Gravity and thermodynamics are more closely related than you might think

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Topics

1. Joule experiment.
2. Tolman's law.
3. Eckart's temperature gradient.
4. Layzer-Irvine equation.
5. Gravo-thermal catastrophe.
6. Last scattering surface.
7. Gravitational lensing.
8. Temperature and entropy of black holes.
9. Equipartition theorem.
10. Generalized second law for black holes.
11. Idem for cosmological horizons.

Joule Experiment

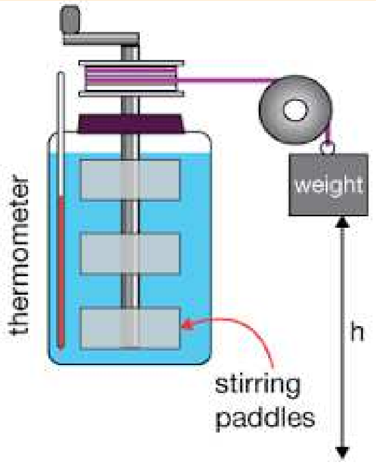


Figure: Gravitational energy becomes heat.

Pocket temperature

$$ds^2 = g_{00}\, c^2 dt^2 \, + \, g_{ij}\, dx^i dx^j$$

Thermal equilibrium in a static ($dx^i = 0$) gravitating fluid,

$$T\,\sqrt{-g_{00}} = \mathrm{C}$$

“... the proper temperature of a fluid as measured by local observers using ordinary thermometric methods would not be constant through a fluid sphere which has come to thermal equilibrium but would vary with gravitational potential, increasing with depth as we go toward the centre of the sphere." (Tolman 1934).

At the Earth surface, $d\log T/dr \simeq -10^{-18}\mathrm{cm}^{-1}$.

Eckart's temperature gradient (1940)

$$q^\mu = -\kappa\,[h^{\mu\nu}(\partial T/\partial x^\nu) + T\,a_\nu]$$

In the local and instantaneous rest frame,

$$q_0 = 0 \quad \& \quad \vec{q} = -\kappa[\nabla T + Tc^{-2}\vec{a}]$$

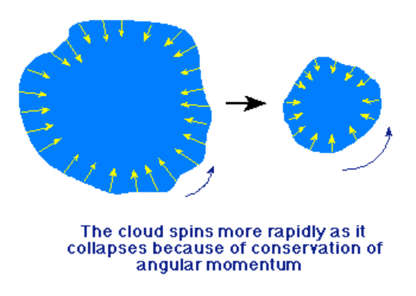
In an inviscid, heat conducting fluid under an external and stationary weak gravitational field $(g_{00} = -(1 + 2c^{-2}\phi))$

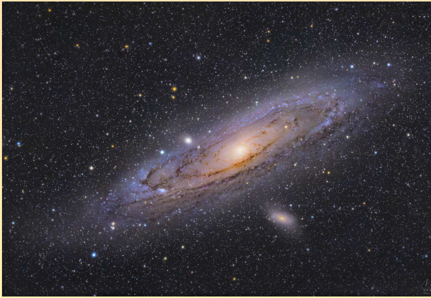
$$\sigma = -\vec{q}\cdot[\nabla T + T\nabla(c^{-2}\phi)]/T^2 \geq 0$$

$$\vec{q} = -\kappa\,[\nabla T + T\nabla(c^{-2}\phi)]$$

The equivalence principle validates the presence of the acceleration term (Pavón et al., 1980).

Incomplete collapse of a gas cloud





Initially $\langle \mathcal{K} \rangle \ll | \langle V \rangle |$.

Typical spiral galaxy.

Layzer-Irvine equation (1961)

Virial theorem, $\langle \mathcal{K} \rangle = -\langle V \rangle / 2$

Approach to dynamical equilibrium

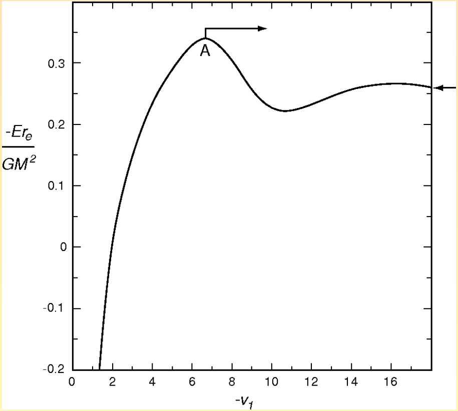
$$\frac{d}{dt}(\langle \mathcal{K} \rangle + \langle V \rangle) + \frac{\dot{a}}{a}(2\langle \mathcal{K} \rangle + \langle V \rangle) = 0$$

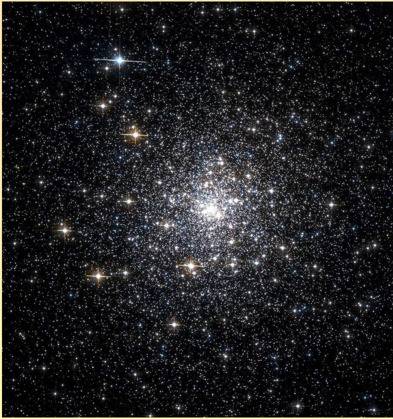
$$\dot{a} > 0 \quad \text{and} \quad \frac{d}{dt}(\langle \mathcal{K} \rangle + \langle V \rangle) > 0$$

Energy dissipation $\Rightarrow$ generation of entropy

$$\dot{S} = \frac{\dot{a}}{a} \frac{|2\langle \mathcal{K} \rangle + \langle V \rangle|}{T} \geq 0.$$

Gravo-thermal catastrophe (Lynden-Bell & Wood 1968)





Dimensionless radius vs $-\log$ density contrast.

Globular cluster M70.

Pound-Rebka Experiment

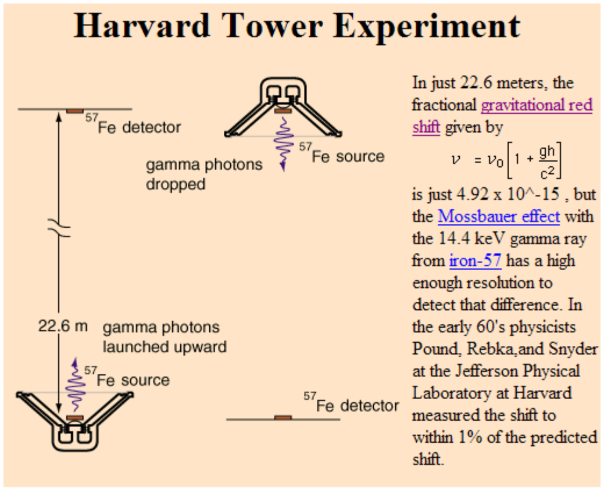


Figure: By the Equivalence principle, wavelengths are affected by the gravitational field.

CMB

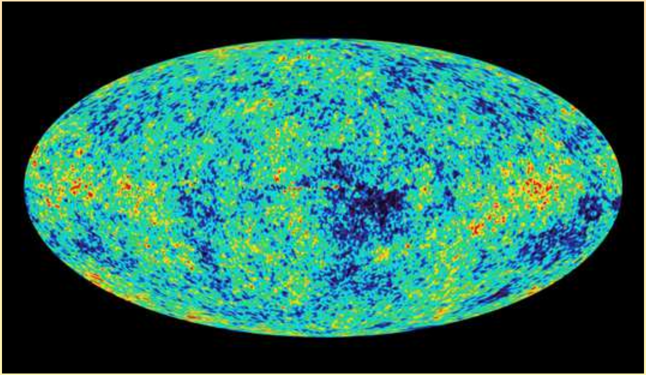


Figure: Intrinsic anisotropies on the last scattering surface.

The bending of light

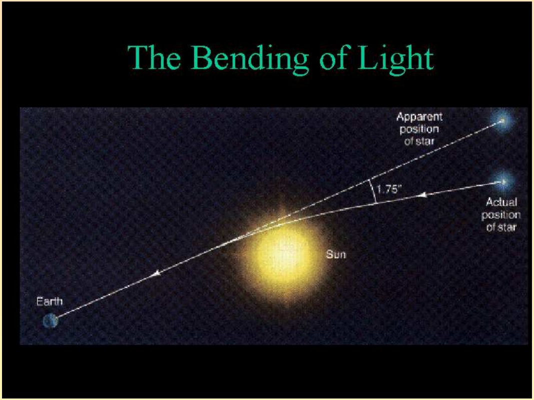


Figure: Gravitational lensing.

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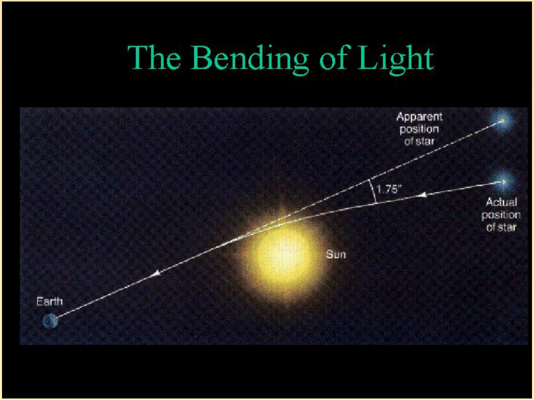


Figure: Gravitational lensing.

“Seeing" a MACHO

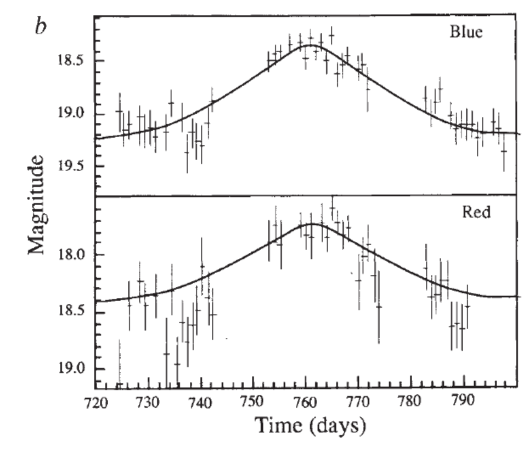
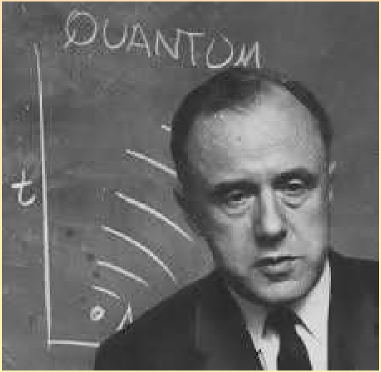
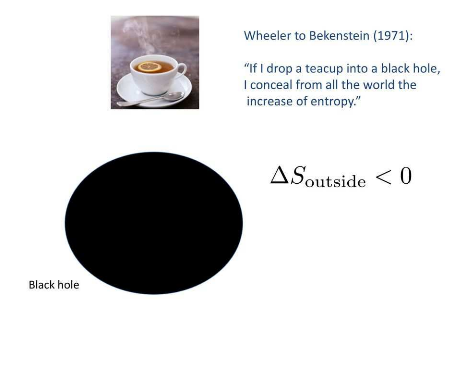


Figure: Light intensity enhancement.

Hints of BH entropy





$$ds^2 = -(1 - 2M/r)\, dt^2 + dr^2/(1 - 2M/r) + r^2(d\theta^2 + \sin^2\theta\, d\phi^2).$$

$\mathcal{A} \propto M^2 \quad \Rightarrow \quad \mathcal{A} > \mathcal{A}_1 + \mathcal{A}_2.$ So we guess $S \propto \mathcal{A}$.

BH thermodynamics (Hawking 1974, 1976)

The first law of BH Mechanics: $d(Mc^2) = \frac{\kappa c^2}{8\pi G} d\mathcal{A} + \Omega dJ + \Phi dQ$

$$\mathcal{A} = 4\pi r_+^2 = 4\pi \left\{2M^2 - Q^2 + 2\sqrt{M^4 - J^2 - M^2Q^2}\right\} \qquad (G = c = 1)$$

$$< n >= \frac{\Gamma}{\exp[(\hbar\omega - \hbar m\Omega - e\Phi)/(k_B\, T)] \mp 1}$$

$$\Rightarrow S = k_B \mathcal{A}/4\ell_p^2, \qquad T = \hbar\kappa/2\pi k_B$$

Equipartition theorem for BHs

Smarr's formula for Kerr BHs:

$$M = \frac{\kappa \mathcal{A}}{4\pi} + 2\Omega J \qquad \left(\kappa = \frac{1}{2M} \frac{\sqrt{1 - J^2/M^4}}{1 + \sqrt{1 - J^2/M^4}}, \quad J^2 \leq M^4 \right)$$

Upon the identifications $T = \kappa/2\pi, \quad \mathcal{A} = \mathcal{N}$ (in Planck's units)

$$M = \xi \mathcal{N} T$$

Each degree of freedom contributes

$$\xi = \frac{\pi}{2\pi - \Omega^2 \mathcal{N}}$$

to the BH mass.

($\xi = 1/2$ for non-rotating BHs ($\Omega = 0$))

Equipartition theorem for Kerr-Newman BHs

Smarr's formula for charged rotating BHs:

$$M = \frac{\kappa \mathcal{A}}{4\pi} + 2\Omega J + \Phi Q$$

$$\left(\Phi = \frac{1}{M} \left[\frac{Q}{2} + \frac{2\pi Q^3}{\mathcal{A}} \right] \right)$$

For $Q^2 \ll M^2 \;\Rightarrow\; M \simeq \frac{\kappa}{4\pi} \mathcal{A} + \frac{\Omega^2 \mathcal{A}}{2\pi} M + \frac{Q^2}{2M}$

$$M = \xi \mathcal{N} T$$

$$\xi = \frac{\pi}{2\pi - \Omega^2 \mathcal{N}} \left[1 + \frac{2\pi - \Omega^2 \mathcal{N}}{4\,\pi \left(\frac{\kappa \mathcal{N}}{4\pi} \right)^2} Q^2 \right] + \mathcal{O}(Q^4)$$

GSL for black holes I (Bekenstein 1977)

$$\Delta S_{bh} = (\Delta M - \Omega \Delta J - \Phi \Delta Q)/T_{bh}$$

$$< n >= \frac{\Gamma}{\exp(x) \mp 1} \qquad (x = \hbar\omega - \hbar m \Omega - e\Phi)$$

$$\Delta M = -\sum < n > \hbar\omega \,, \qquad \Delta J = -\sum < n > \hbar m \,,$$
$$\Delta Q = -\sum < n > e$$

$$\langle \Delta S_{bh} \rangle = -\sum x \, \Gamma/(e^x \mp 1)$$

$$\delta \left[-\sum p_{\{n\}} \ln p_{\{n\}} - \sum \beta \sum n' p_{\{n\}} - (\alpha - 1) \sum p_{\{n\}} \right] = 0$$

$$p_{\{n\}} = e^{-\alpha} \prod e^{-\beta}$$

$$\langle \Delta S \rangle = \langle \Delta S_{bh} \rangle + \Delta S_{rad} = \alpha + \sum \Gamma(\beta - x)/(e^x \mp 1) > 0.$$

GSL for black holes II (Unruh & Wald 1982)

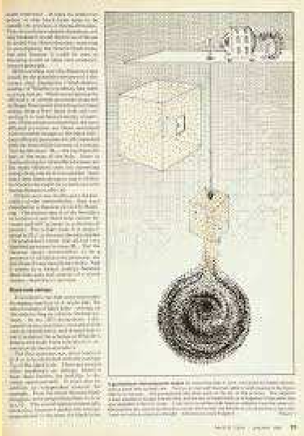


Figure: Energy is lowered onto a bh at the floating point $(E = \varepsilon V) \quad \Rightarrow$ $\Delta S_{bh} = s(\varepsilon)V \geq S_{box}$.

Event Horizon

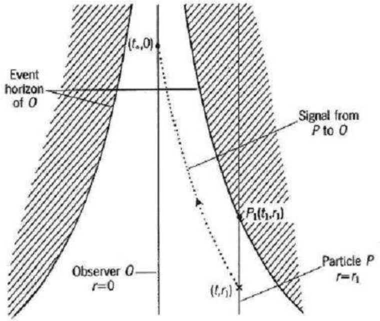


Figure 5.2: The event horizon of an observer in an accelerated FRW universe (a De Sitter universe in this case). The signal emitted by a particle P at the instant $t < t_1$ arrives to the observer at the finite instant t_*. Therefore, as $t \to t_1$, it follows that $t_* \to \infty$ and the light signal takes longer and longer to arrive from the particle to the observer. Signals emitted once the particle is beyond the horizon will never reach the observers position. (Adapted from Fig. 23.5 of Ref. [67]).

Figure: The event horizon of the de Sitter universe has an entropy proportional to its area and a temperature proportional to the rate of expansion (Gibbons & Hawking 1977).

GSL for cosmic horizons

$$S_h = 2\pi\mathcal{A}\,, \quad \mathcal{A} = 4\pi/H^2 \qquad (8\pi G = c = k_B = 1)$$

$$\dot{S}_f = \frac{a^3}{T_f\alpha\rho_f}(\tau\dot{\sigma} + 3\alpha\rho_f H)^2\,, \qquad \dot{S}_h = \frac{8\pi^2}{H^3}\left[(\gamma - 3\alpha H)\rho_f - \tau\dot{\sigma}\right]$$

$$\dot{S} = \dot{S}_f + \dot{S}_h = \frac{8\pi^2}{H^3}\left[\gamma\rho_f + \frac{\tau^2\dot{\sigma}^2}{3\alpha\rho_f H} + \tau\dot{\sigma}\right]$$

If the DEC ($\rho + P \geq 0 \;\Rightarrow\; \gamma\rho_f \geq \tau\dot{\sigma} + 3\alpha\,\rho_f\,H$) holds,

$$\dot{S} \geq \frac{8\pi^2}{3\alpha\rho_f H^4}\left[\tau\dot{\sigma} + 3\alpha\rho_f H\right]^2 \geq 0.$$

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- Thermal energy and gravity can be mutually inter-converted (e.g. radiation $\leftrightarrow$ black hole) while respecting the aforesaid laws.
- The latter set constraints on gravity driven processes.
- We may speculate that the GSL holds also at large scales.
- Gravity and thermodynamics are not so unrelated disciplines after all.