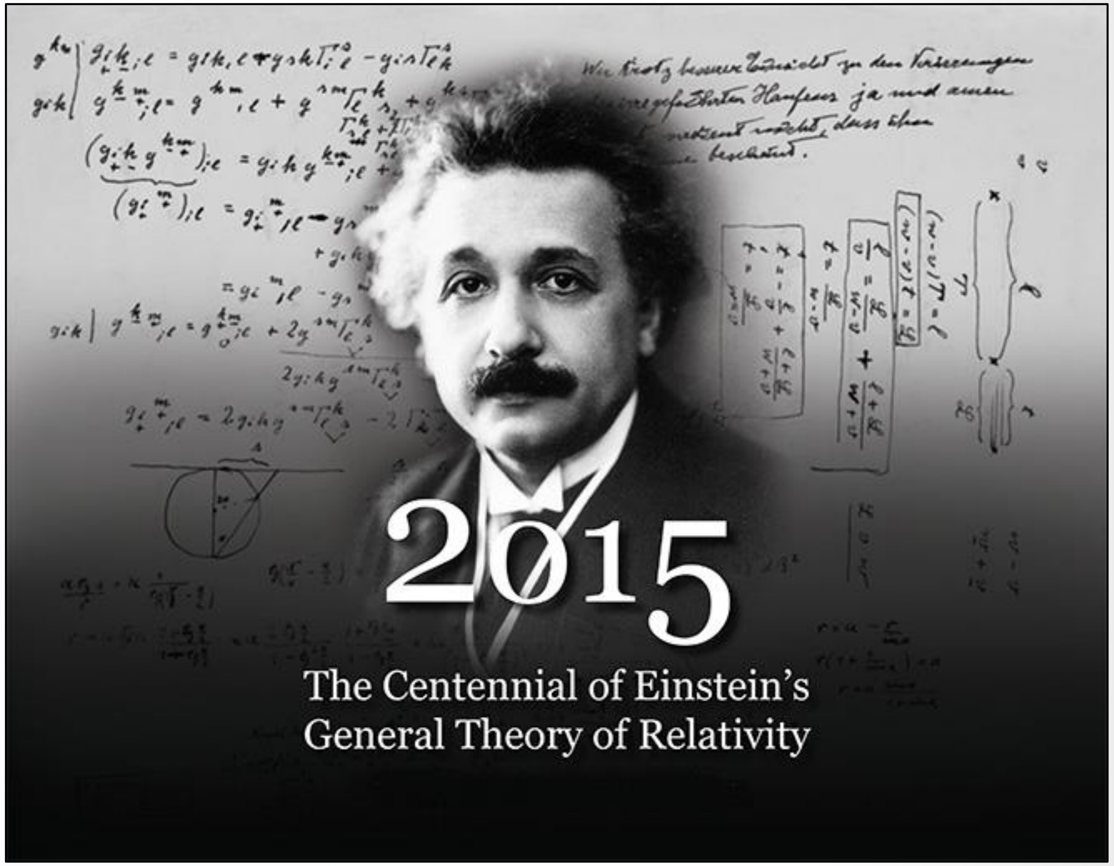
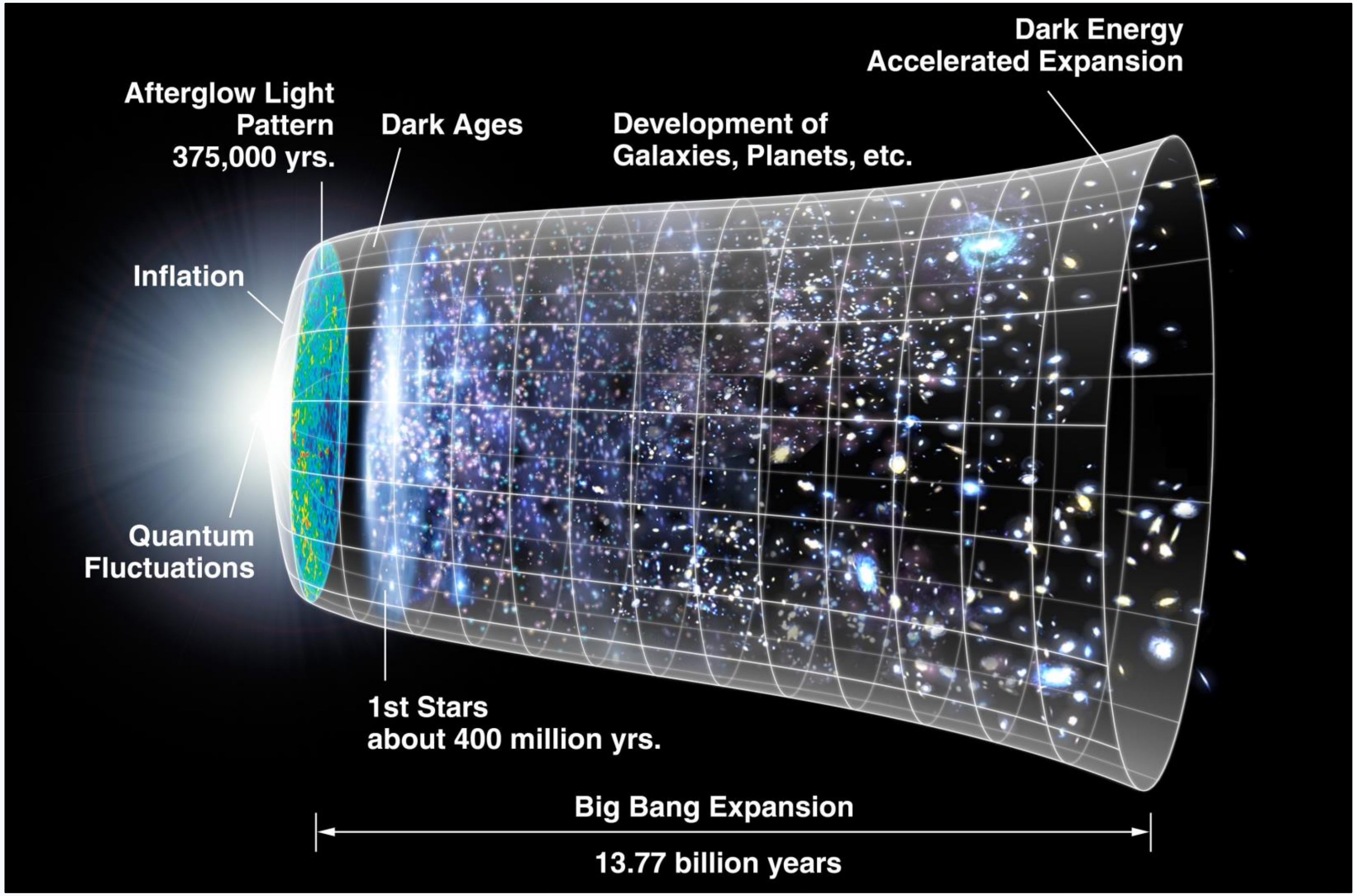
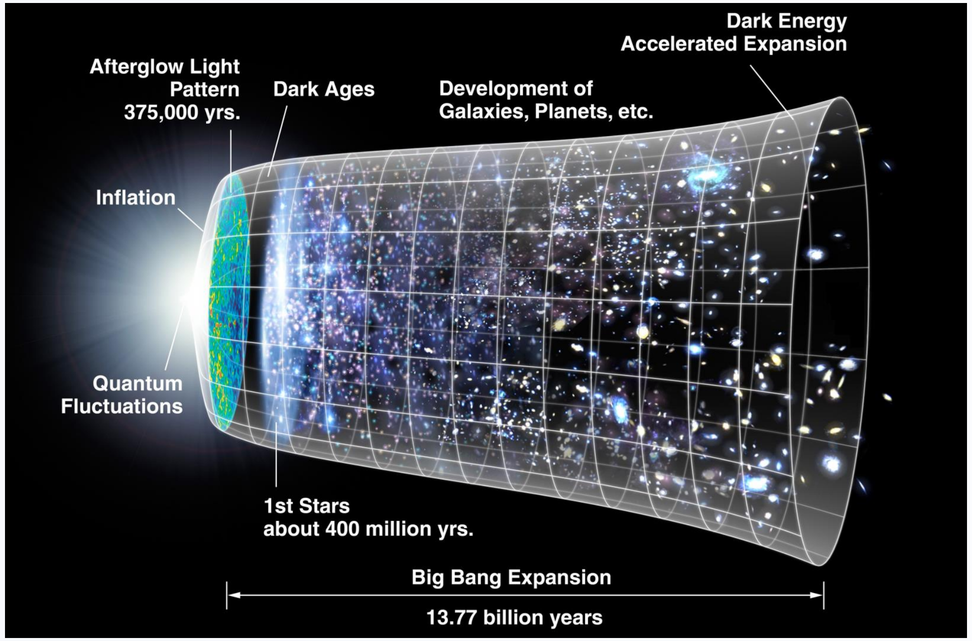
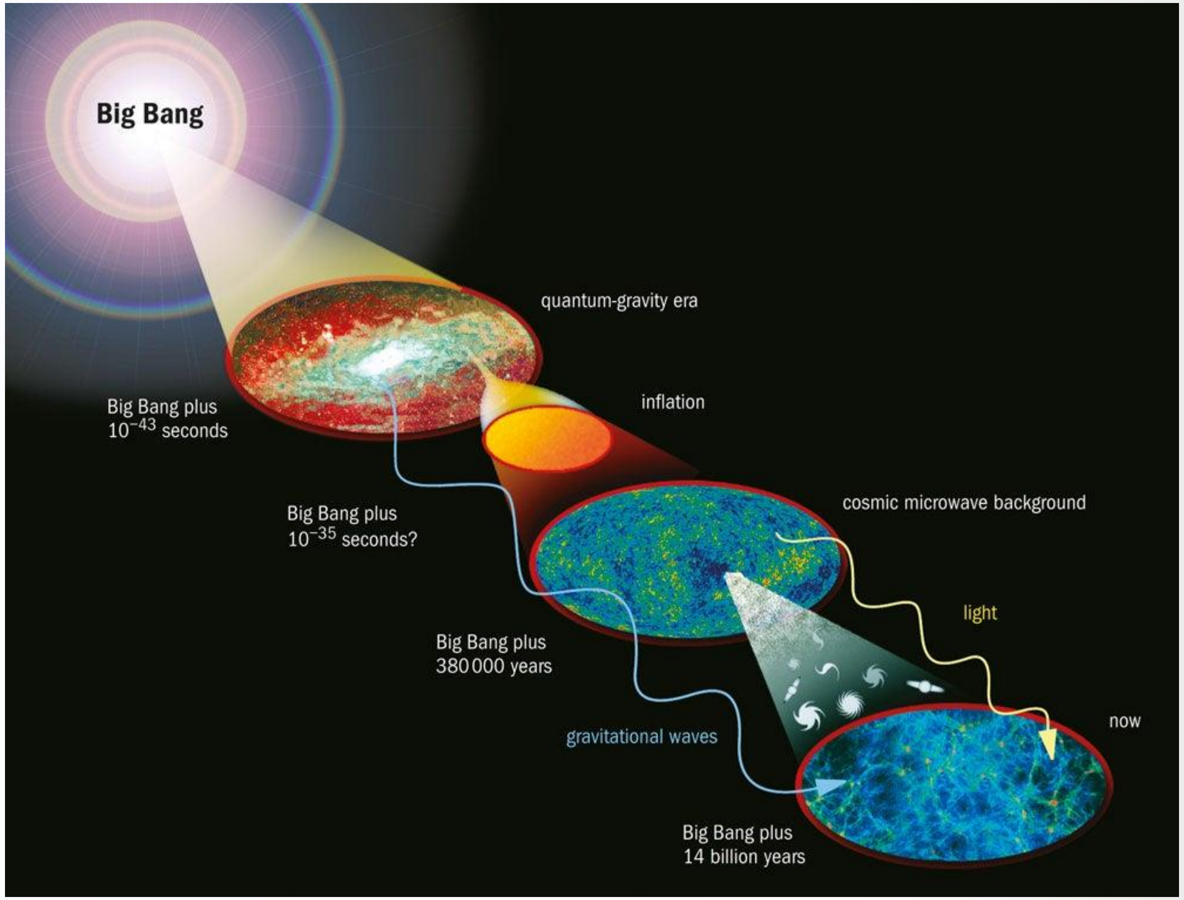


Cosmological History



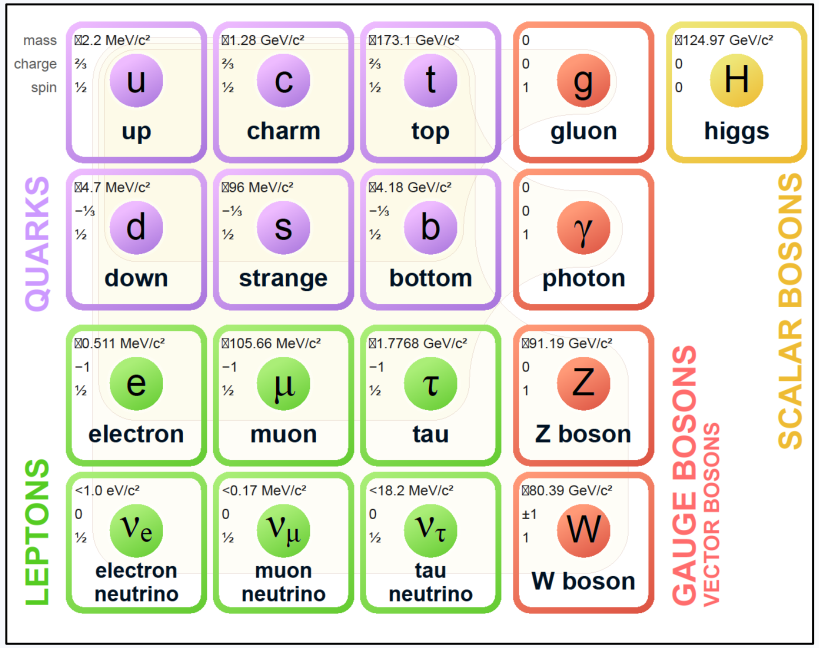
Can we describe the early universe?





No!

Adding Inflation



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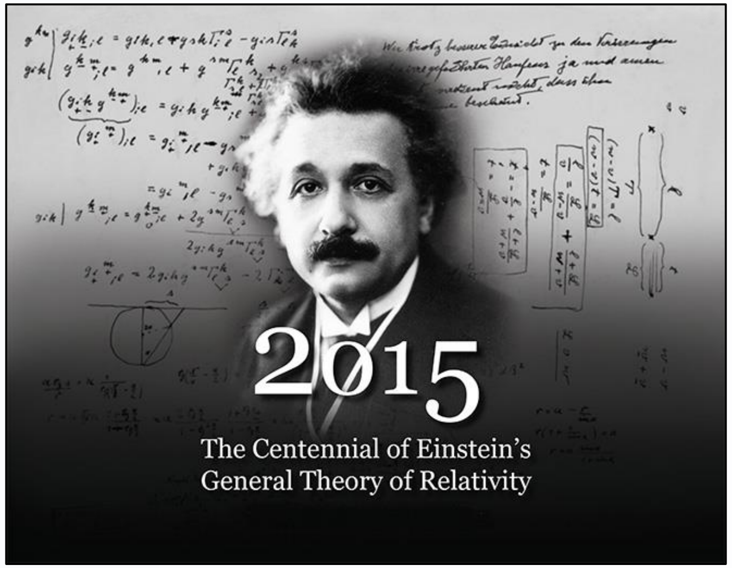

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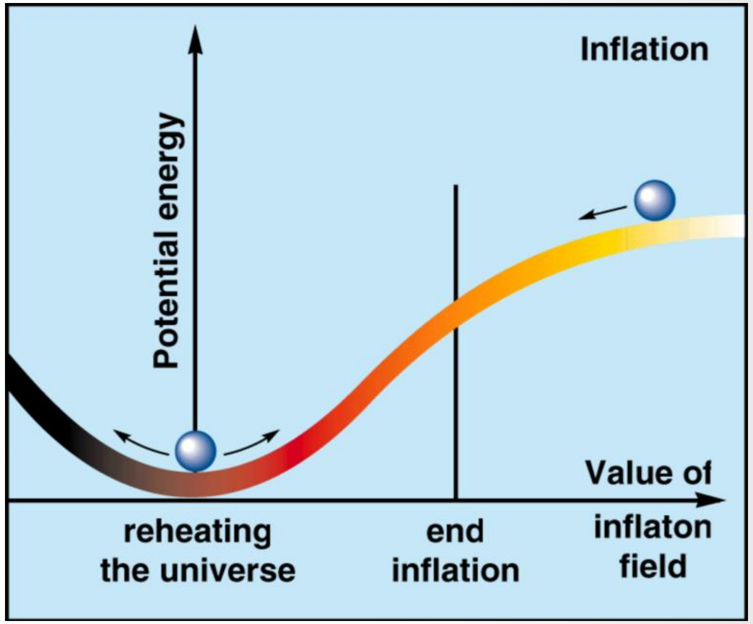


arXiv:astro-ph/9906497

General Relativity and Standard Modifications

- Einstein 1915 – **GR**:

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}[\mathcal{R}] + \int d^4x\sqrt{-g}\mathcal{L}_m(g_{\mu\nu},\psi)$$

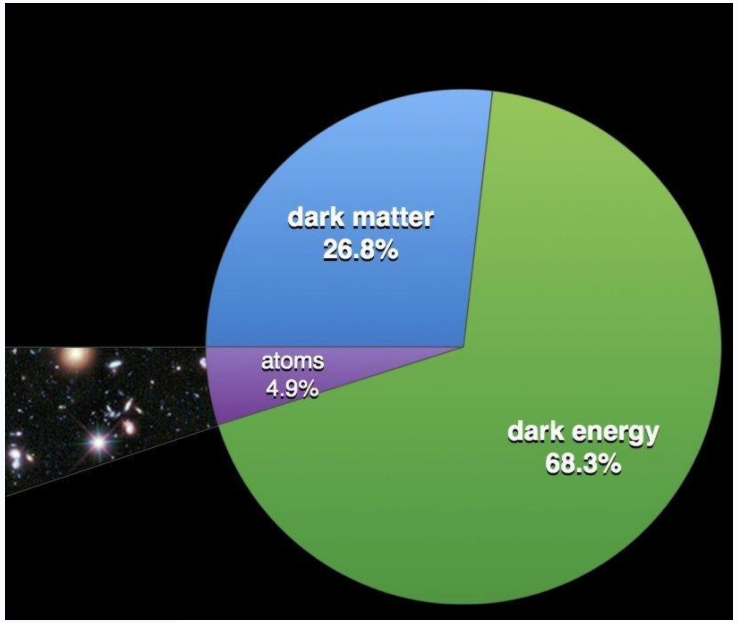
$$\Rightarrow \quad \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = 8\pi G\, T_{\mu\nu}$$

with $T_{\mu\nu} \coloneqq \frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_m}{\delta g_{\mu\nu}}$

Late-time reasons for modified gravity

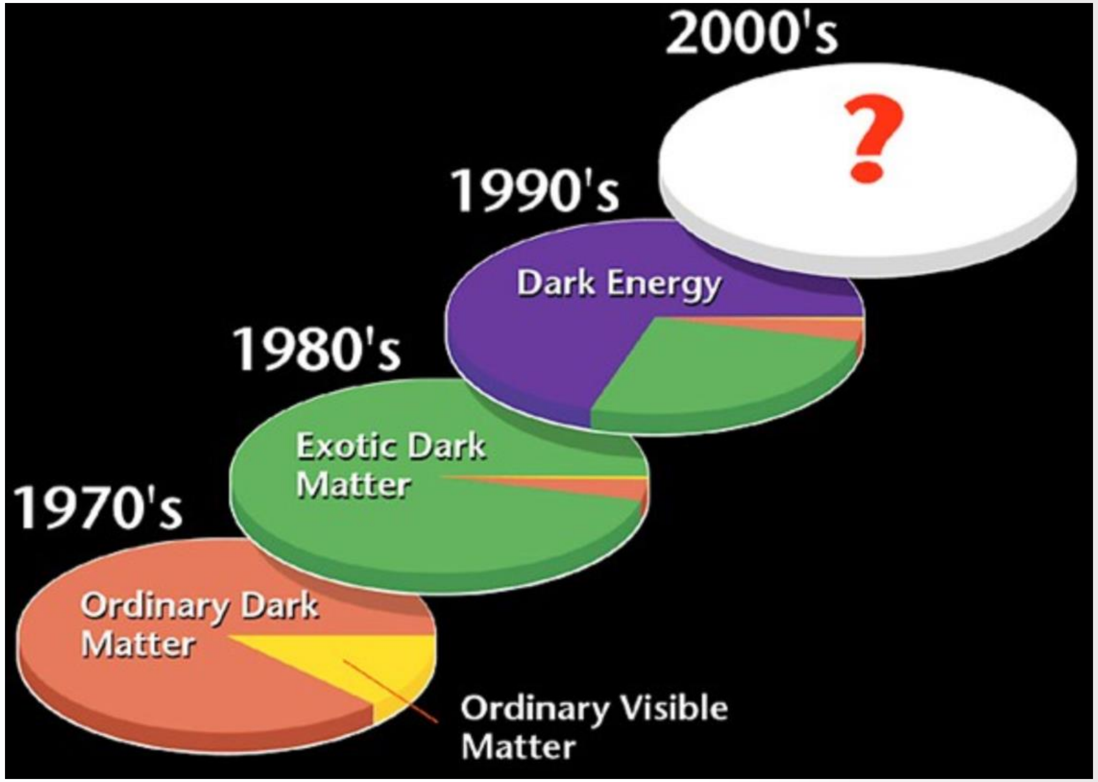
Galactic Dynamics: Flat rotation curve problem

Dark Energy: Late-time acceleration



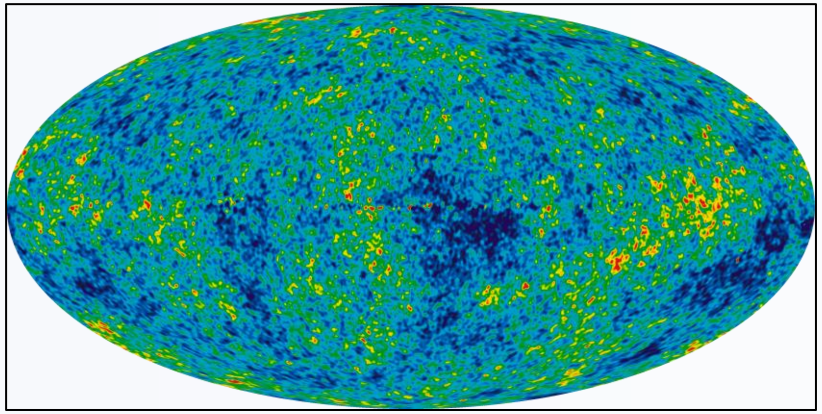
Planck Collaboration

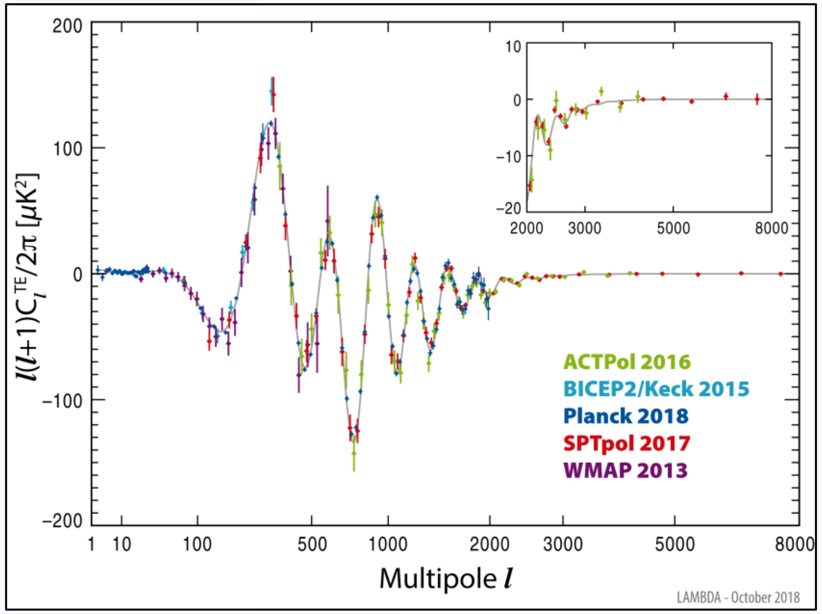
Energy budget of the universe over the decades



WMAP Science Team

Modified Matter I





Dark Matter Detection Attempts:

2000 – MACHO: MACHO microlensing

2010 – DAMA/LIBRA: WIMP particle interactions

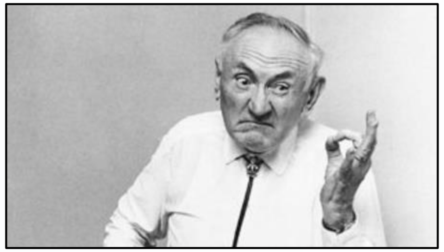
2014-2016 – LUX: WIMP particle interactions

2015 – The Axion Dark Matter eXperiment (ADMX)

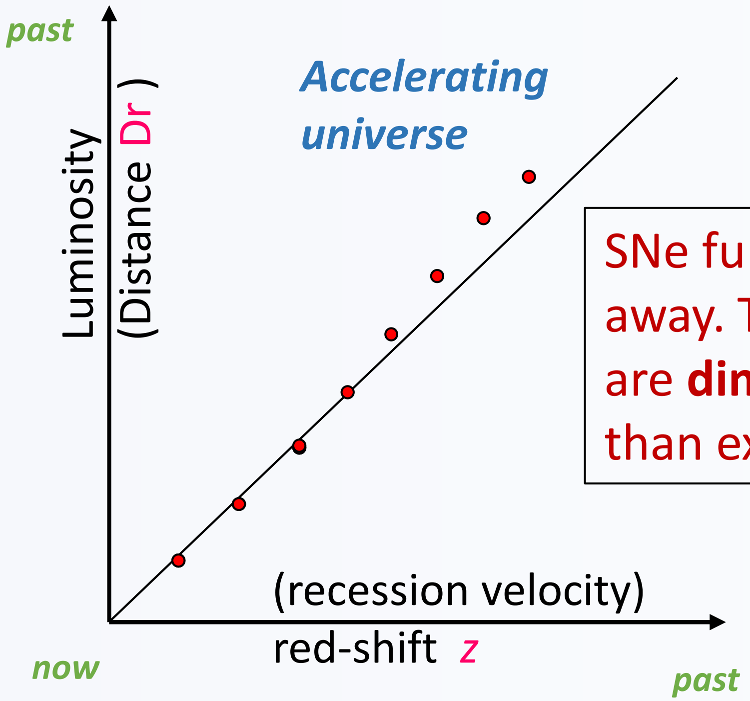
2016 – IceCube: Sterile neutrinos

2016 – LHC: Supersymmetric particles

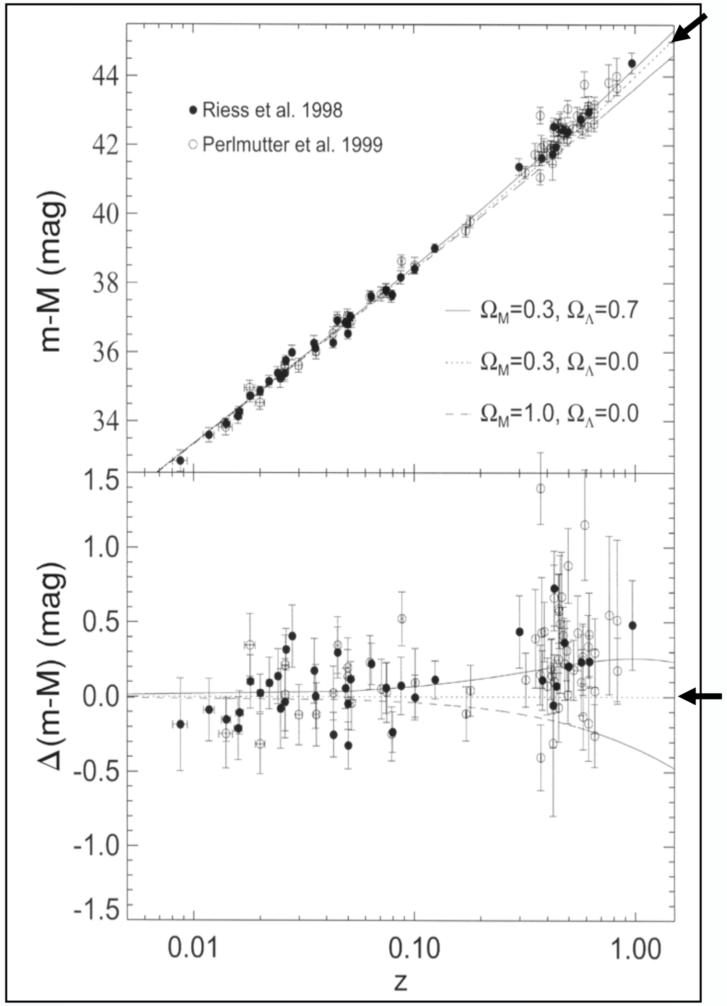
2017 – ANAIS Experiment

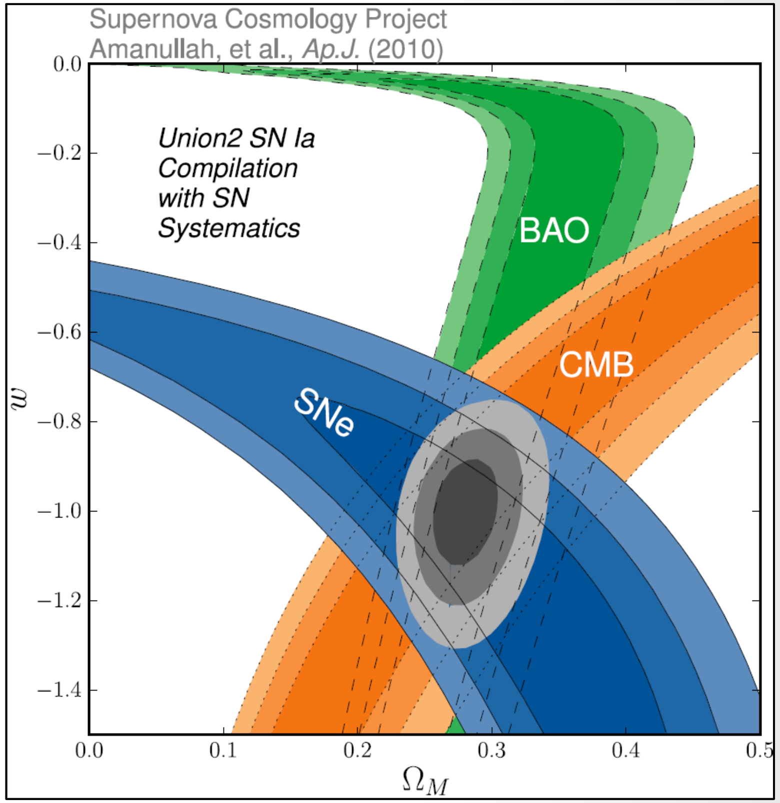


Dark Energy as the cosmological constant

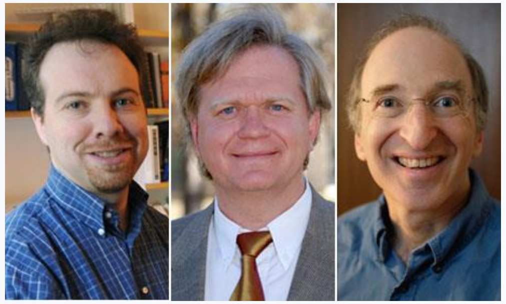


SNe further away. They are **dimmer** than expected



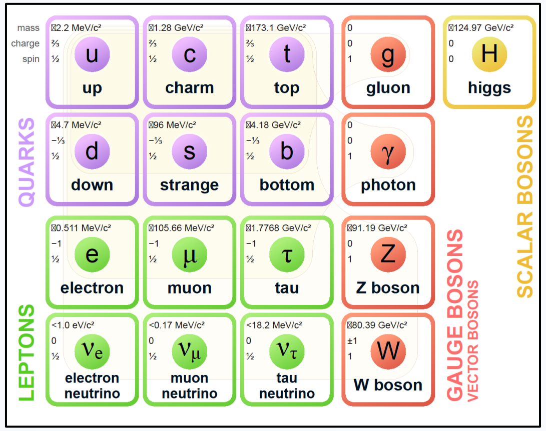


$$w_{DE} = -1.03 \pm 0.03$$



2011 Nobel Prize in Physics

Modified Matter II



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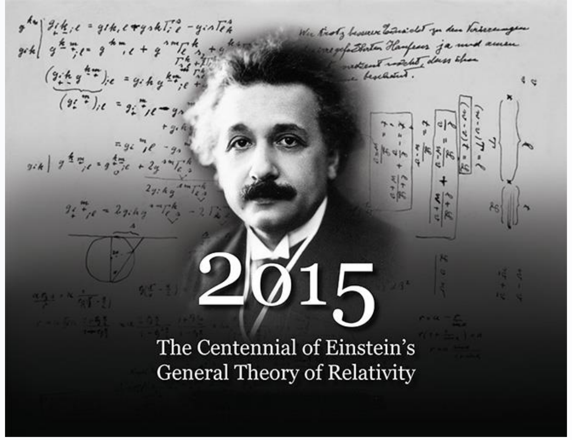

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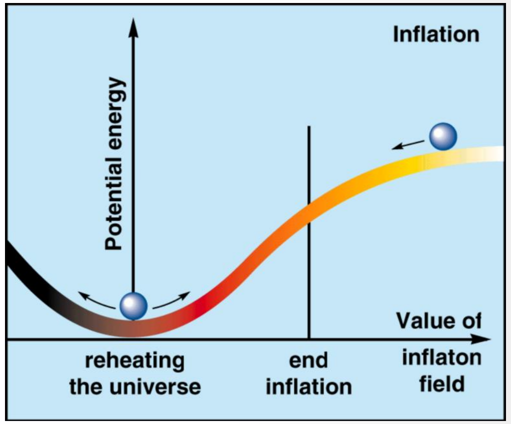

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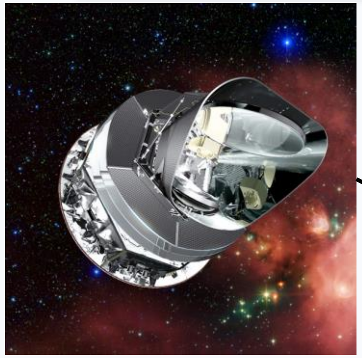
$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,[\mathcal{R} - 2\Lambda]$$

$$+\int d^4x\sqrt{-g}\,\mathcal{L}_m(g_{\mu\nu},\psi)$$

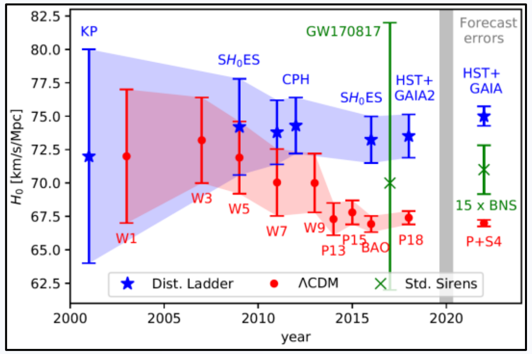
The H_0 Tension

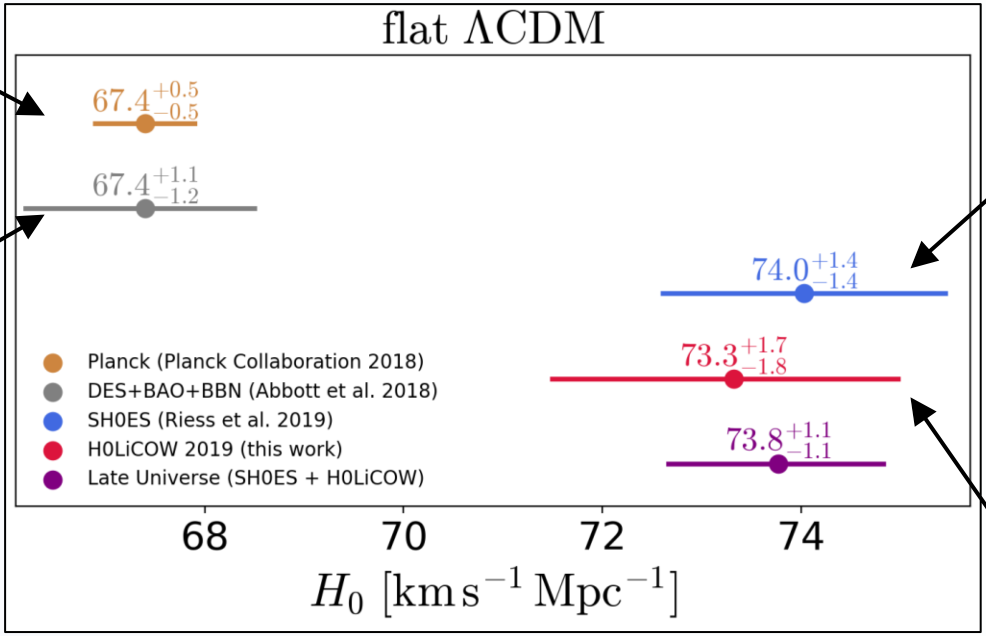


Planck Mission (predictions from the CMB)

THE DARK ENERGY SURVEY

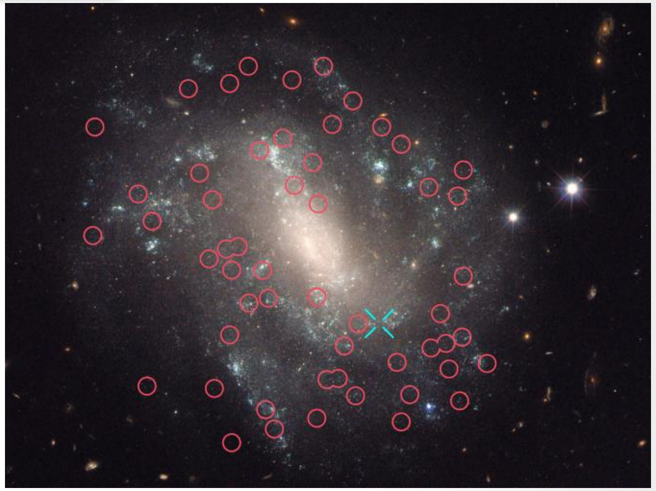
(visible and near-infrared survey)



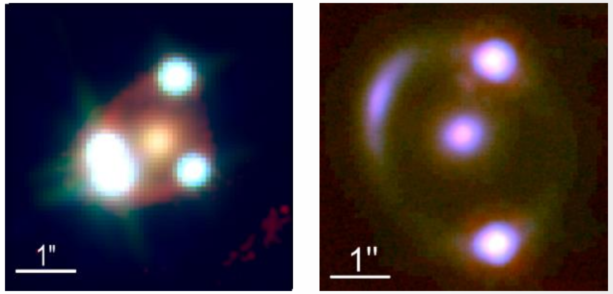


Wong et al. MNRAS 498, 1 (2020)

5.3σ tension

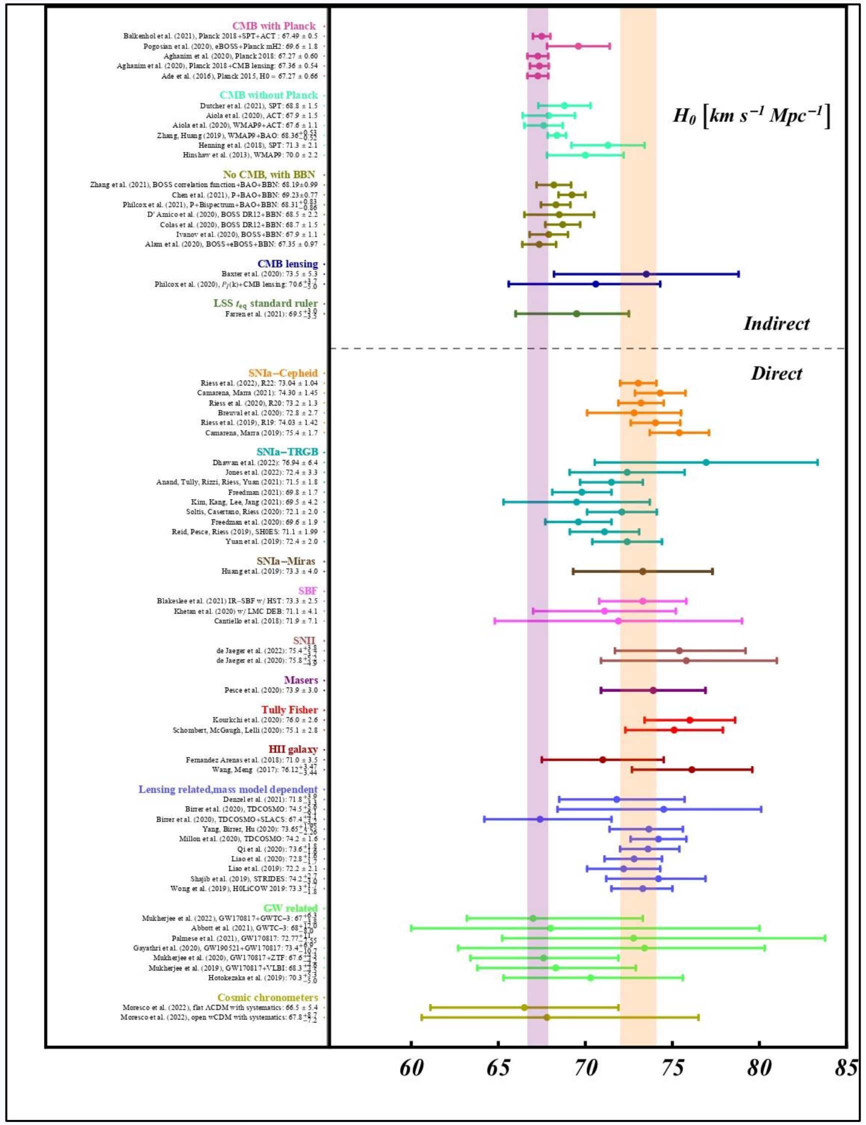


The SH0ES Project (calibrated with Cepheid variables)



H0LICOW (strong lensing - cosmography)

The H_0 Tension



Di Valentino et al. CQG, 38 (15) (2021)

Cosmology Intertwined, JHEAp. 2204, 002 (2022)

Modified Gravity through Lovelock's Theorem

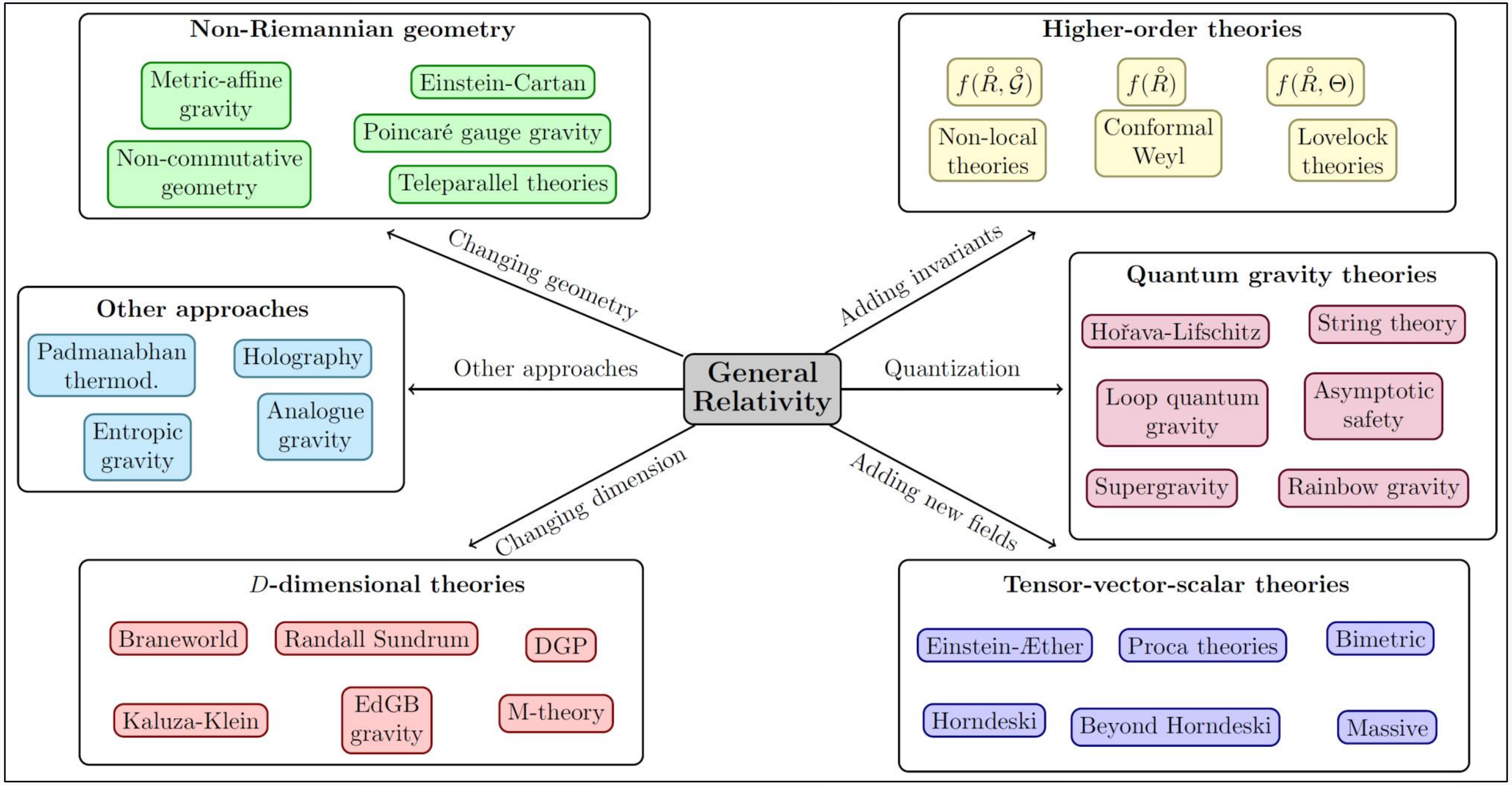
Lovelock's Theorem

For **second order** vacuum field equations $E_{\mu\nu} = 0$, if $E_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$, then

$$E_{\mu\nu} \coloneqq G_{\mu\nu} + \Lambda g_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R} g_{\mu\nu} + \Lambda g_{\mu\nu}$$

- Adding new fields (scalar, vector, tensor)
- Use more/less than four dimensions of spacetime
- Take an emergent form of gravity
- Changing the gravitational connection
- Consider non-local terms
- Adding more than second-order derivatives of the metric

The Modified Gravity Landscape



What inspiration can we get from other branches of physics?

Inspiration from Particle Physics

- **Gauge Principle**: Replace **global symmetries** by **local ones**
- **Group generators** produce compensating **fields**
- This results in the **standard model forces**

Can we apply this to gravity?

Gauge theory of gravity

- Formulating a **gauge theory of gravity (1956 onwards)**
- Starting from **special relativity (SR)**
  - Applying **Yang-Mills theory** to **SR**
  - Result is **Poincaré gauge theory** (**curvature** and **torsion** appear as field strengths)
- **Torsion** is the **field strength** of the **translation group**

Hehl et al. Phys.Rept. 258, 1 (1995) [arXiv:gr-qc/9402012]

Modified Gauge Gravity

- One can always **modify gravity** (supergravity, conformal, metric affine,…)
- In all of them, **torsion** is related to the **gauge structure** of the theory
- Here, torsion opens the possibility of having a **quantum theory** of gravity

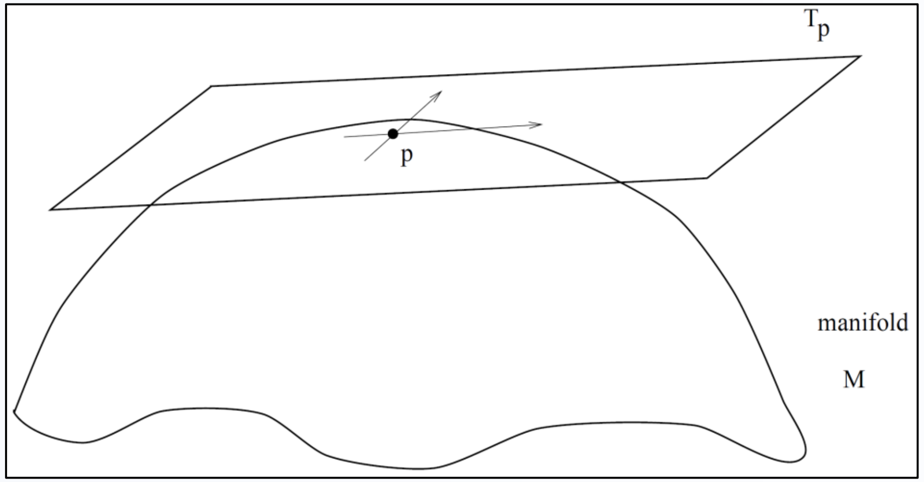
Modifying Gravity

- **Accelerating Universe (1998)**: Thousands of works in modified gravity ($f(\mathcal{R})$, Horndeski, Galileon, Lovelock, massive, Weyl,…)
- These are almost all **curvature-based**
- Can we **modify gravity** using **torsion**?

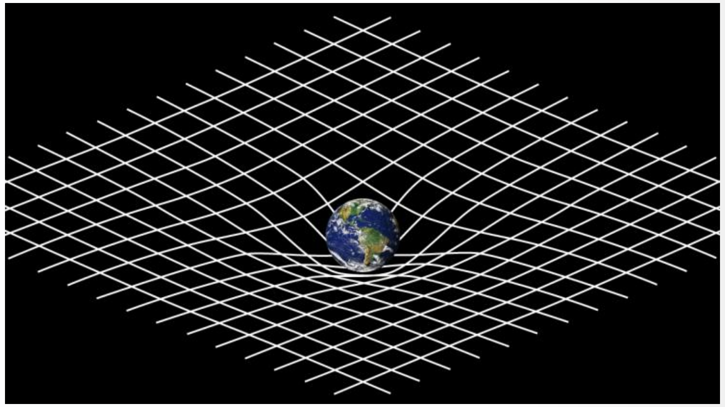
Saridakis et al. [The Cantata Consortium], Modified Gravity and Cosmology: An Update by the CANTATA Network. Springer, Cham (2021) [arXiv:2105.12582]

Rethinking the connection

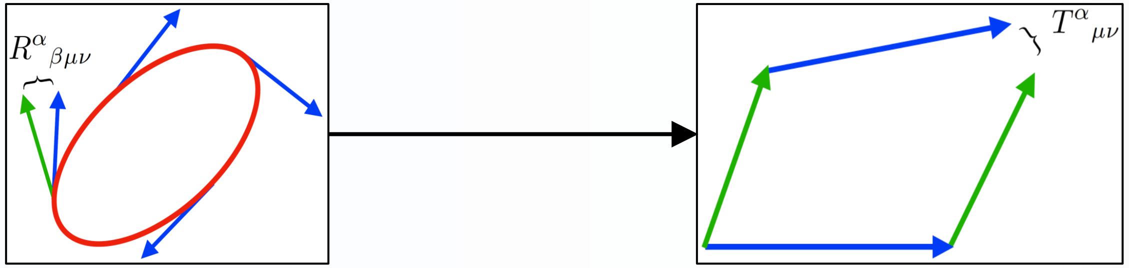
Spacetime tells matter how to move; matter tells spacetime how to curve



John Archibald Wheeler



<u>Connection of gravity</u>: Curvature is a property of the **connection**, not of the **spacetime**



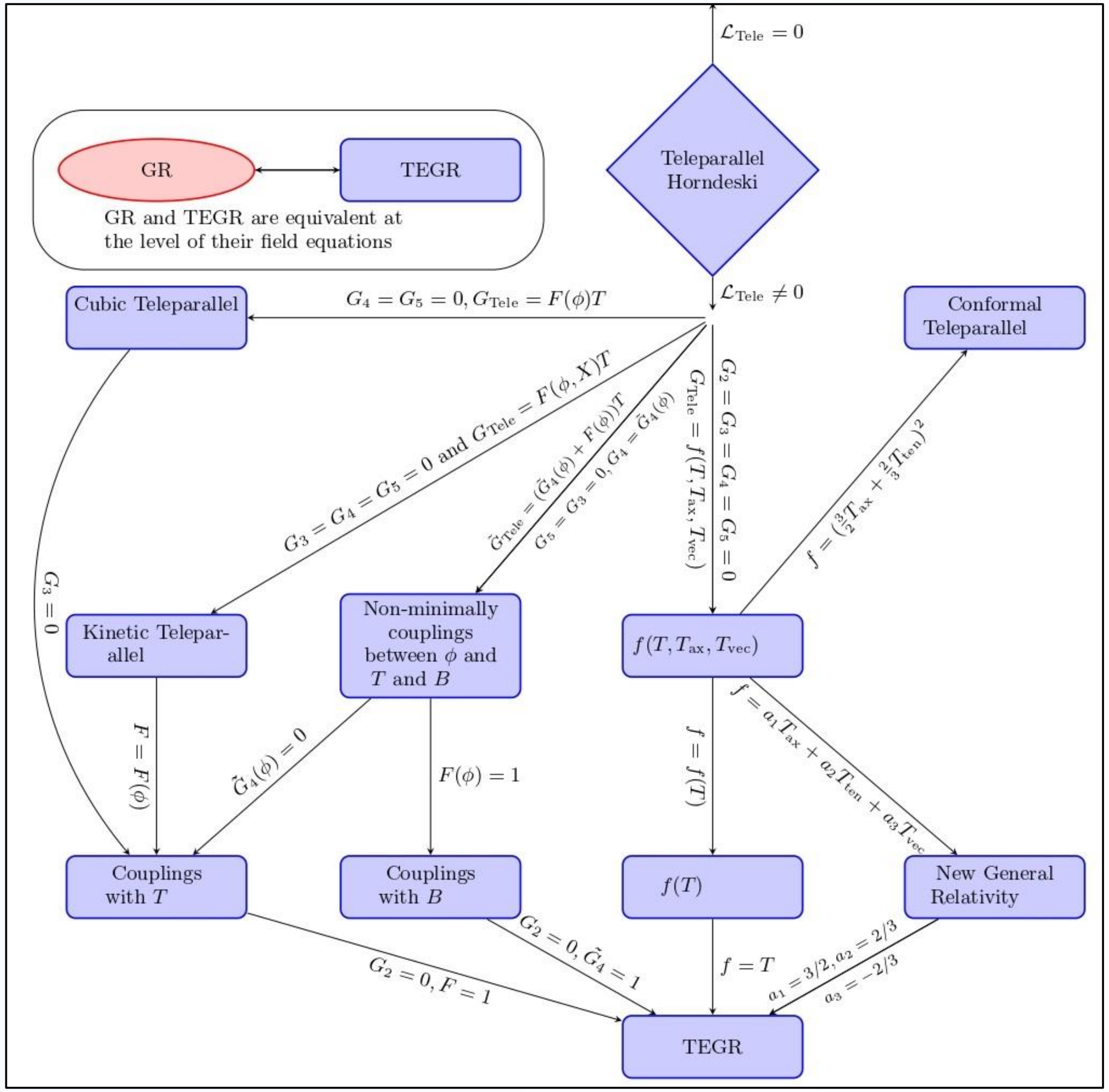
The Teleparallel Equivalent of GR (TEGR)

- **TEGR**: This is the simplest torsional theory of gravity
- Tetrad ($e^a{}_\mu$): Relate the tangent space ($g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$)
- Use the **teleparallel connection** ($\Gamma^\sigma_{\mu\nu} = e_a{}^\sigma \partial_\nu e^a{}_\mu + e_a{}^\sigma \omega^a{}_{\nu\mu}$) instead of the **Levi-Civita connection** (Christoffel symbols)
- **Torsion tensor**: Measures torsion ($T^\sigma{}_{\mu\nu} = \Gamma^\sigma_{\nu\mu} - \Gamma^\sigma_{\mu\nu}$)
- TEGR Action:

$$S = -\frac{1}{16\pi G}\int d^4x\, e[T]$$

where $T \equiv \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}{}^{\rho} T^{\nu\mu}{}_{\nu}$

Modified Teleparallel Gravity



Bahamonde et al. RoPP 86 026901 (2023) [arXiv:2106.13793]

Modified Teleparallel Gravity

- **Curvature-Torsion Relation:** $\mathcal{R} = -T + B$

$$B \propto \nabla^{\mu} T^{\lambda}{}_{\lambda\mu}$$

- $f(T)$ **Gravity:** Inspire by $f(\mathcal{R})$ gravity

$$S = \frac{1}{16\pi G} \int d^4x\, e[-T + f(T)] + S_{\text{mat}}$$

- Taking a flat (**FLRW**) cosmology: $g_{\mu\nu} = \text{diag}\left(-1, a(t)^2, a(t)^2, a(t)^2\right)$

- **Friedmann equations:**

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} + \frac{T}{3} f_T$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 - f_T - 2T f_{TT}}$$

$$T = 6H^2 = 6\left(\frac{\dot{a}}{a}\right)^2$$

$f(T)$ Effective Dark Energy

- Interpreting the modification to TEGR as a **dark fluid**

$$8\pi G\,\rho_{DE} \coloneqq T f_T - \frac{f}{2}$$

$$8\pi G\,(p_{DE} + \rho_{DE}) \coloneqq -\dot{H}(f_T + 2T f_{TT})$$

- The **effective Equation-of-State** (EoS) turns out to be

$$\omega_{DE} \coloneqq \frac{p_{DE}}{\rho_{DE}} = -1 + (1+\omega)\frac{(f - T - 2T f_T)(f_T + 2T f_{TT})}{(1 - f_T - 2T f_{TT})(f - 2T f_T)}$$

Scalar Perturbations

- $f(T)$ **gravity** leaves imprints at the **perturbative level**

$$e^0{}_\mu = \delta^0_\mu(1+\psi)\,, e^i{}_\mu = \delta^i_\mu a(1-\phi) \;\Rightarrow\; ds^2 = (1+2\psi)dt^2 - a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

- **Matter over-density perturbations** also contribute through

$$\delta_m = \frac{\delta\rho_m}{\rho_m}$$

- **Matter perturbation evolution** equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m + 4\pi G_{\text{eff}}\rho_m\delta_m = 0$$

Identifying the **effective gravitational constant**

$$G_{\text{eff}} = \frac{G_N}{1+f_T}$$

What do observations tell us about modified teleparallel gravity?

$f(T)$ Gravity Models

Popular models of $f(T)$ gravity

1. Power-law Model: $f_1(T) = \alpha_1 (T)^{b_1}$
2. Linder Model: $f_2(T) = \alpha_2 T_0 \left(1 - e^{-b_2\sqrt{T/T_0}}\right)$
3. Exponential Model: $f_3(T) = \alpha_3 T_0 \left(1 - e^{-b_3 T/T_0}\right)$

Observational Data and Priors

- Cosmic Chronometers (CC): Spectroscopic dating and **independent of cosmological models** ($z \sim 2$)
- Supernovae Type Ia (SN): **Pantheon Sample**
- Baryonic Acoustic Oscillations: **Acoustic perturbations** in early Universe plasma
- $\Delta\alpha/\alpha$ from Quasar Absorption lines: Keck (K) observatory, VLT (V), 21 literature measurements (N) and Oklo nuclear reactor (O)

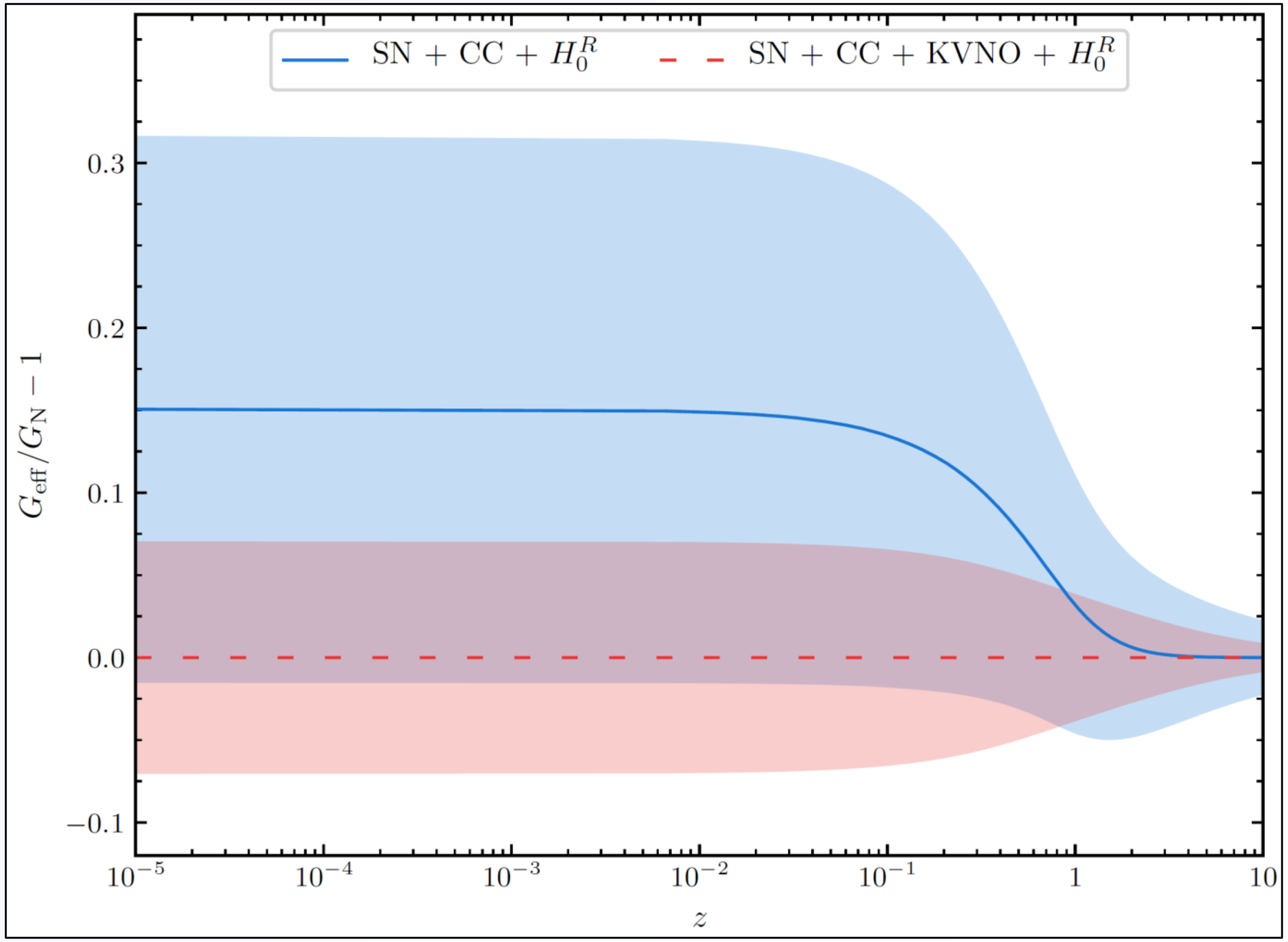
H_0 Priors

- SH0ES Survey [R19]: Riess et al. (2019) mainly using **Cepheid** calibrated **SNe Ia** $\rightarrow H_0^{\mathrm{R19}} = 74.03 \pm 1.42\ \mathrm{km\ s^{-1} Mpc^{-1}}$

- Tip of the Red Giant Branch [TRGB]: Freedman et al. (2019) reports $H_0^{\mathrm{TRGB}} = 69.8 \pm 1.9\ \mathrm{km\ s^{-1} Mpc^{-1}}$

- H0licow [HW]: Based on **strong lensing** $\rightarrow H_0^{\mathrm{HW}} = 73.3 \pm 1.8\ \mathrm{km\ s^{-1} Mpc^{-1}}$

$f_1(T)$ Model



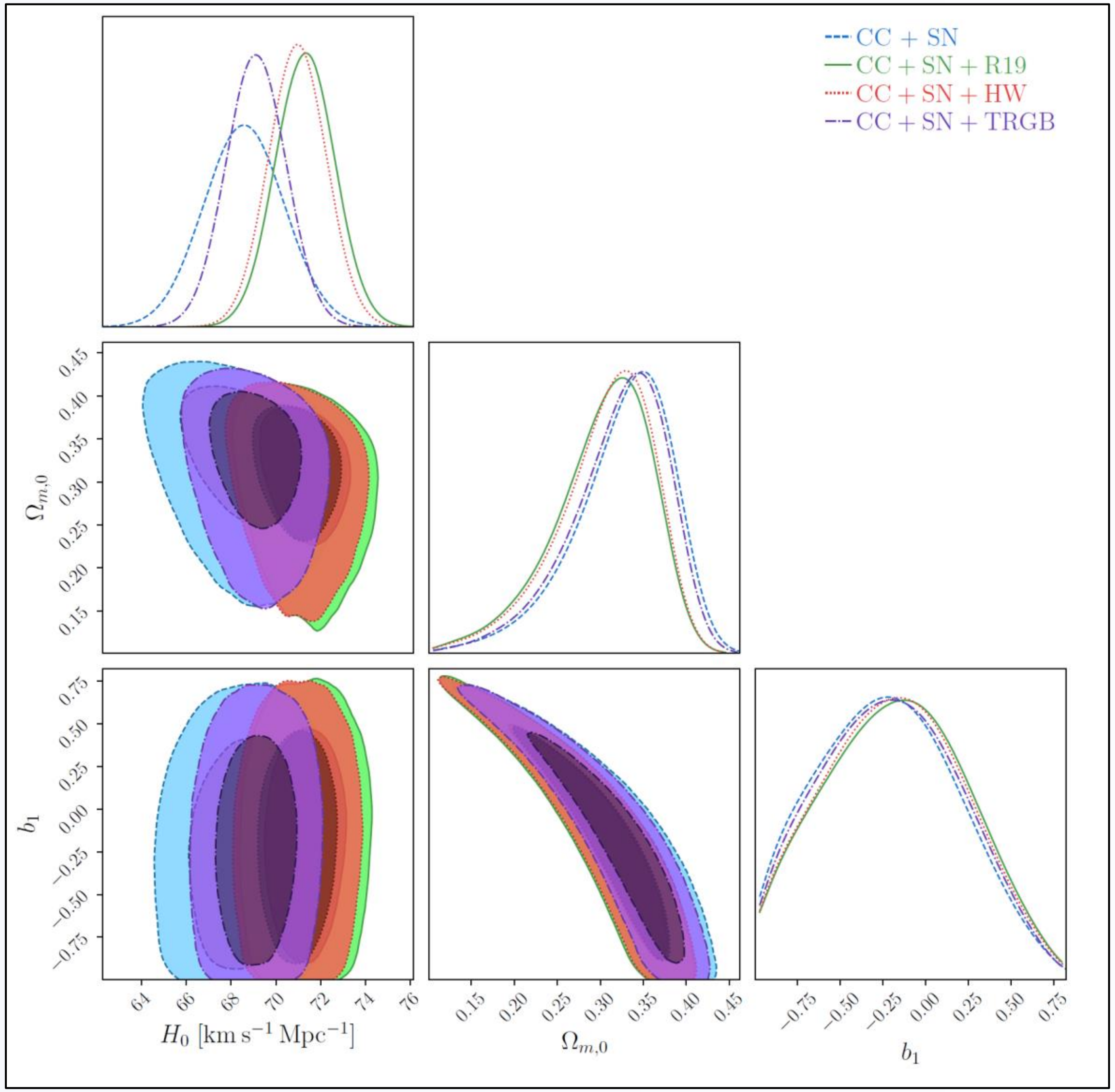
Model: $f_1 = \alpha_1 T^{b_1}$

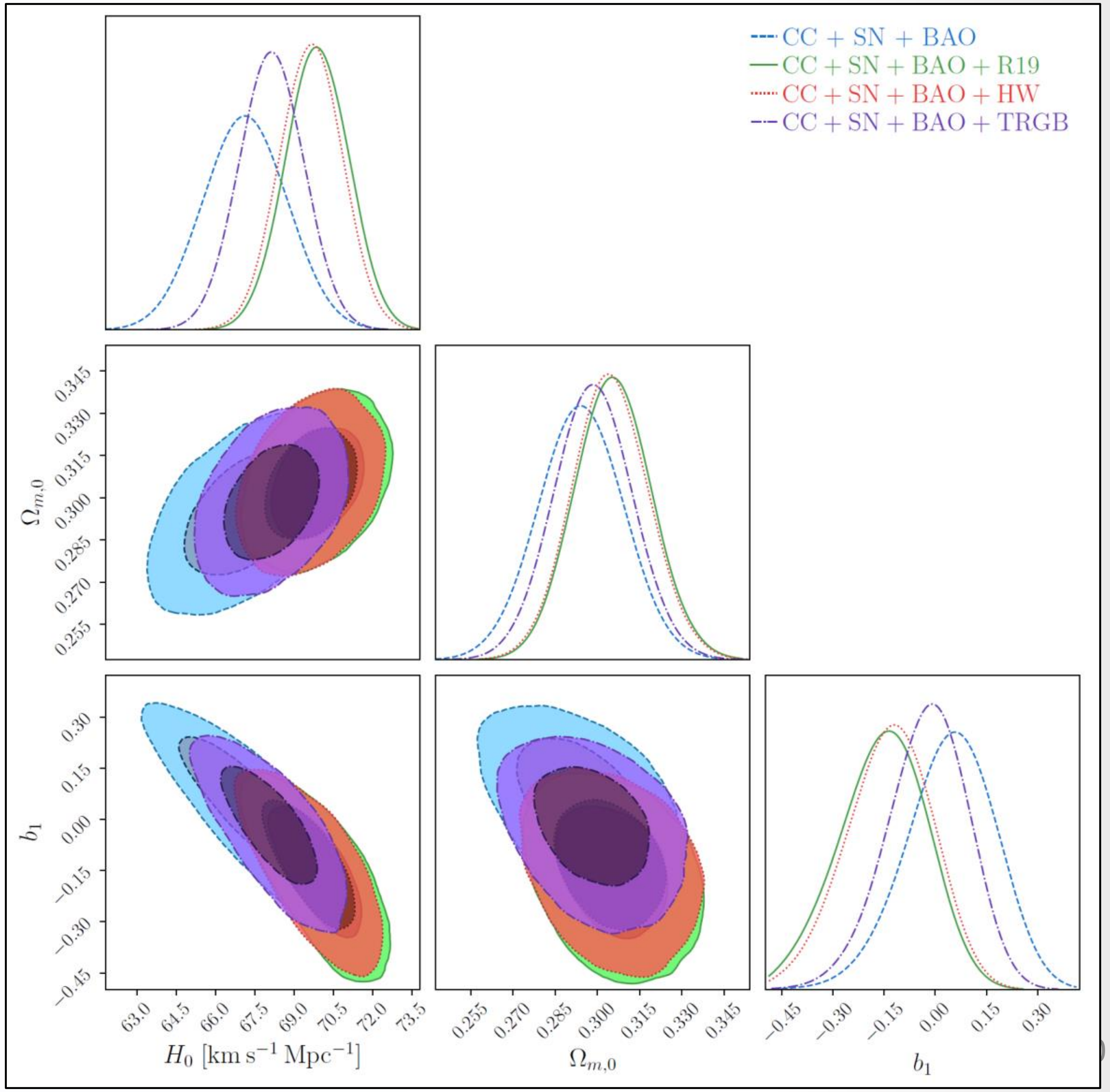
where $\alpha_1 = \left(6H_0^2\right)^{1-b_1} \frac{1-\Omega_0^{\rm m}}{2b_1-1}$

$$G_{eff}(z \simeq 0) \simeq G_N$$

Jackson Levi Said et al. JCAP 11, 047 (2020)

Precision Cosmology Constraints for f_1CDM





Results for f_1CDM

Data Sets	H_0 $\left[\mathrm{km\ s^{-1}\ Mpc^{-1}}\right]$	$\Omega_{\mathrm{m},0}$	b_1	ΔAIC	ΔBIC
CC+SN	68.5 ± 1.8	$0.350^{+0.045}_{-0.064}$	$-0.22^{+0.41}_{-0.48}$	1.45	6.43
CC+SN+R19	$\mathbf{71.3^{+1.3}_{-1.4}}$	$\mathbf{0.326^{+0.045}_{-0.065}}$	$\mathbf{-0.13^{+0.40}_{-0.50}}$	**1.51**	**6.50**
CC+SN+HW	71.0 ± 1.3	$0.329^{0.045}_{0.062}$	$-0.16^{+0.41}_{-0.48}$	1.51	6.50
CC+SN+TRGB	$69.1^{+1.4}_{-1.3}$	$0.344^{+0.045}_{-0.063}$	$-0.20^{+0.42}_{-0.47}$	1.87	6.85
CC+SN+BAO	$\mathbf{67.1 \pm 1.6}$	$\mathbf{0.294 \pm 0.015}$	$\mathbf{0.06 \pm 0.13}$	**1.68**	**6.68**
CC+SN+BAO+R19	69.9 ± 1.2	$0.305^{+0.014}_{-0.013}$	$-0.14^{+0.12}_{-0.13}$	0.56	5.56
CC+SN+BAO+HW	69.7 ± 1.2	$0.304^{+0.014}_{-0.012}$	$-0.12^{+0.12}_{-0.13}$	0.89	5.89
CC+SN+BAO+TRGB	68.1 ± 1.2	0.298 ± 0.014	$-0.01^{+0.11}_{-0.12}$	2.00	7.00

$$\mathrm{AIC} = 2k - 2\ln L$$

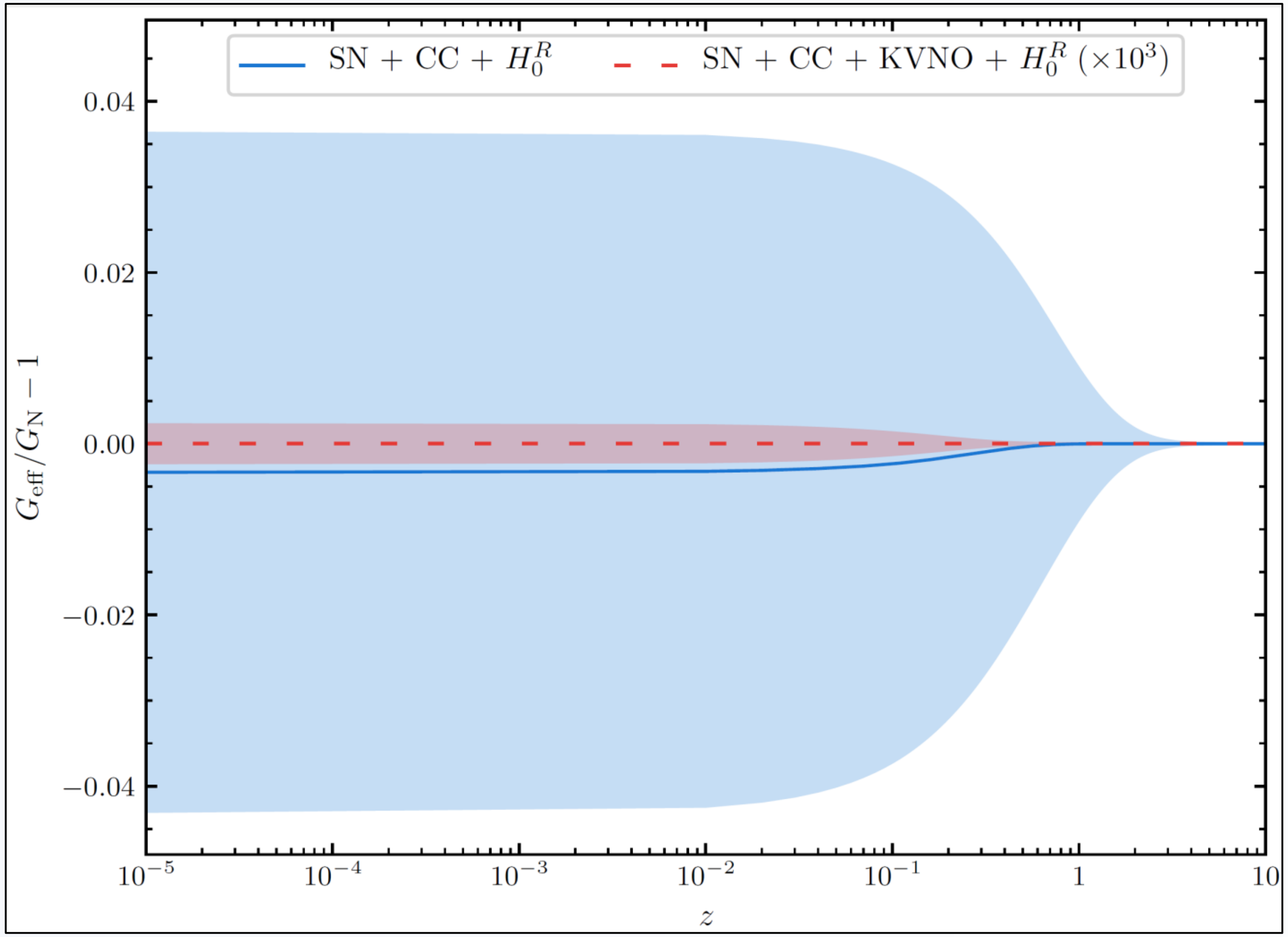
Number of Model parameters

Maximum likelihood

$$\mathrm{BIC} = k\ln n - 2\ln L$$

Number of points in a data set

$f_2(T)$ Model

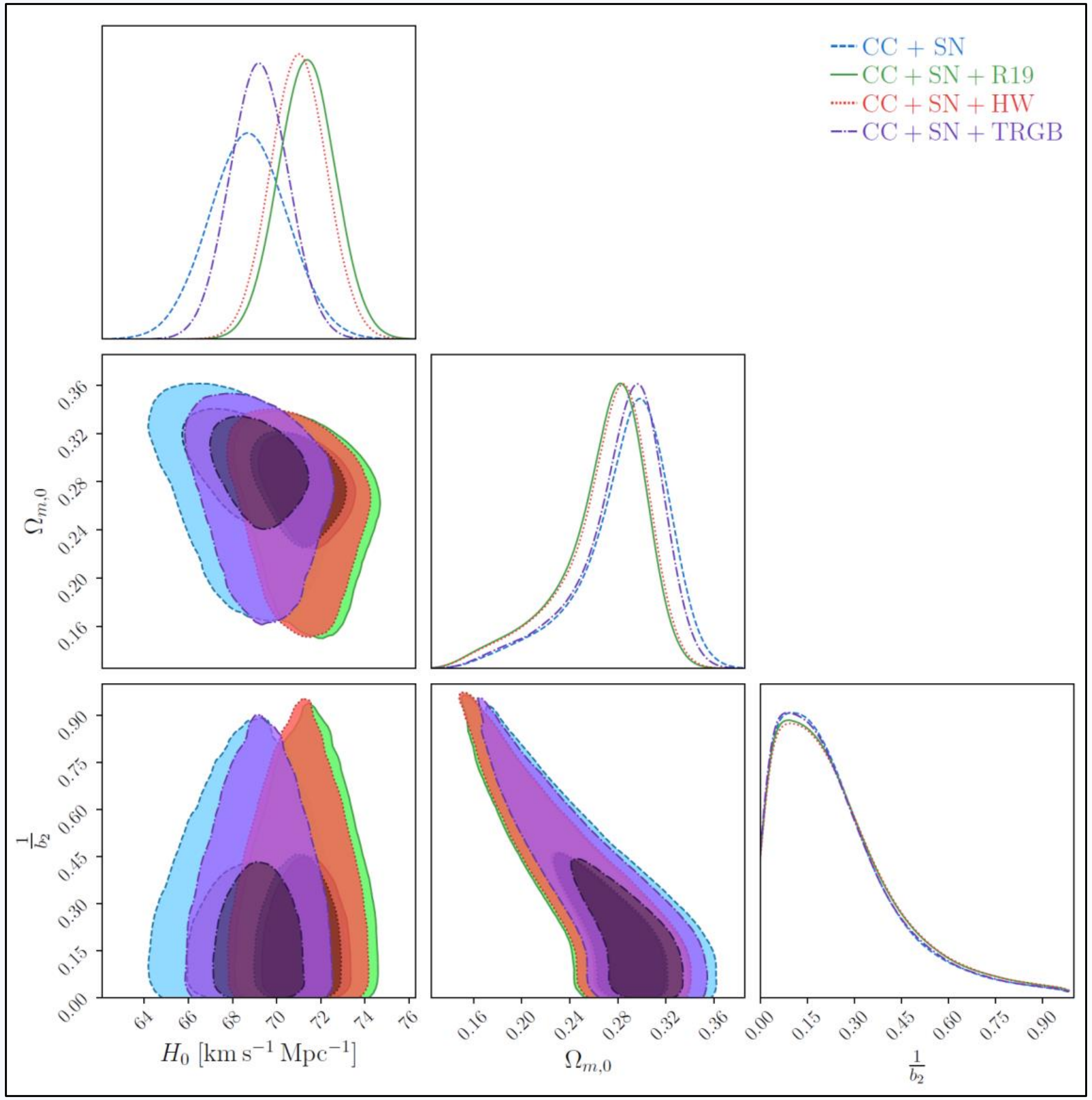


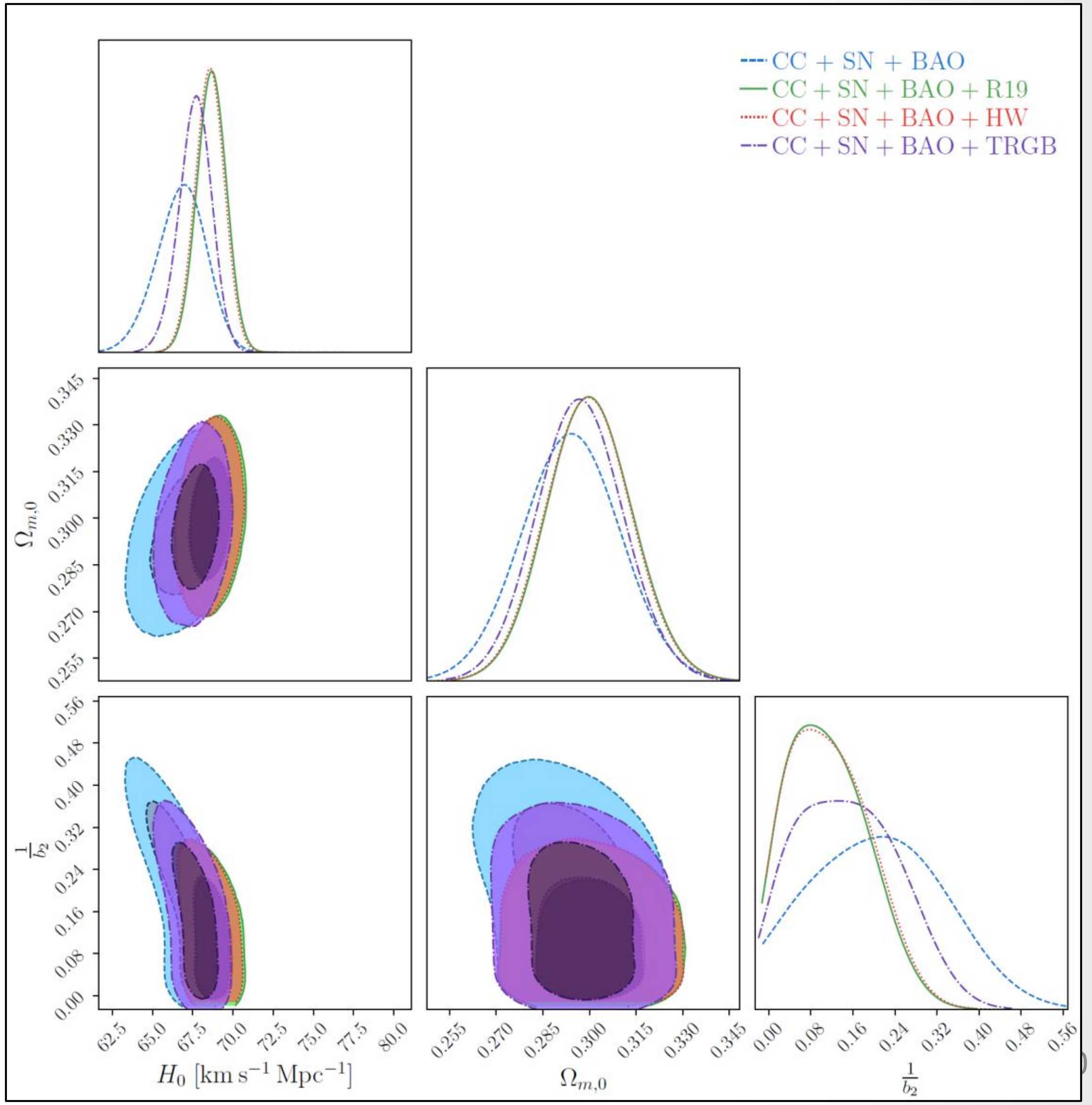
<u>Model:</u> $f_2 = \alpha_2 T_0 \left(1 - \text{Exp}\left[-b_2\sqrt{T/T_0}\right]\right)$

where $\alpha_2 = -\frac{1-\Omega_0^{\rm m}}{1-(1+b_2)e^{-b_2}}$

$$G_{eff}(z \simeq 0) \simeq G_N$$

Precision Cosmology Constraints for f_2CDM





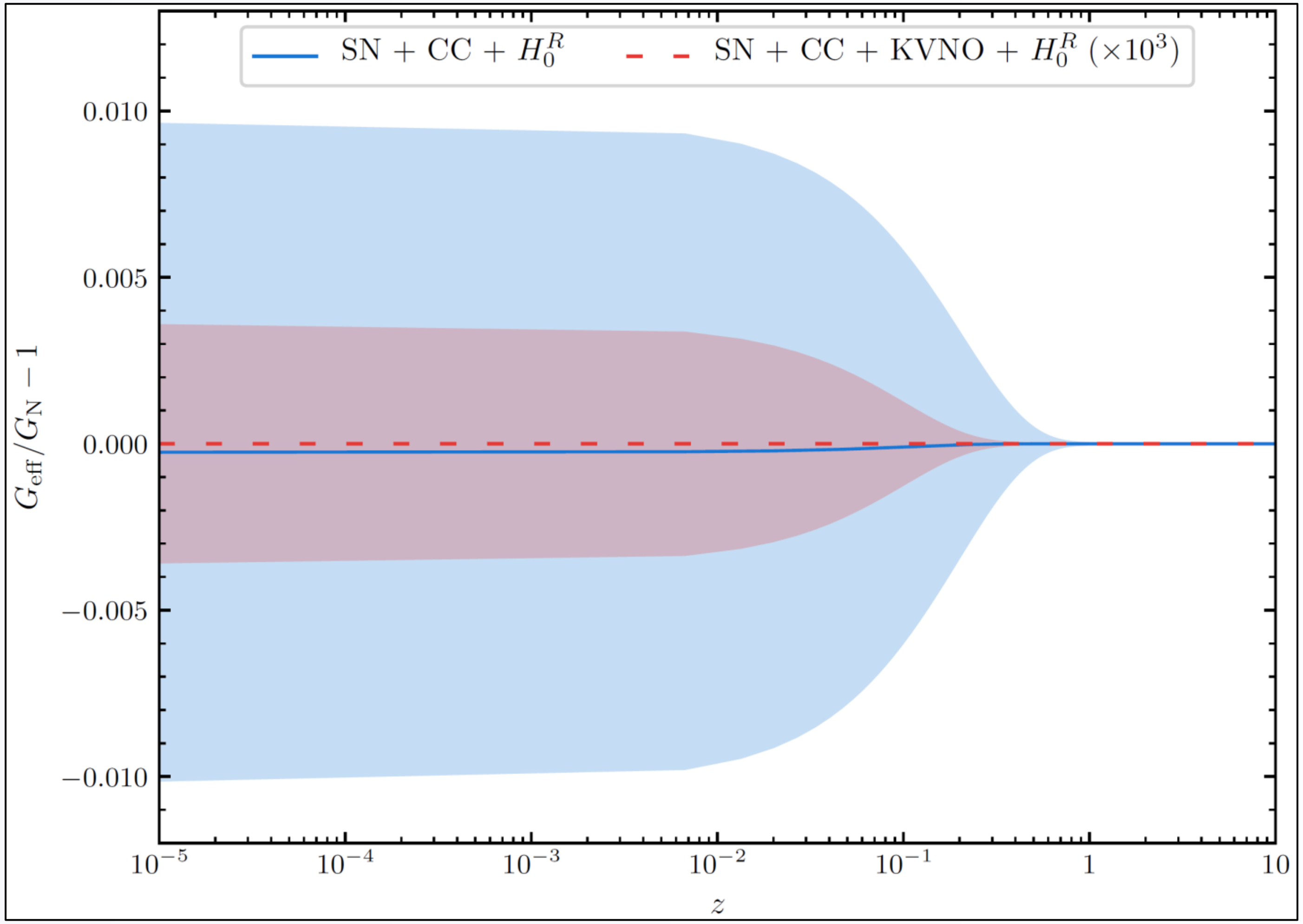
Results for f_2CDM

Data Sets	H_0 $[\mathrm{km\ s^{-1}\ Mpc^{-1}}]$	$\Omega_{\mathrm{m},0}$	$1/b_2$	ΔAIC	ΔBIC
CC+SN	$68.7^{+1.8}_{-1.7}$	$0.298^{+0.031}_{-0.035}$	$0.101^{+0.227}_{-0.098}$	2.00	6.98
CC+SN+R19	$\mathbf{71.4 \pm 1.3}$	$\mathbf{0.283^{+0.027}_{-0.036}}$	$\mathbf{0.088^{+0.252}_{-0.086}}$	**2.00**	**6.99**
CC+SN+HW	$71.0^{+1.3}_{-1.2}$	$0.285^{+0.027}_{-0.036}$	$0.096^{+0.245}_{-0.093}$	2.00	6.99
CC+SN+TRGB	$71.0^{+1.3}_{-1.2}$	$0.296^{+0.028}_{-0.085}$	$0.088^{+0.239}_{-0.085}$	2.00	6.99
CC+SN+BAO	$\mathbf{66.90^{+1.5}_{-1.6}}$	$\mathbf{0.294 \pm 0.016}$	$\mathbf{0.22^{+0.12}_{-0.15}}$	**1.06**	**6.06**
CC+SN+BAO+R19	$68.71^{+0.88}_{-0.96}$	0.300 ± 0.014	$-0.079^{+0.098}_{-0.064}$	2.00	7.00
CC+SN+BAO+HW	$68.58^{+0.89}_{-0.92}$	$0.300^{+0.013}_{-0.014}$	$0.076^{+0.105}_{-0.060}$	2.00	7.00
CC+SN+BAO+TRGB	67.70 ± 1.00	0.297 ± 0.014	$0.128^{+0.111}_{-0.099}$	1.90	6.90

$$\mathrm{AIC} = 2k - 2\ln L$$

$$\mathrm{BIC} = k\ln n - 2\ln L$$

$f_3(T)$ Model

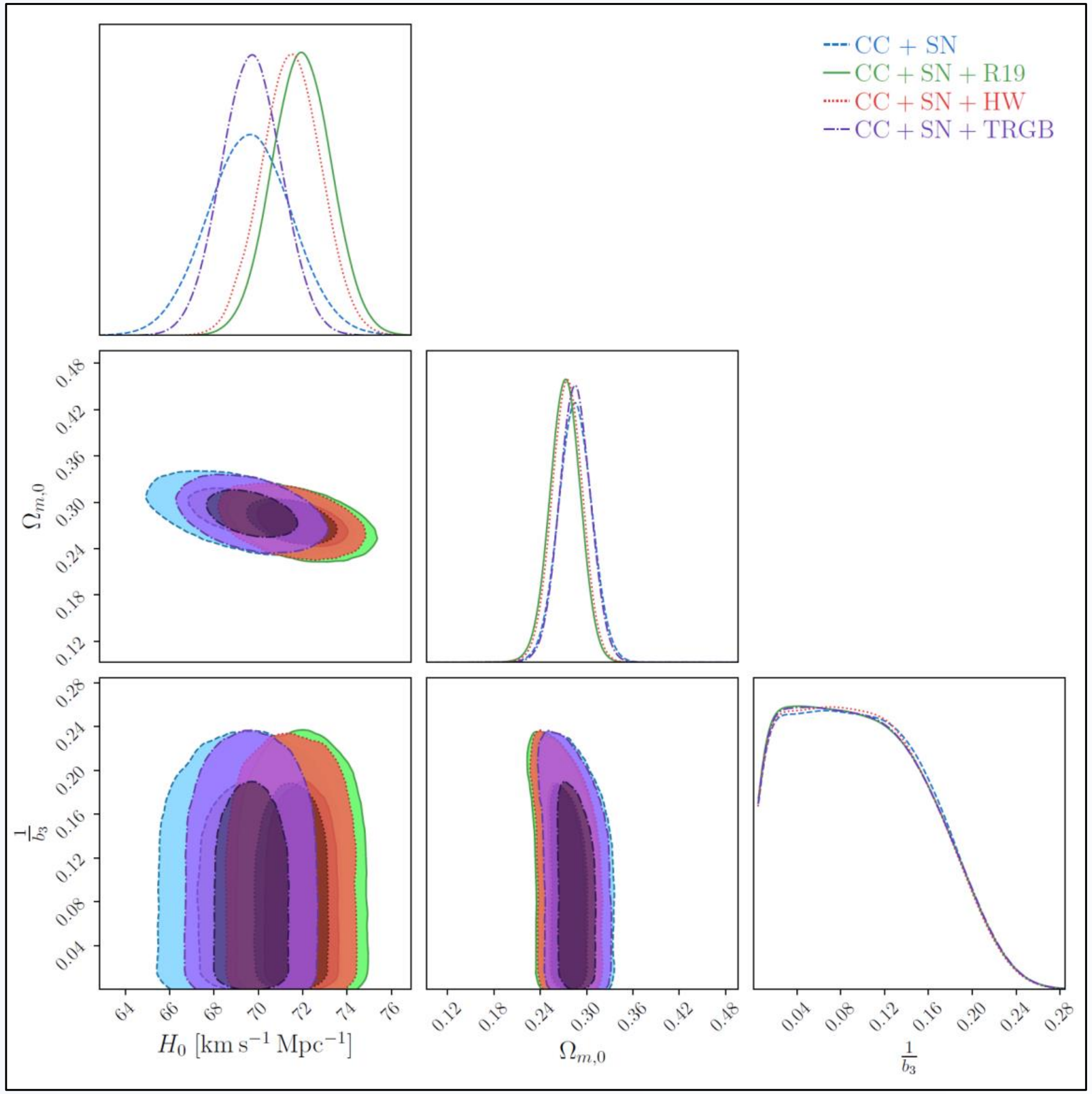


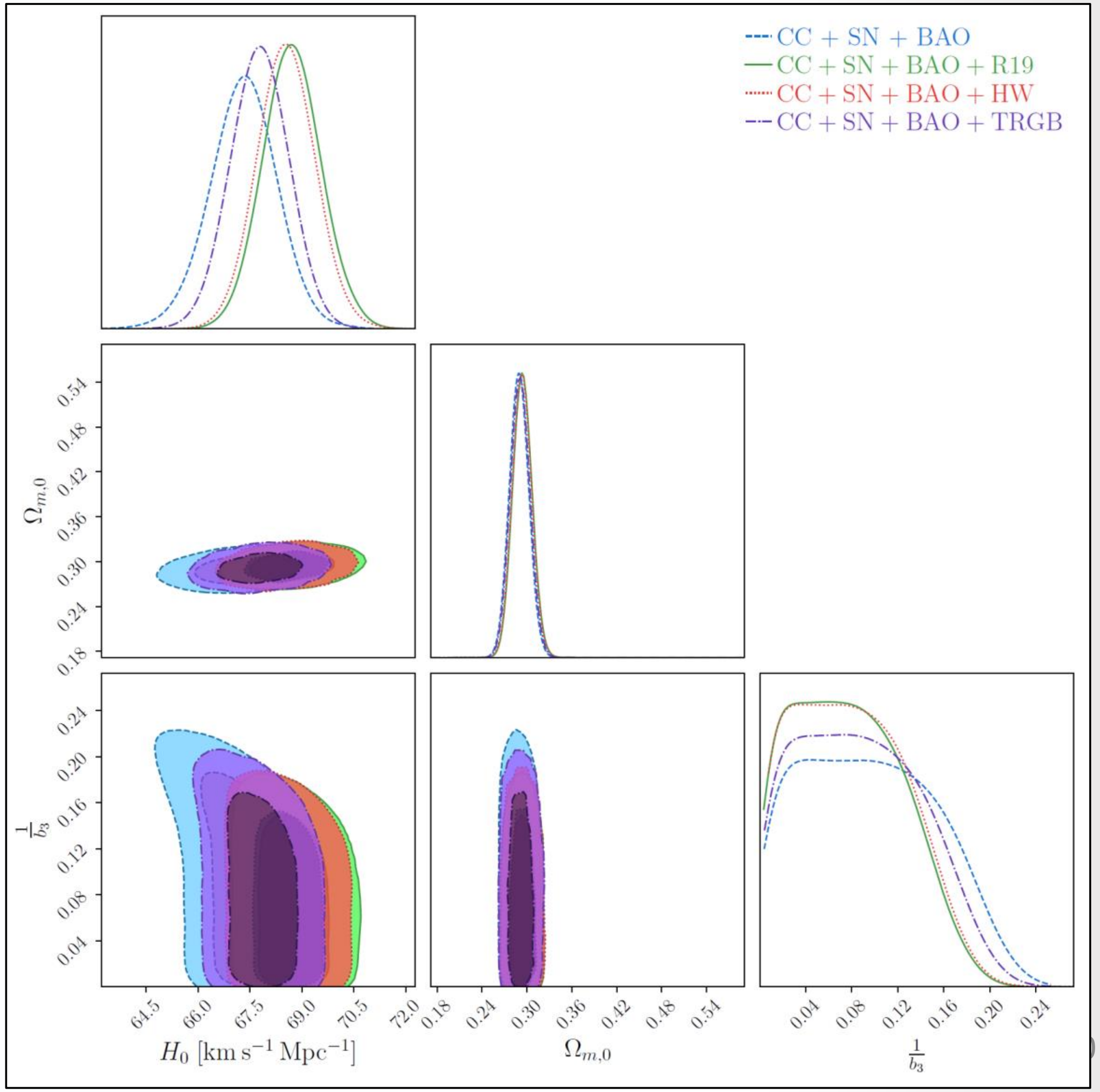
<u>Model:</u> $f_3 = \alpha_3 T_0 (1 - \mathrm{Exp}[-b_3 T/T_0]\,)$

where $\alpha_3 = \dfrac{1-\Omega_0^{\rm m}}{-1+(1+2b_3)e^{-b_3}}$

$$G_{eff}(z \simeq 0) \simeq G_N$$

Precision Cosmology Constraints for f_3CDM





Results for f_3CDM

Data Sets	H_0 $\left[\mathrm{km\ s^{-1}\ Mpc^{-1}}\right]$	$\Omega_{\mathrm{m},0}$	$1/b_3$	ΔAIC	ΔBIC
CC+SN	$68.7^{+1.8}_{-1.7}$	$0.298^{+0.031}_{-0.035}$	$0.101^{+0.227}_{-0.098}$	2.00	6.98
CC+SN+R19	$\mathbf{71.4 \pm 1.3}$	$\mathbf{0.283^{+0.027}_{-0.036}}$	$\mathbf{0.088^{+0.252}_{-0.086}}$	$\mathbf{2.00}$	$\mathbf{6.99}$
CC+SN+HW	$71.0^{+1.3}_{-1.2}$	$0.285^{+0.027}_{-0.036}$	$0.096^{+0.245}_{-0.093}$	2.00	6.99
CC+SN+TRGB	$71.0^{+1.3}_{-1.2}$	$0.296^{+0.028}_{-0.085}$	$0.088^{+0.239}_{-0.085}$	2.00	6.99
CC+SN+BAO	$\mathbf{66.9^{+1.5}_{-1.6}}$	$\mathbf{0.294 \pm 0.016}$	$\mathbf{0.22^{+0.12}_{-0.15}}$	$\mathbf{1.06}$	$\mathbf{6.06}$
CC+SN+BAO+R19	$68.71^{+0.88}_{-0.96}$	0.3000 ± 0.014	$-0.079^{+0.098}_{-0.064}$	2.00	7.00
CC+SN+BAO+HW	$68.58^{+0.89}_{-0.92}$	$0.300^{+0.013}_{-0.014}$	$0.076^{+0.105}_{-0.060}$	2.00	7.00
CC+SN+BAO+TRGB	67.7 ± 1.0	0.297 ± 0.014	$0.128^{+0.111}_{-0.099}$	1.90	6.90

$$\mathrm{AIC} = 2k - 2\ln L$$

$$\mathrm{BIC} = k\ln n - 2\ln L$$

Can we do this in a model-independent way?

Gaussian Processes Regression

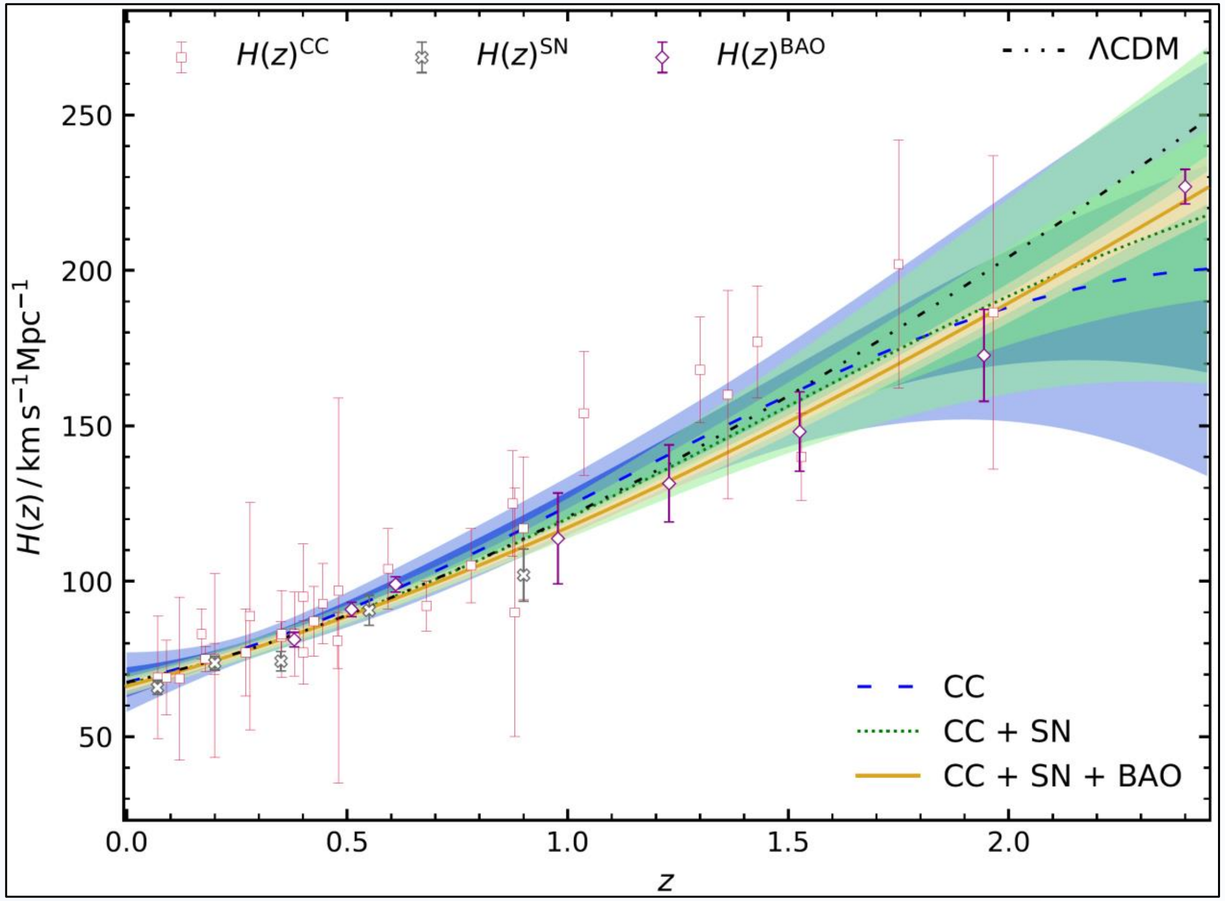
- The covariance function contains **non-physical hyperparameters** θ which define the distribution $k(\theta, x, x')$
- Iterating over these values using **Bayesian inference** (or others) can produce better hyperparameters
- The result is a (physics) **model independent reconstruction** of the behavior of some parameter
- This is superior to regular fitting because it is nonparametric and so **assumes no physical model** whatsoever

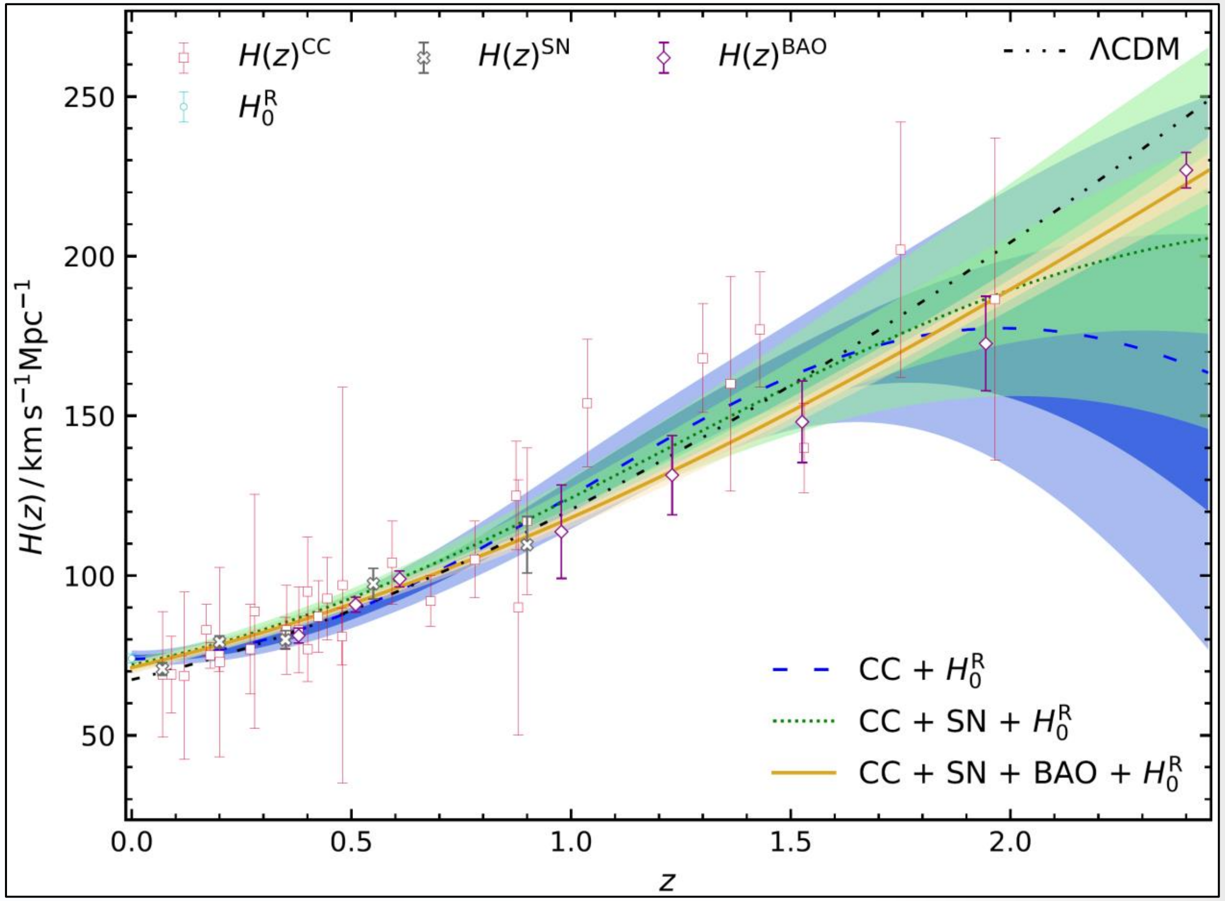
The Covariance Functions

Squared Exponential (Gaussian):

$$k(x, x') = \sigma_f^2 \, \mathrm{Exp}\left[-\frac{1}{2}\left(\frac{x - x'}{l_f}\right)^2\right]$$

Square Exponential H_0 GP





$H_0 = 67.539 \pm 4.772 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 67.001 \pm 1.653 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 66.197 \pm 1.464 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 73.782 \pm 1.374 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 72.022 \pm 1.076 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 71.180 \pm 1.025 \mathrm{km\ s^{-1} Mpc^{-1}}$

Square Exponential Covariance for H_0

Distance (in σ units) between the H_0 arguments:

$$d(H_{0,i}, H_{0,j}) = \frac{H_{0,i} - H_{0,j}}{\sqrt{\sigma_i^2 + \sigma_j^2}}$$

Data set(s)	$H_0 [\mathrm{km\ s^{-1}\ Mpc^{-1}}]$	$d(H_0, H_0^{\mathrm{R19}})$	$d(H_0, H_0^{\mathrm{TRGB}})$	$d(H_0, H_0^{\mathrm{HW}})$
CC	67.539 ± 4.772	-1.304	-0.441	-1.133
CC+$\boldsymbol{H_0^{\mathrm{R}}}$	$\mathbf{73.782 \pm 1.374}$	**-0.126**	**1.711**	**0.217**
CC+SN	67.001 ± 1.653	-3.225	-1.118	-2.617
CC+SN+H_0^{R}	72.022 ± 1.076	-1.128	1.026	-0.622
CC+SN+BAO	$\mathbf{66.197 \pm 1.464}$	**-3.841**	**-1.513**	**-3.113**
CC+SN+BAO+H_0^{R}	71.18 ± 1.025	-1.628	0.645	-1.046

Boundary Conditions

ΛCDM (or $f(T) = \Lambda$) at works at late cosmological times

This implies that

$$f_T(z \simeq 0) \simeq 0$$

$$\Rightarrow f(z \simeq 0) = 6H_0^2\left(\Omega_{m_0} - 1\right)$$

Briffa et al. CQG 38 055007 (2020)

$$S = \frac{1}{16\pi G}\int d^4x\, e[-T + \boldsymbol{f(T)}] + S_{\text{matter}}$$

Propagating $f(T(z))$

- The Friedmann equation contains f_T which **need to be eliminated finite difference** methods

- Using a **central differencing** approach (error $\sim \mathcal{O}(\Delta z^2)$), we can assume

$$f'(z_i) \simeq \frac{f(z_{i+1}) - f(z_{i-1})}{z_{i+1} - z_{i-1}}$$

- Therefore, we can remove the $f_T(T) = f'(z)/T'(z)$

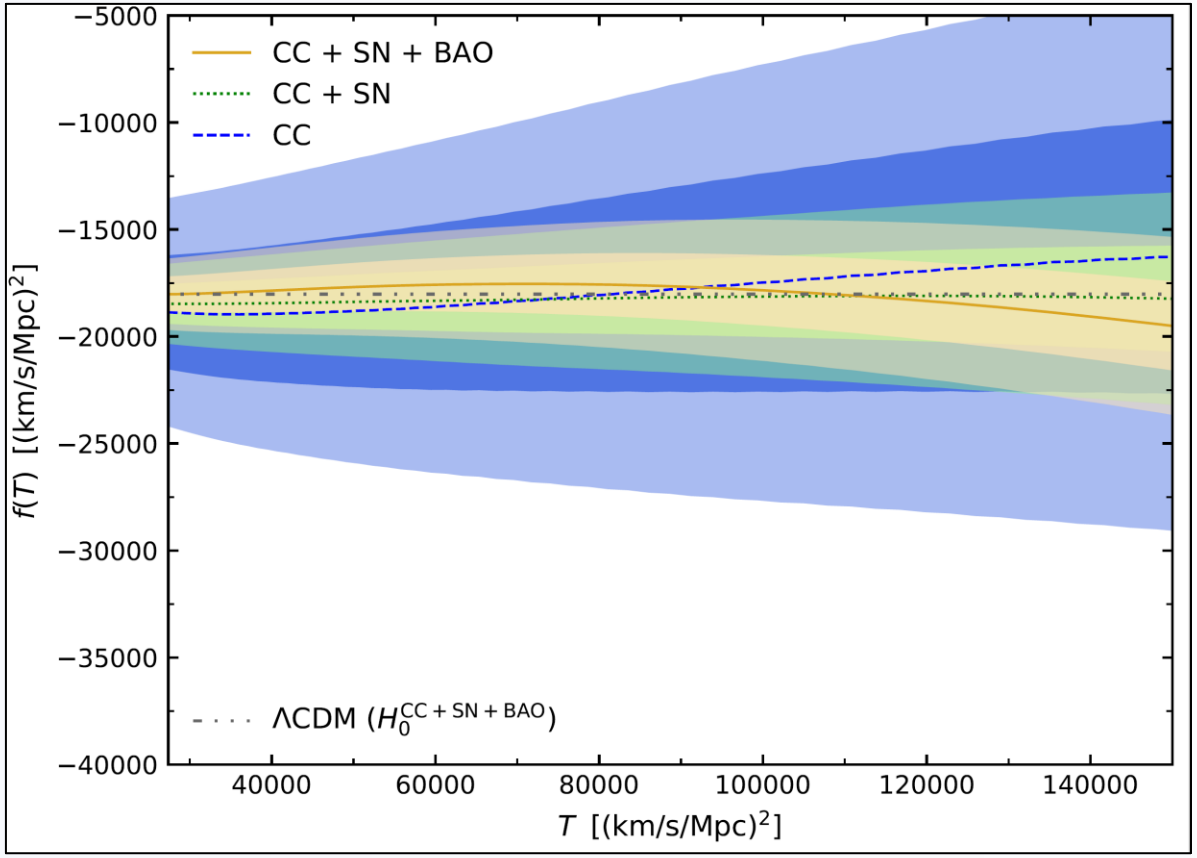
$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} + \frac{T}{3}\boldsymbol{f_T}$$

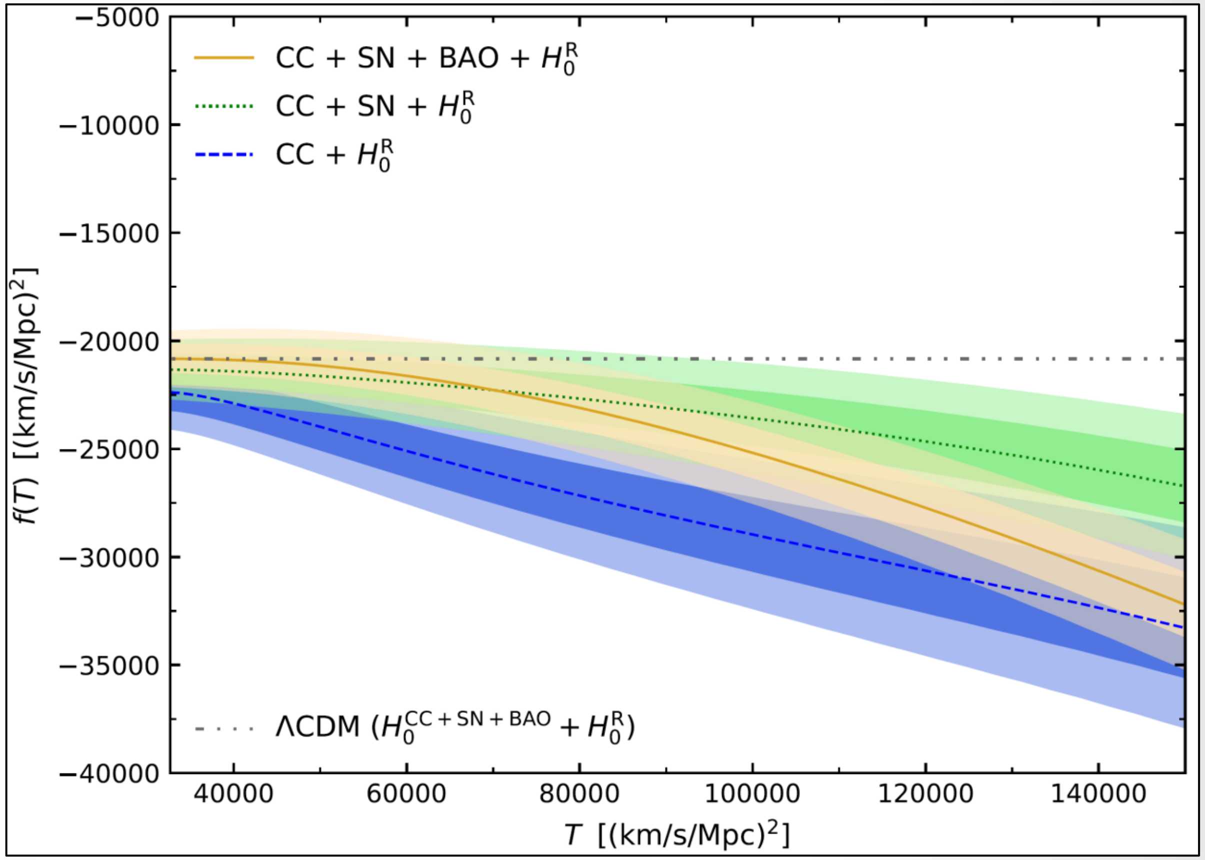
- This then gives a **propagation equation**

$$f(z_{i+1}) = f(z_{i-1}) + 2(z_{i+1} - z_{i-1})\frac{H'(z_i)}{H(z_i)}\left(3H(z_i)^2 + \frac{f(z_i)}{2} - 3H_0^2\Omega_{m_0}(1+z_i)^3\right)$$

- Using **forward differencing**, we can produce a second boundary condition

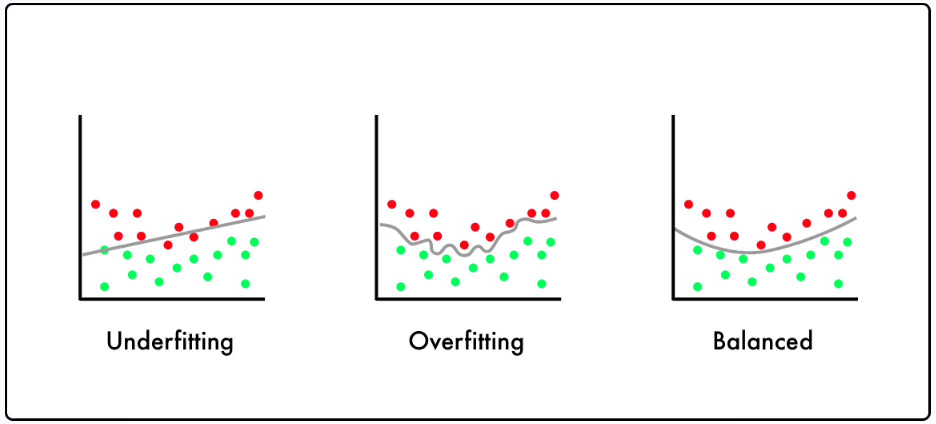
Square Exponential $f(T)$ GP



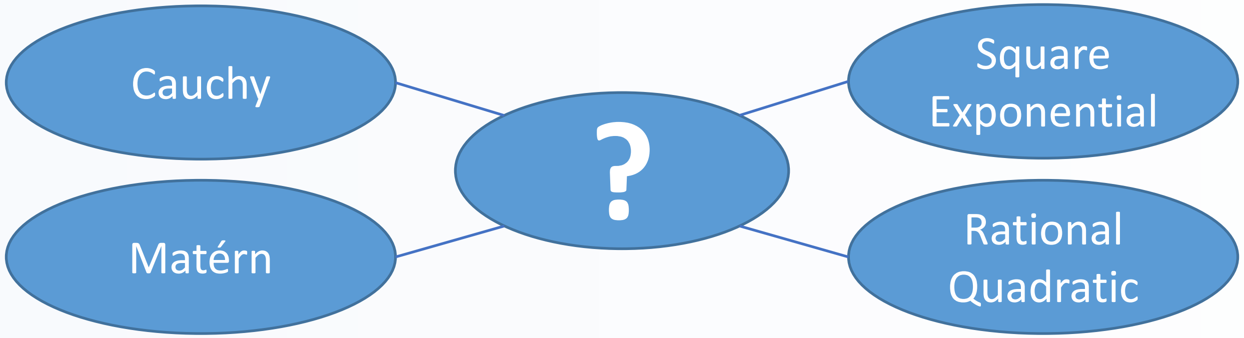


Open Problems with GP Reconstructions

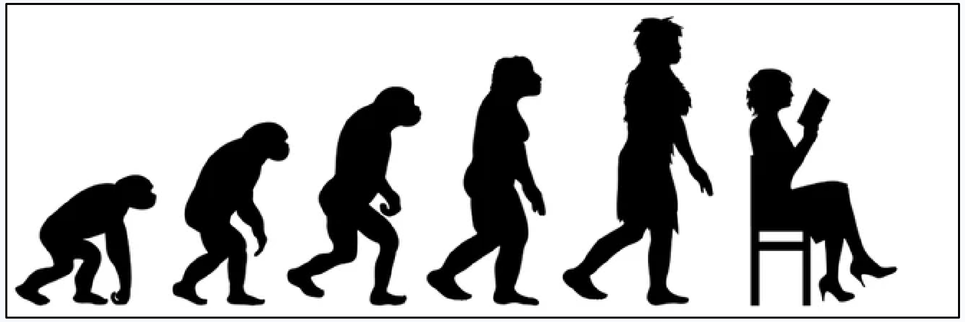
- **<u>Overfitting at origin</u>:** GP is very prone to overfitting for small data sets, which is especially pronounced at the origin, i.e. Hubble constant

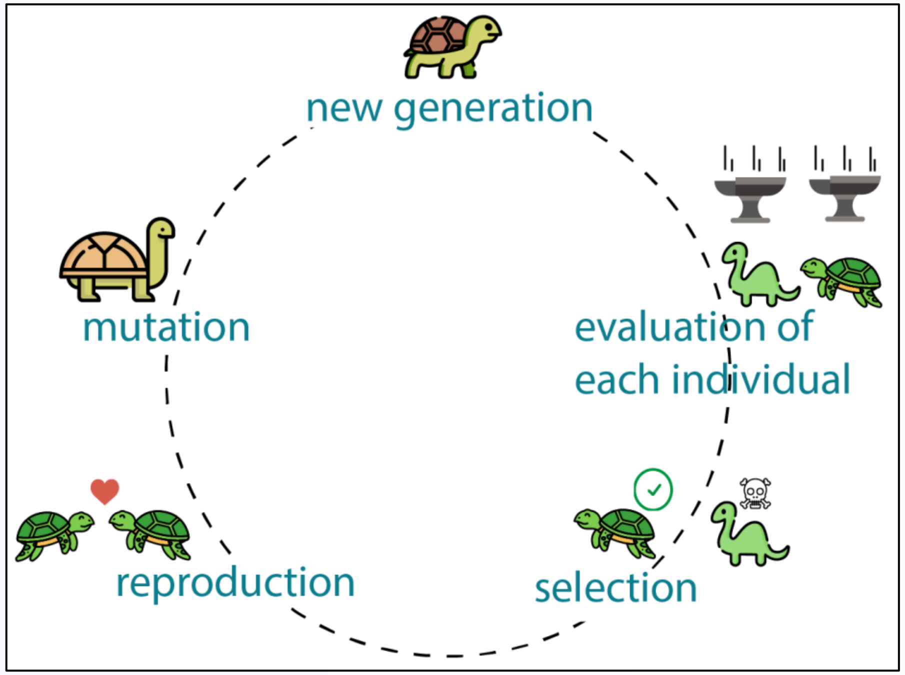


- **<u>Kernel Selection Problem</u>:** There is no natural kernel for cosmology



Genetic Algorithms (GAs)



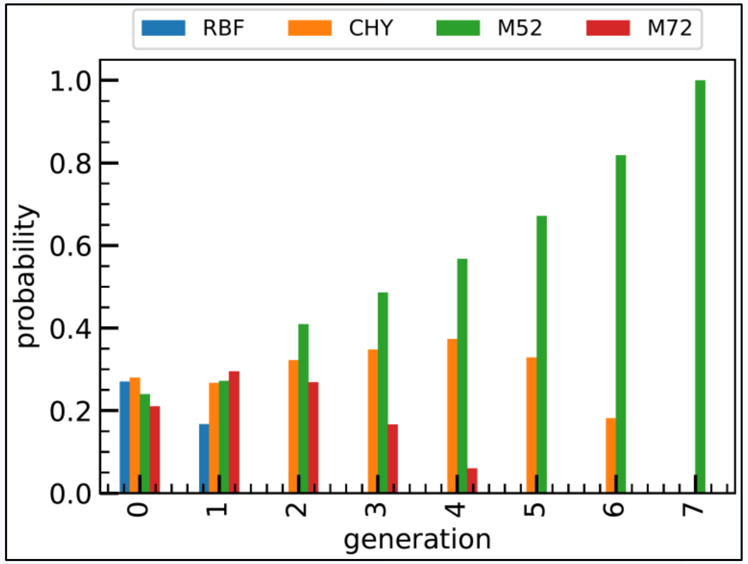


Fitness function: Score to characterize the performance of each generation (BIC inspired)

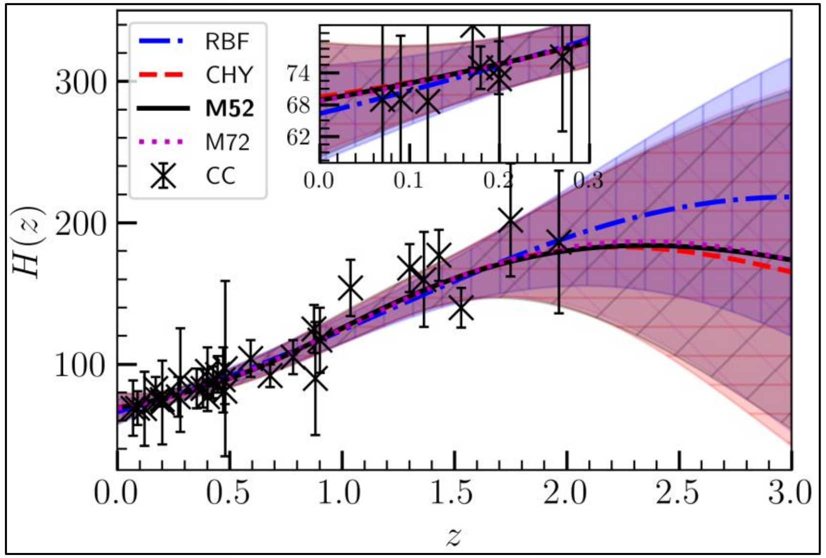
$$\mathcal{F} = \ln \mathcal{L} - \frac{k_{\text{eff}} \ln N}{2}$$

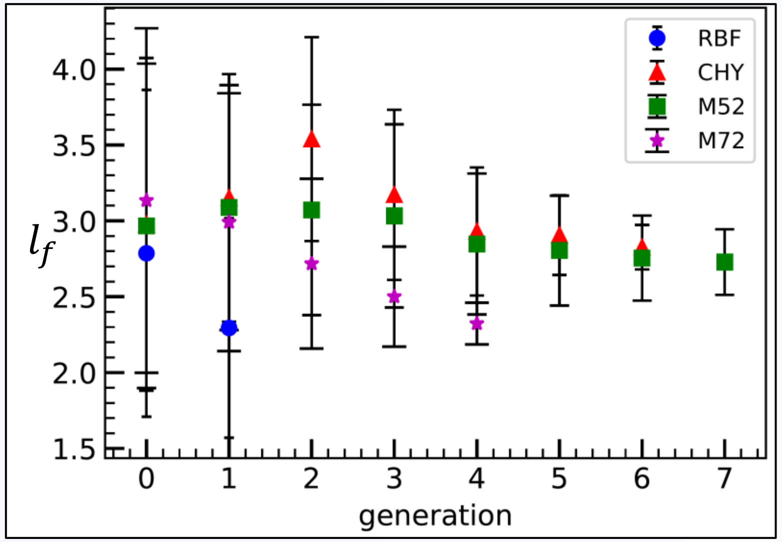
Selection: Population that will survive
Crossover: Inheritance of kernels
Mutation: Changes in addition to crossover

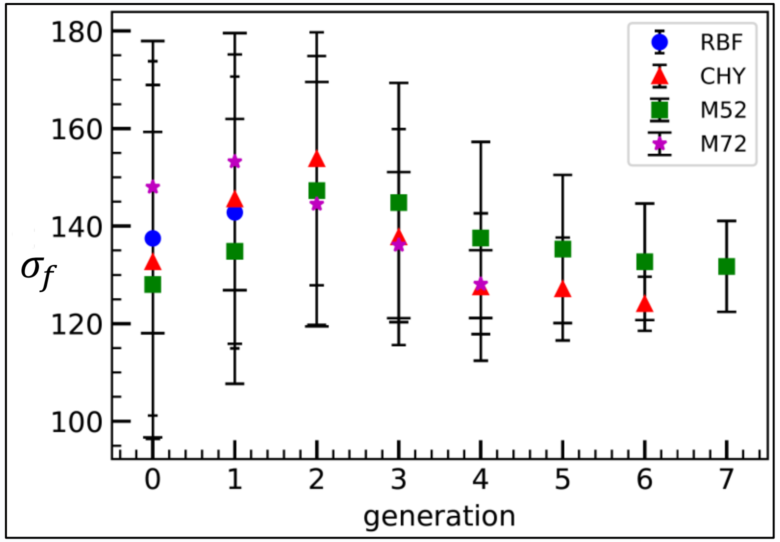
Genetic Algorithms (GAs)



CC dataset



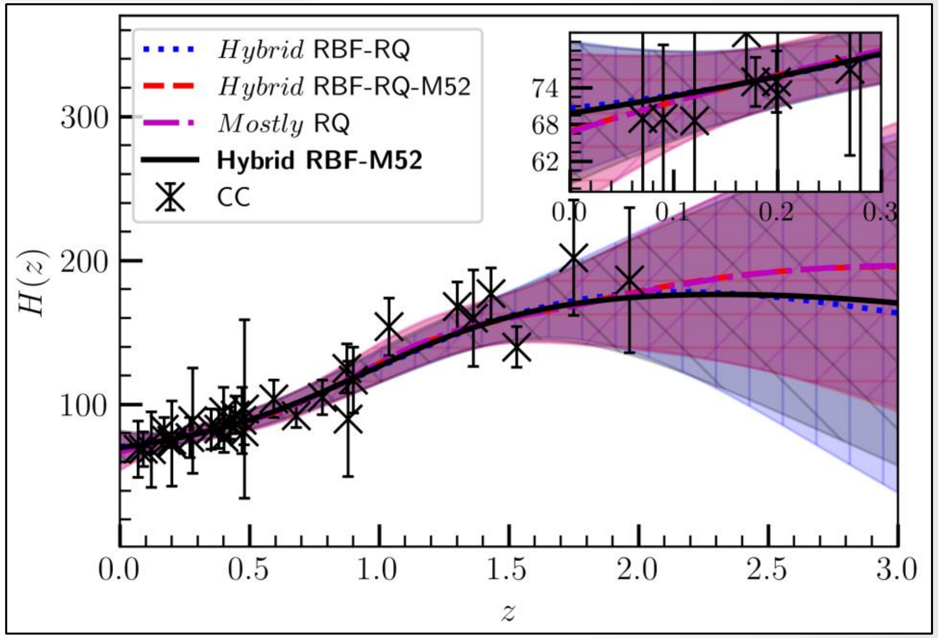




Trials for GAs

Trial	Population size	Selection rate	Mutation rate	No. of generations	Best fitness
1	10^4	0.5	0.15	10^1	-143.5
2	10^4	0.3	0.30	10^1	-148.5
3	10^3	0.1	0.10	10^2	-143.4
4	$\mathbf{10^3}$	**0.3**	**0.50**	$\mathbf{10^2}$	$\mathbf{-141.8}$

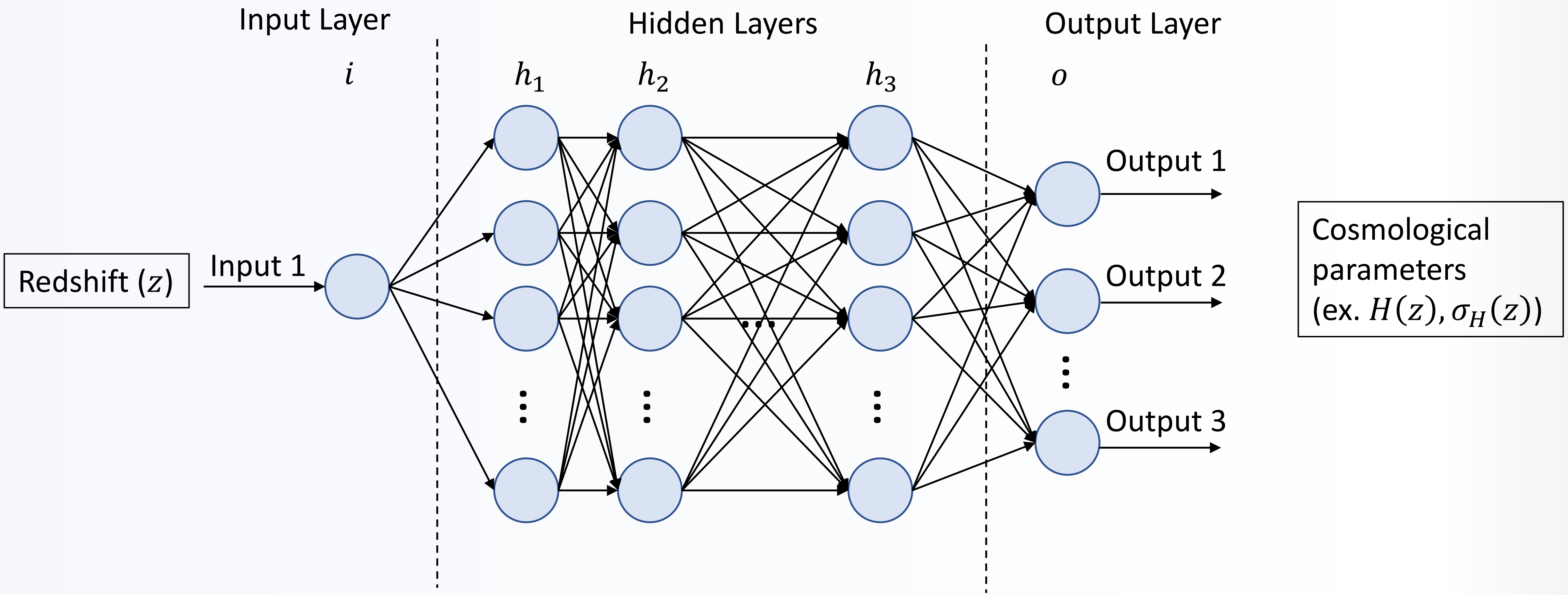
Kernel	H_0	$\ln \mathcal{L}$	χ	fitness	Penalty
Hybrid RBF-RQ	70.6 ± 5.5	-131.49	13.1	-143.5	12.0
Hybrid RBF-RQ-M52	66.9 ± 6.3	-131.38	12.0	-148.5	17.2
Mostly RQ	66.7 ± 6.4	-131.36	11.7	-143.4	12.0
***Hybrid* RBF-M52**	$\mathbf{69.8 \pm 5.8}$	$\mathbf{-131.48}$	**12.7**	$\mathbf{-141.8}$	**10.3**



$$\text{Penalty} = \frac{k_{\text{eff}} \ln N}{2}$$

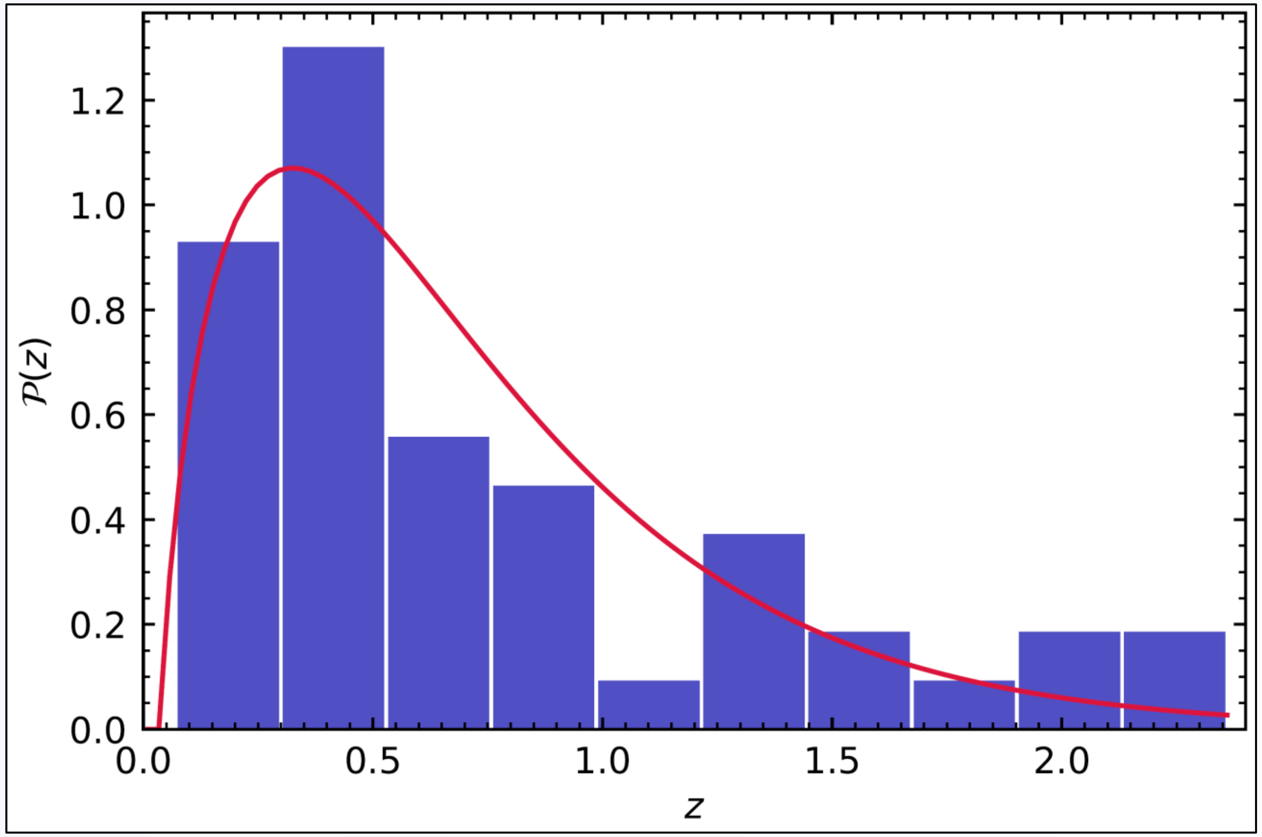
Bernardo et al. JCAP 08, 027 (2021)

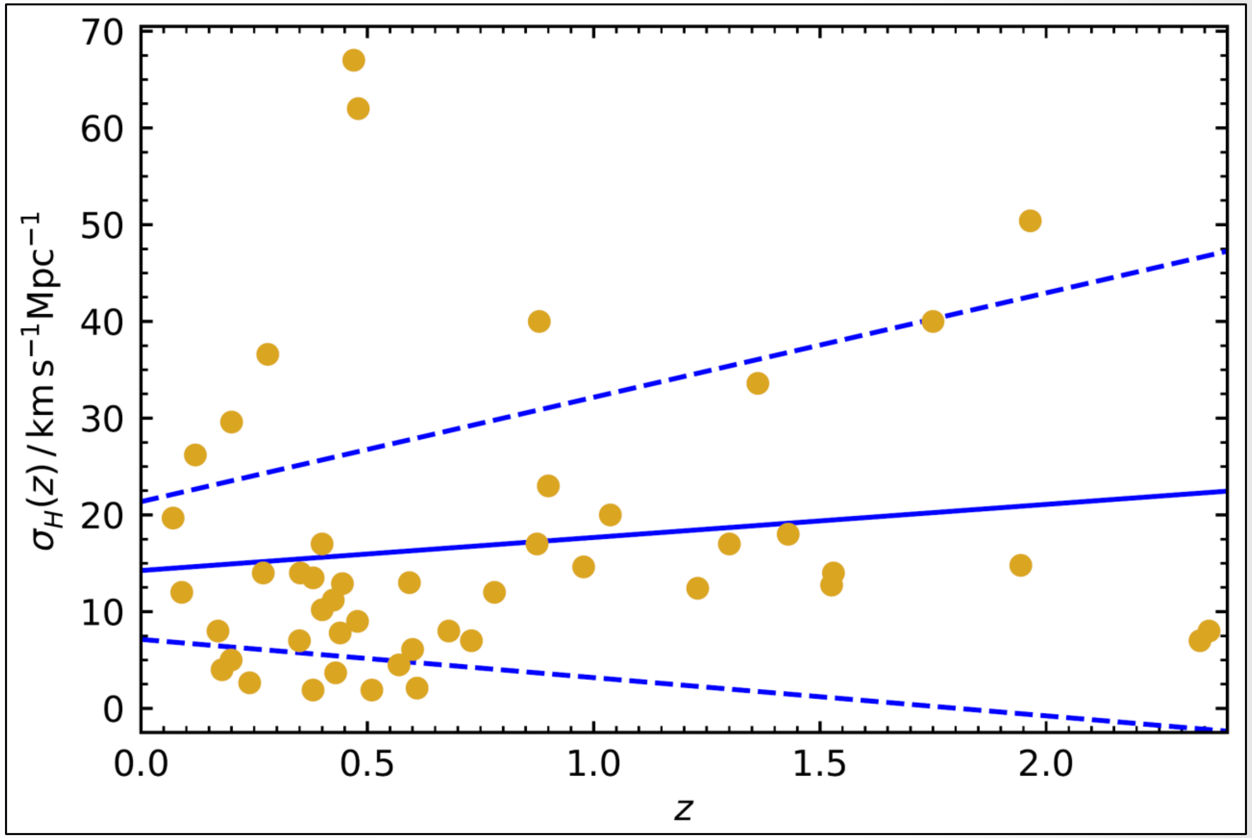
Artificial Neural Networks (ANNs)



Training Data for the ANN

CC+BAO dataset





This observes the gamma distribution:

$$\mathcal{P}(z,\alpha,\lambda) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} z^{\alpha-1} e^{-\lambda z}$$

Mean: $\sigma_H = 14.25 + 3.42z$

Upper error: $\sigma_H = 21.37 + 10.79z$

Lower error: $\sigma_H = 7.14 - 3.95z$

Designing the ANN

- **Risk** – Optimizes the **number of hidden layers and neurons** in an ANN

$$\text{risk} = \sum_{i=1}^{N} (\text{Bias}_i^2 + \text{Variance}_\text{i}) = \sum_{i=1}^{N} \left(\left[H_{Obs}(z_i) - H_{pred}(z_i) \right]^2 + \sigma_H^2(z_i) \right)$$

- **Loss** – Balances the **number of iterations** a system needs to predict the observational data

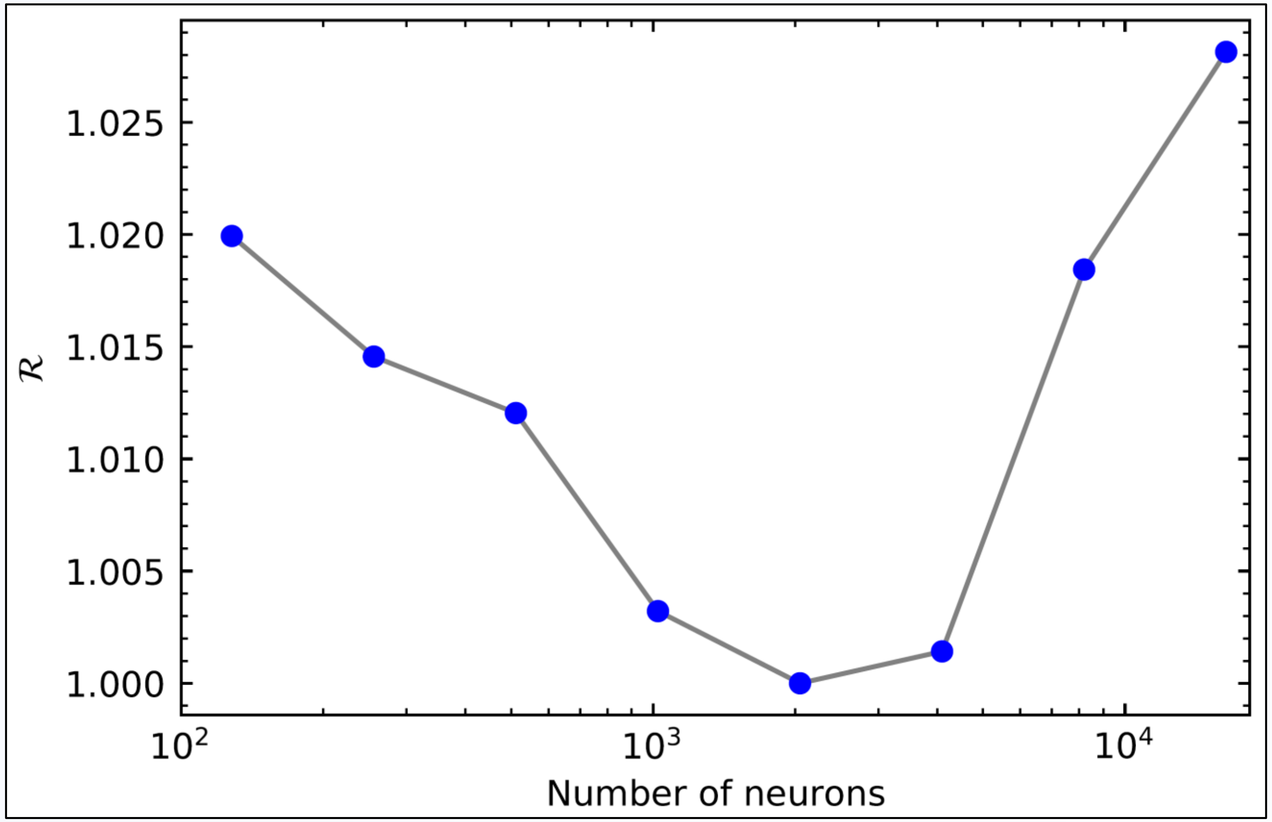
  1. **L1** (Least absolute deviation)

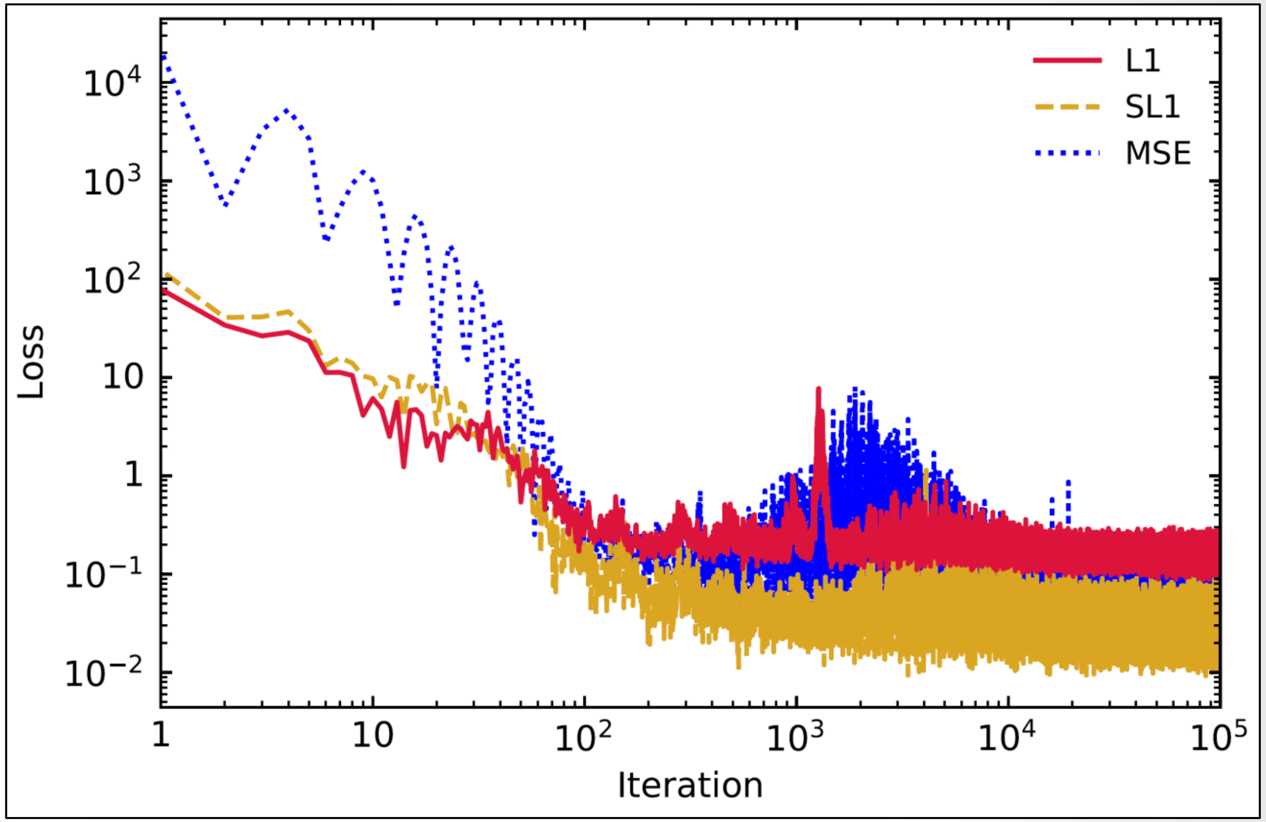
$$\text{L1} = \sum_{i=1}^{N} \left| H_{Obs}(z_i) - H_{pred}(z_i) \right|$$

  2. Smoothed L1 (**SL1**)
  3. Mean Square Error (**MSE**)

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} \left(H_{Obs}(z_i) - H_{pred}(z_i) \right)^2$$

Building the ANN

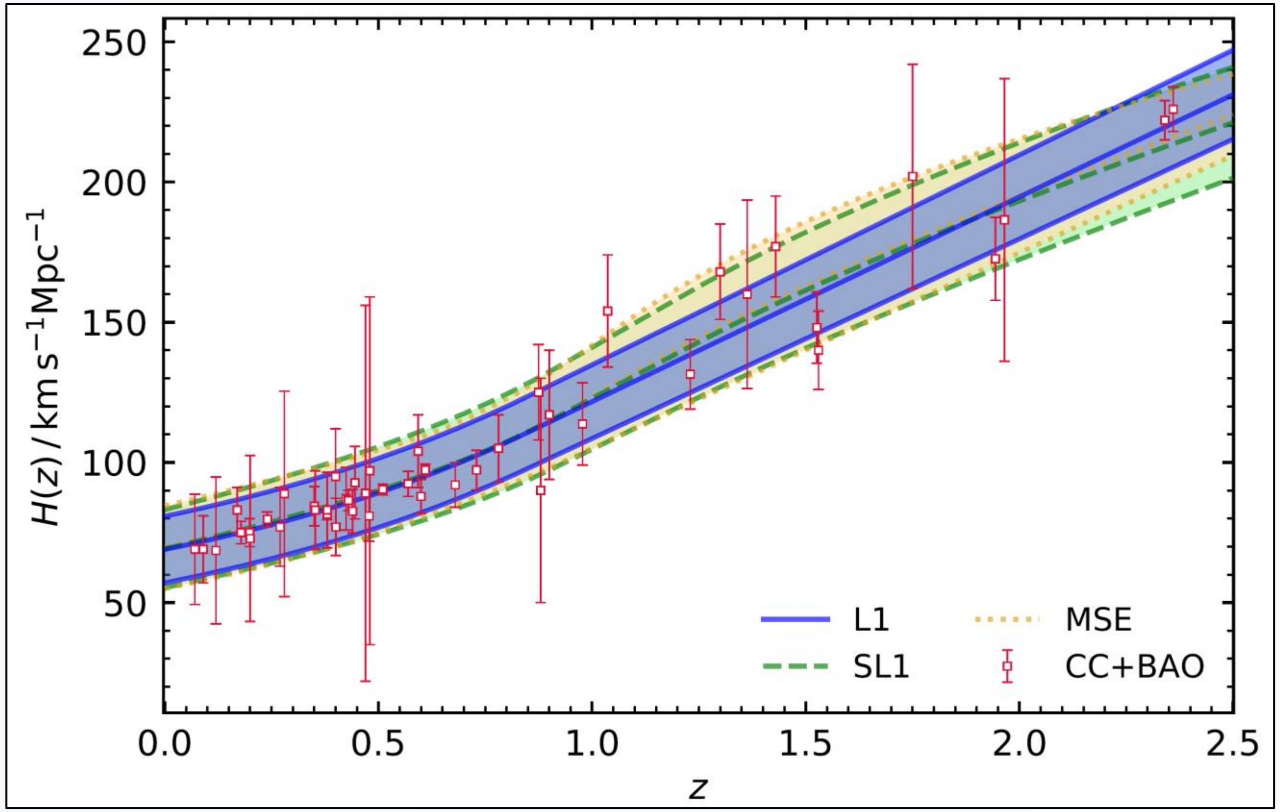


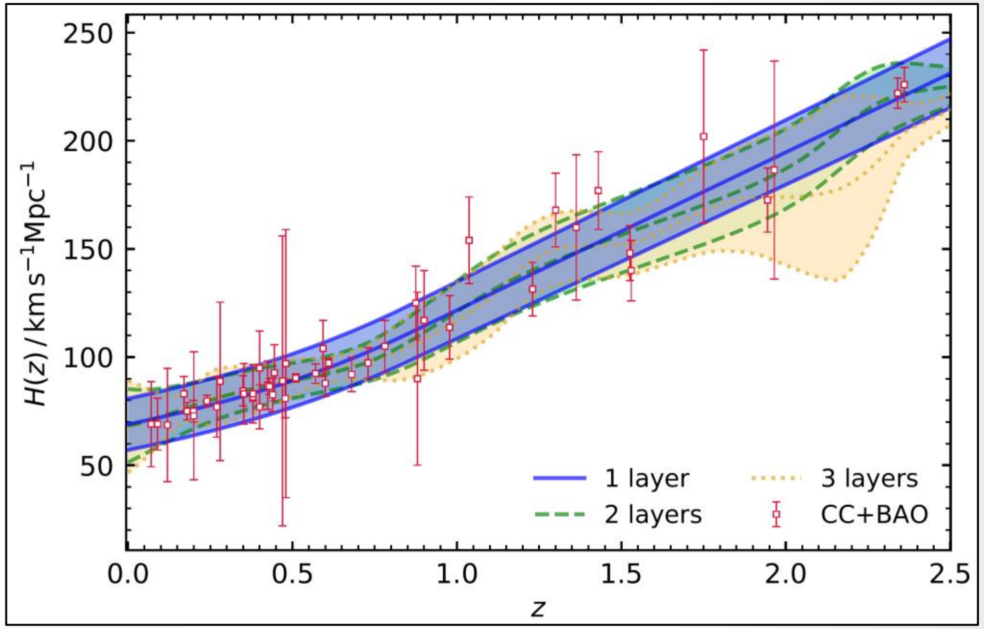


Risk function for **one layer** ($number\ of\ neurons = 2^n$
$n \in \{7, ..14\}$)

Dialektopoulos et al. JCAP 02, 023 (2022)

Using the ANN





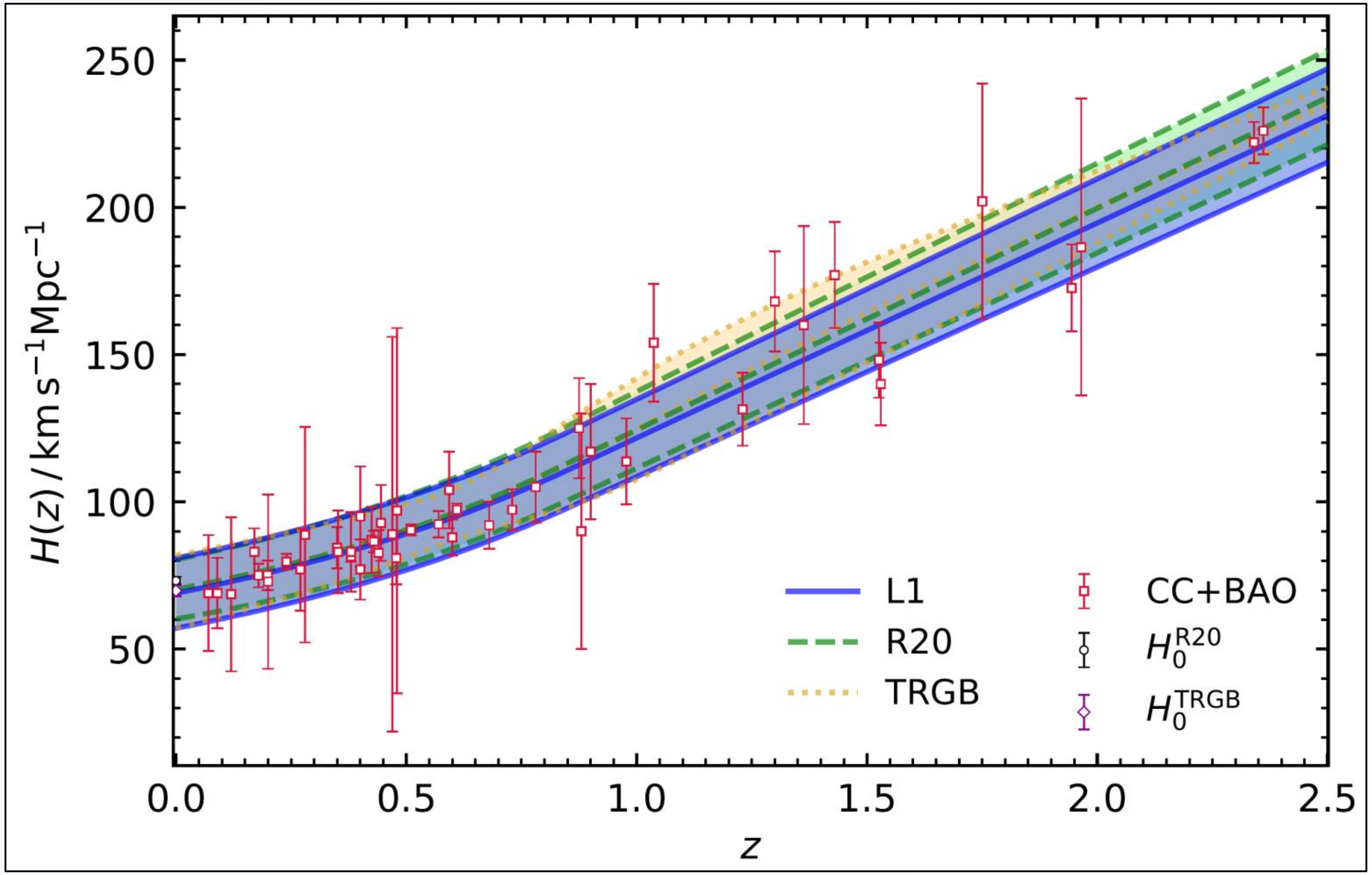
One layer is preferred

MSE: $H_0 = 69.76 \pm 14.82 \text{ km s}^{-1}\text{Mpc}^{-1}$

L1: $H_0 = 68.93 \pm 11.90 \text{ km s}^{-1}\text{Mpc}^{-1}$

SL1: $H_0 = 69.18 \pm 13.92 \text{ km s}^{-1}\text{Mpc}^{-1}$

What about priors?



Priors:

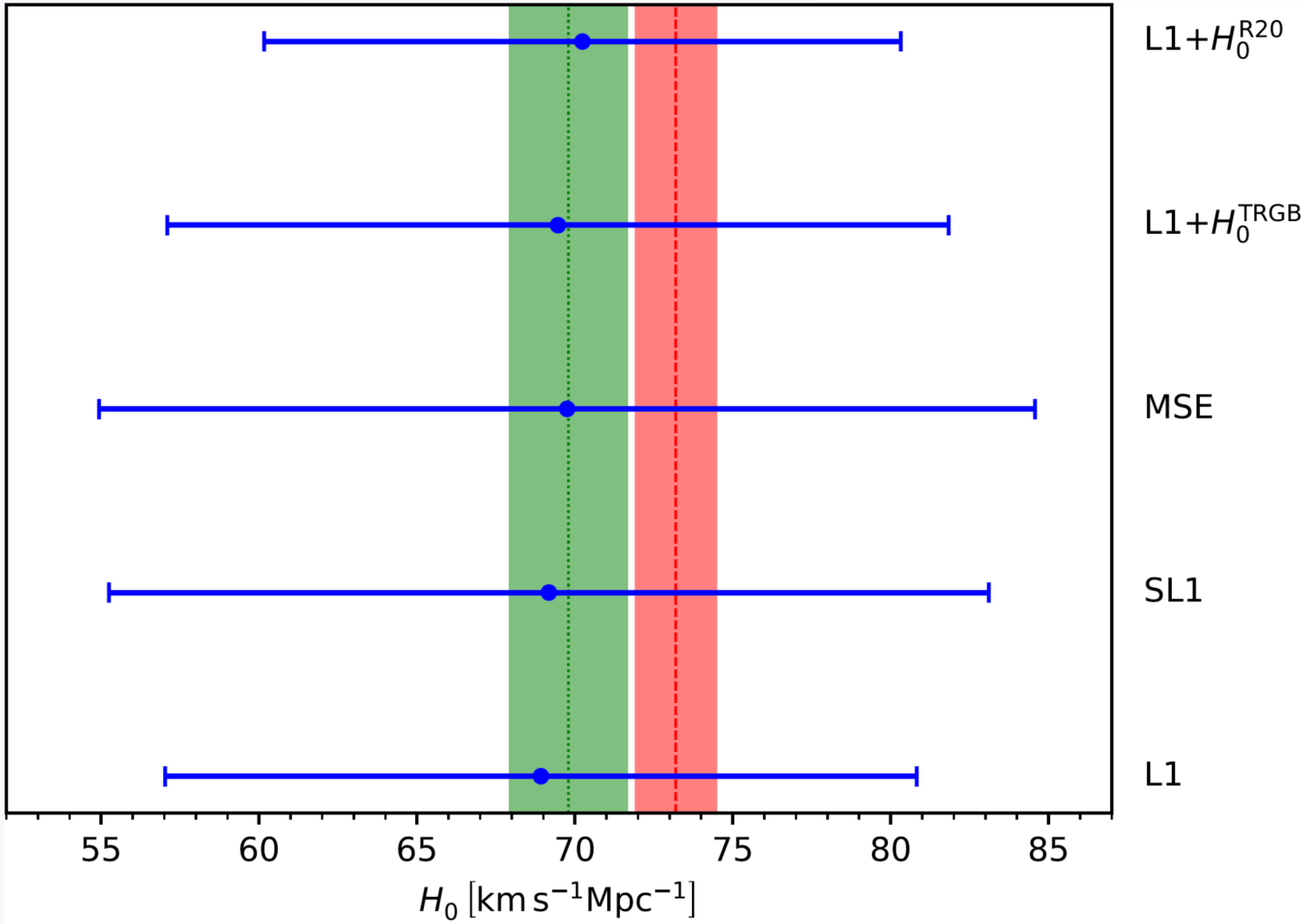
$H_0^{\mathrm{R20}} = 73.2 \pm 1.3\ \mathrm{km\ s^{-1}Mpc^{-1}}$

$H_0^{\mathrm{TRGB}} = 69.8 \pm 1.9\ \mathrm{km\ s^{-1}Mpc^{-1}}$

$\boldsymbol{H_0^{\mathrm{R20}}}$: $H_0 = 70.24 \pm 10.08\ \mathrm{km\ s^{-1}Mpc^{-1}}$

$\boldsymbol{H_0^{\mathrm{TRGB}}}$: $H_0 = 69.47 \pm 12.37\ \mathrm{km\ s^{-1}Mpc^{-1}}$

Whisker Plot of Results



Priors:

$$H_0^{\mathrm{R20}} = 73.2 \pm 1.3\ \mathrm{km\,s^{-1}Mpc^{-1}}$$

$$H_0^{\mathrm{TRGB}} = 69.8 \pm 1.9\ \mathrm{km\,s^{-1}Mpc^{-1}}$$

What about gravitational waves?

Horndeski Gravity

Horndeski Gravity: Produces the most **general second-order theory** that contains only **one scalar field** (in **standard gravity**)

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left[\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right]$$

where

$$\mathcal{L}_2 = \mathrm{G}_2(\phi, X)$$
$$\mathcal{L}_3 = \mathrm{G}_3(\phi, X)\Box\phi$$
$$\mathcal{L}_4 = \mathrm{G}_4(\phi, X)\mathcal{R} + \mathrm{G}_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$
$$\mathcal{L}_5 = \mathrm{G}_5(\phi, X)\mathcal{G}_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}\mathrm{G}_{5,X}(\phi, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]$$

Teleparallel Horndeski Gravity (TeleDeski)

- **TeleDeski Goal**: What is the **TG analog** of **Horndeski theory**?
- **Conditions**: (i) Field equations must be second-order; (ii) terms cannot be parity-violating; (iii) contributions can be at most quadratic in torsion
- **Extra contribution**: $\mathcal{L}_{\text{Tele}} = G_{\text{Tele}}(\phi, X, T, T_{Ax}, T_{vec}, I_2, J_i)$ [I_2 - linear coupling with matter, J_i - quadratic coupling with matter]

Bahamonde et al. PRD 100, 064018 (2019)

Tensor Perturbations

- Taking **tensor perturbations** for **tetrads fields**

$$e^0{}_\mu = \delta^0_\mu \,, e^i{}_\mu = \delta^i_\mu + \frac{1}{2}\delta^j_\mu \delta^{ki} h_{jk} \Rightarrow \; ds^2 = dt^2 - a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

- Produces a **gravitational wave propagation equation** (GWPE)

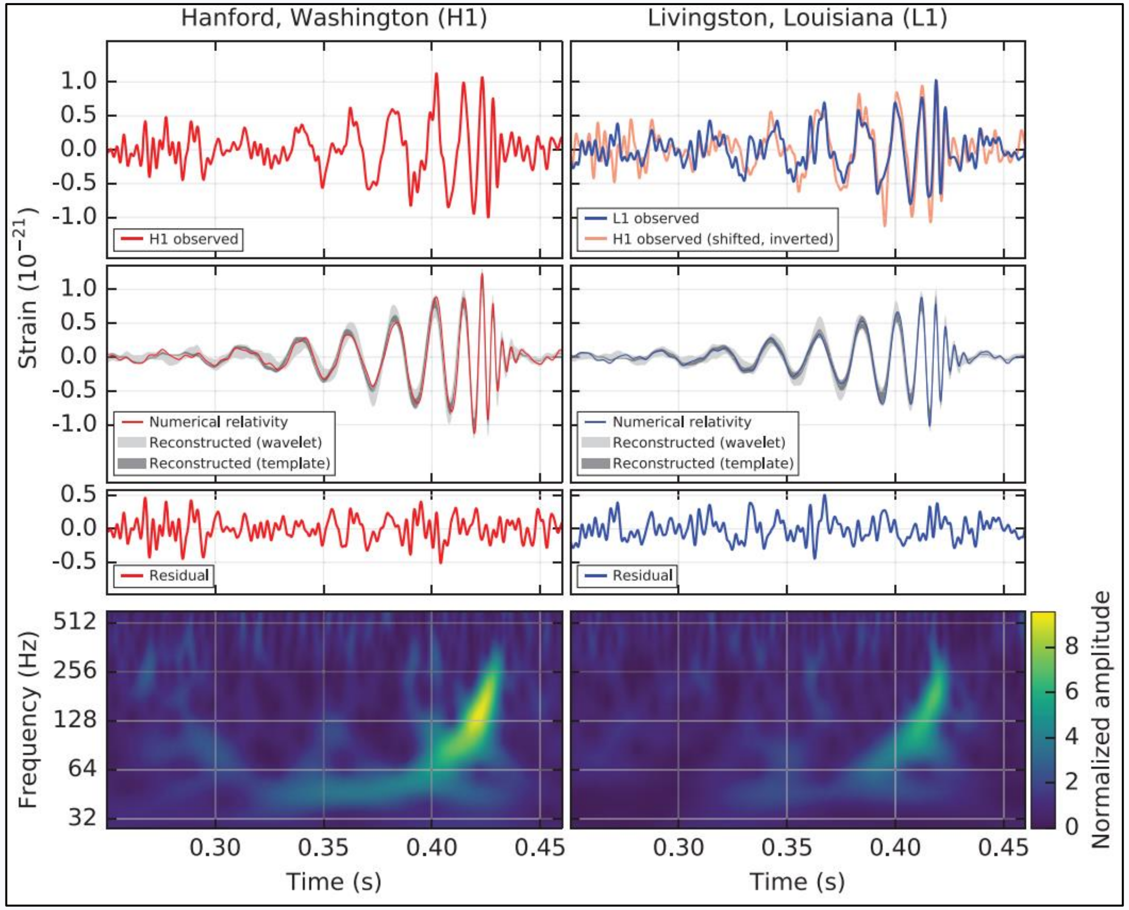
$$\ddot{h}_{ij} + (3 + \alpha_M) H \dot{h}_{ij} - (1 + \alpha_T)\frac{k^2}{a^2} h_{ij} = 0$$

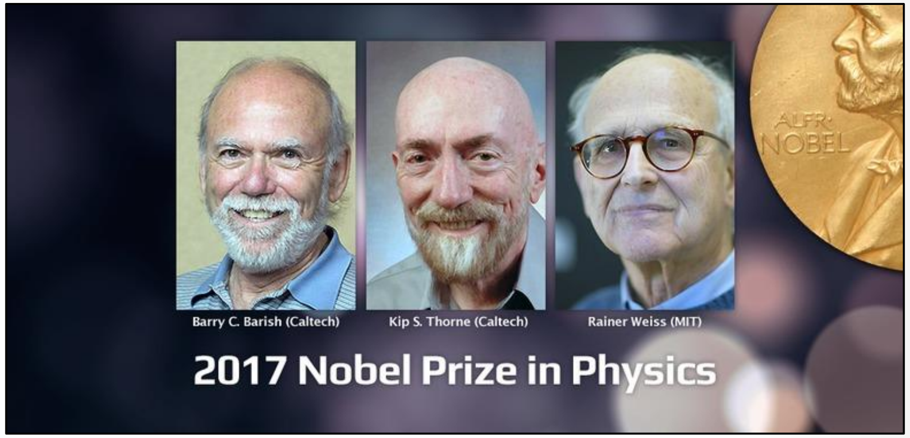
in the Fourier domain

- $\alpha_T = c_T^2 - 1$ is the **tensor excess speed** and $\alpha_M = \frac{1}{HM_*^2}\frac{dM_*^2}{dt}$ is the **Planck mass run rate** (M_*^2 is the effective **Planck mass**)

GW Observations

Can we use **GW observations** to detect **modified gravity**?

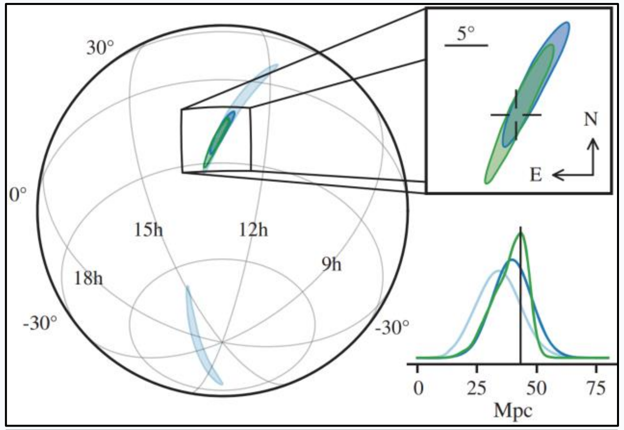




The Era of Multi-messenger Astronomy

GW170817

LIGO-Virgo localization



Virgo observatory

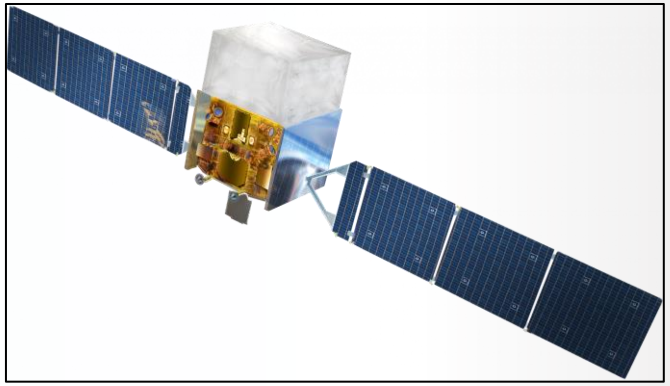
$$M_{\mathrm{Tot}} = 2.74^{+0.04}_{-0.01} M_{\odot}$$

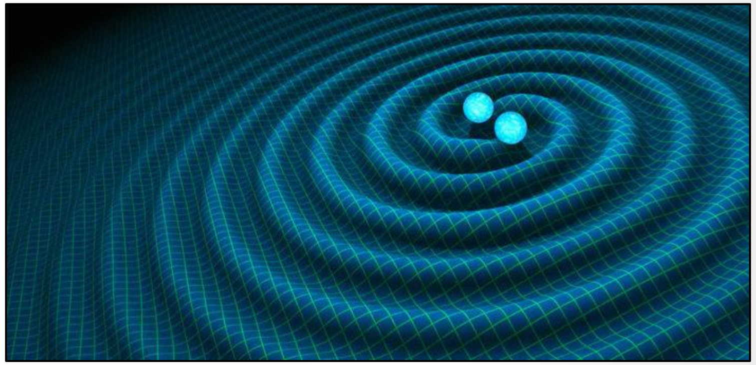
$$\Delta T = 1.7s$$

$$c_T = c^{+7\times10^{-16}}_{-3\times10^{-15}}$$

GRB170817A

Fermi Telescope





GWs in TeleDeski

$$\ddot{h}_{ij} + (3+\alpha_M)H\dot{h}_{ij} - (1+\alpha_T)\frac{k^2}{a^2}h_{ij} = 0$$

- **TeleDeski GWPE:**

$$\alpha_T = \frac{2X}{M_*^2}\left(2G_{4,X} - 2G_{5,\phi} - G_{5,X}(\ddot{\phi} - \dot{\phi}H) - 2G_{\text{Tele,J}_8} - \frac{1}{2}G_{\text{Tele,J}_5}\right) = 0$$

where $M_*^2 = 2\left(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}XHG_{5,X} + 2XG_{\text{Tele,J}_8} + \frac{1}{2}XG_{\text{Tele,J}_5} - G_{\text{Tele},T}\right)$

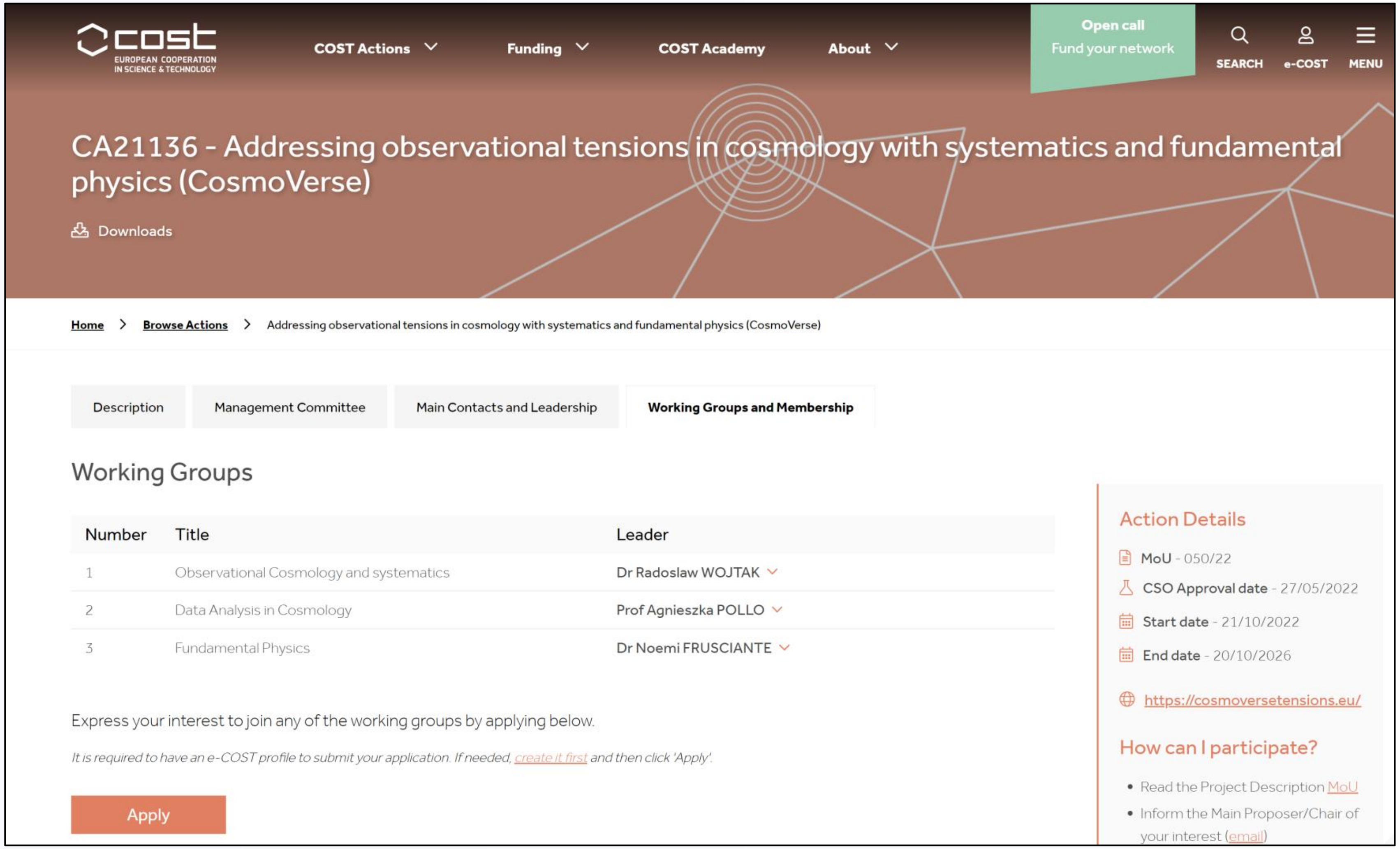
- **Running Planck mass**: Continues to observe $\alpha_M = \frac{1}{HM_*^2}\frac{dM_*^2}{dt}$
- **New possibilities**: Opens new possibilities for reviving Horndeski gravity

Bahamonde et al. PRD 101, 084060 (2020)

Conclusion

- **TG offers an interesting alternative** to traditional ways to modify gravity
- TG satisfies a number of preliminary **observational tests**, and offers a more consistent picture of modified gravity
- TG is compatible with novel methods being developed in conjunction with **machine learning**

CosmoVerse: Join our Working Groups



Number	Title	Leader
1	Observational Cosmology and systematics	Dr Radoslaw WOJTAK
2	Data Analysis in Cosmology	Prof Agnieszka POLLO
3	Fundamental Physics	Dr Noemi FRUSCIANTE

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