



# Tracing beyond GR physics with gravitational waves *qualitative vs quantitative*

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# Approaches in testing strong gravity

## Model independent (e.g. PPN formalism)

- Much simpler
- No prior knowledge of the modified gravity theory is needed
- Mapping to a modified gravity models is not straightforward
- Performing dynamics is not possible

## Model dependent (bounded to a given modified gravity theory)

- Observational implications predicted self consistently from a modified theory
- Gives intuition about what is physically relevant
- Much more involved

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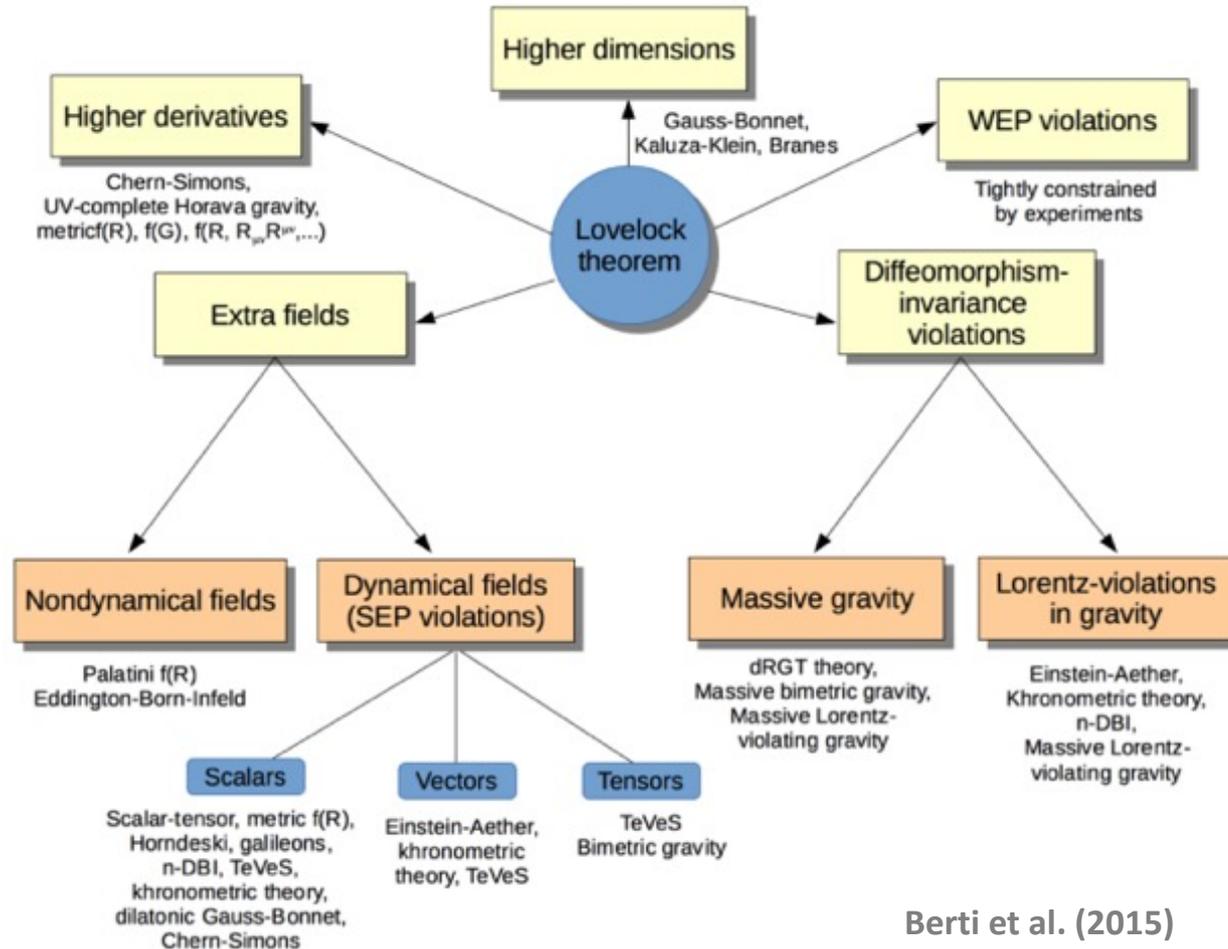
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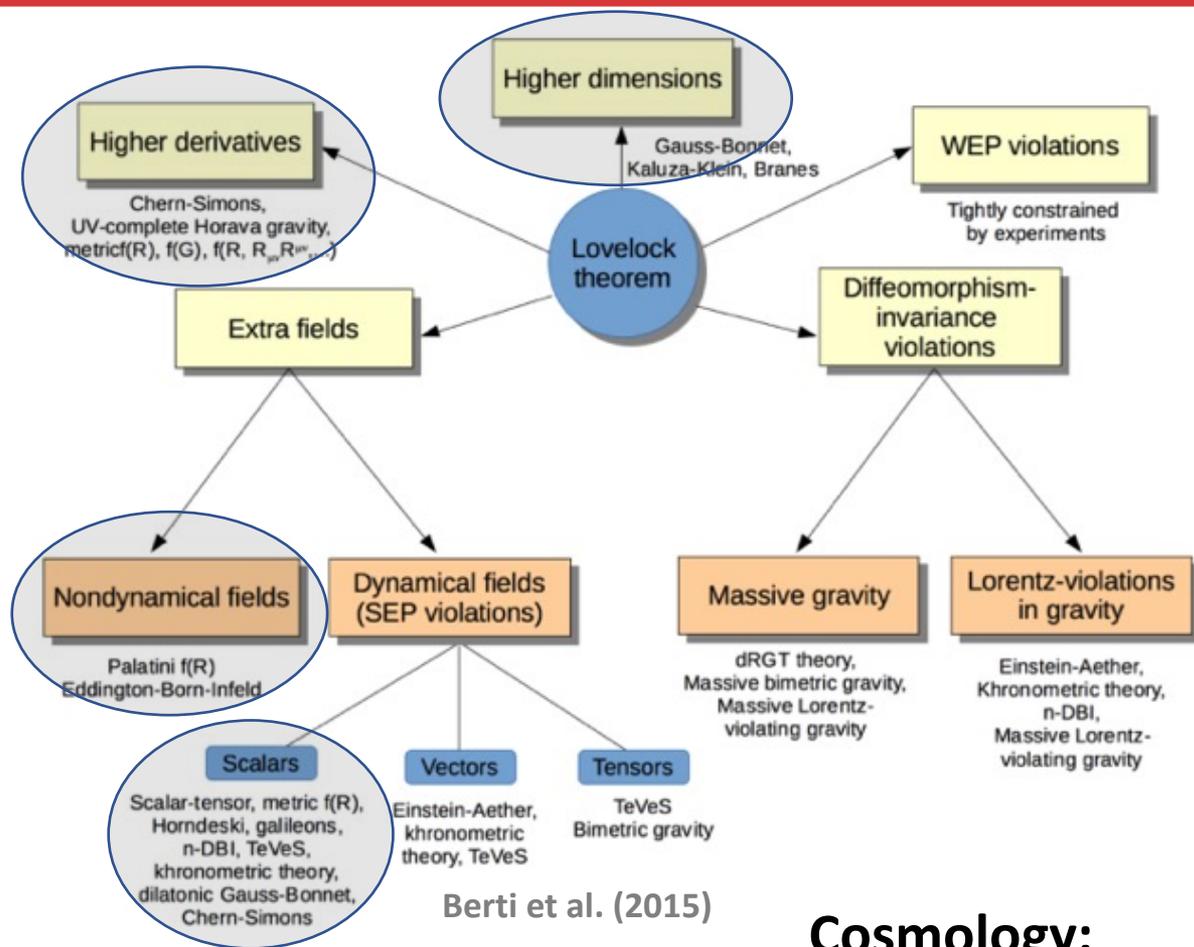
# Lovelock's theorem

Einstein's field equations are **unique** if:

- ✓ we are working in **four dimensions**
- ✓ **diffeomorphism invariance** is respected
- ✓ the **metric** is the **only field** mediating gravity
- ✓ the equations are **second-order differential equations**.



# Extra scalar field(s)



## Quantum gravity motivated:

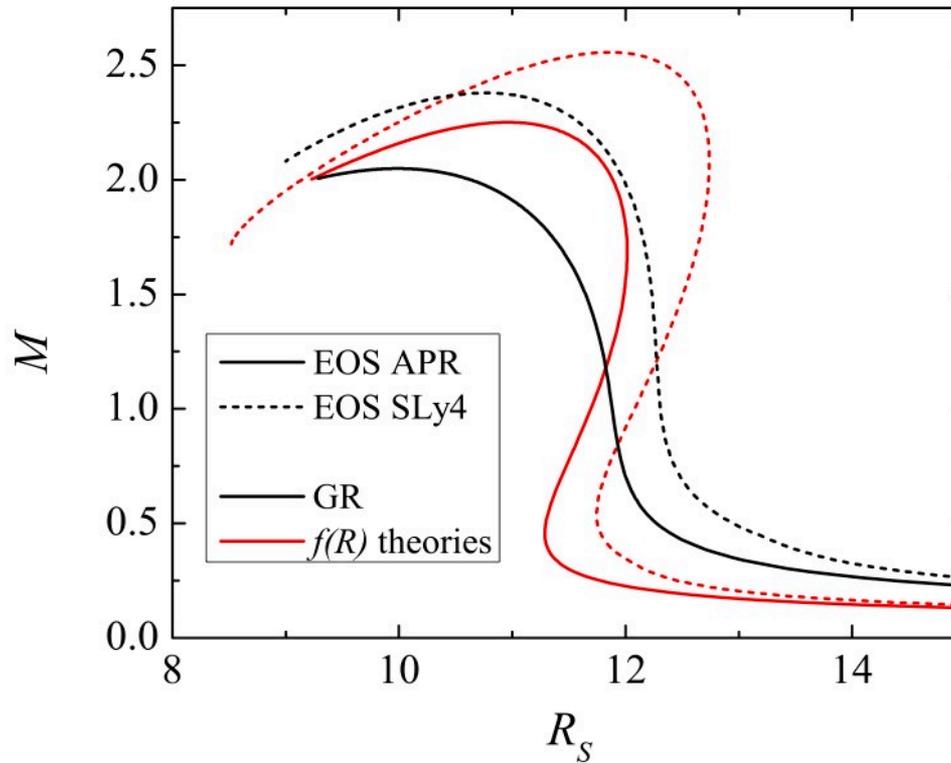
- Gauss-Bonnet gravity
- Chern-Simons gravity

## Cosmology:

- Ultralight axion dark matter
- Inflation scalar field
- $f(R)$ , Horndeski gravity

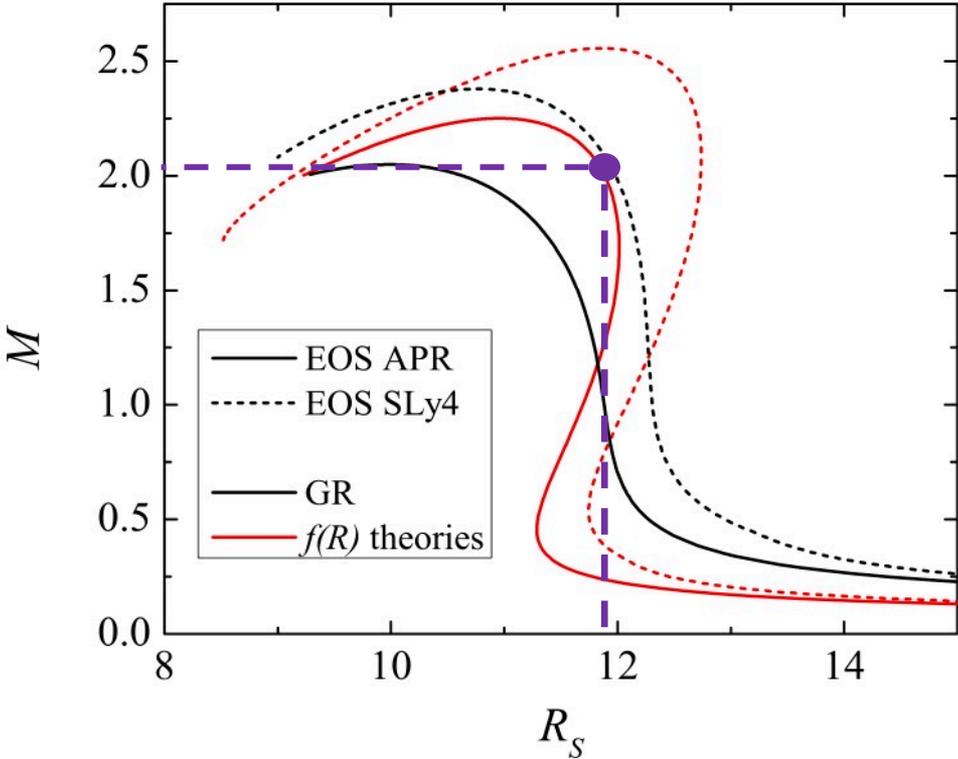
**Quantitative vs. Qualitative**

# Quantitative changes



**Modifying the theory of gravity  $\Leftrightarrow$  EOS uncertainty**

# Quantitative changes



**Modifying the theory of gravity  $\Leftrightarrow$  EOS uncertainty**

**Quantitative vs. Qualitative**

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# Jumps in GW emission during merger

# Gauss-Bonnet gravity - Scalarization

- **Gauss-Bonnet gravity** – the equations are of second order

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - 2\nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) + \lambda^2 f(\varphi) \mathcal{R}_{GB}^2 \right].$$

Gauss-Bonnet invariant:

$$\mathcal{R}_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

- With a proper choice of  $f(\varphi)$ :
  - ✓ **Perturbatively equivalent to GR in the weak field**
  - ✓ **Nonlinear effects for strong fields – scalarization**

# Gauss-Bonnet gravity - Scalarization

- **Scalar field equation :**

$$\nabla_\alpha \nabla^\alpha \varphi = \frac{1}{4} \frac{dV(\varphi)}{d\varphi} - \frac{\lambda^2}{4} \frac{df(\varphi)}{d\varphi} \mathcal{R}_{GB}^2,$$

- **Conditions for the existence** of scalarized solutions

$$(\square - \mu_{\text{eff}}^2) \delta\varphi = 0 \text{ with } \mu_{\text{eff}}^2 = -\frac{\lambda^2}{4} \frac{d^2 f}{d\varphi^2}(\varphi) \mathcal{R}_{GB}^2 < 0$$

- If  $\mu_{\text{eff}}^2 < 0$  a **tachyonic instability** is present leading to development of the scalar field. DD, Yazadjiev PRL (2018), Antoniou et al. PRL (2018), Silva et al. PRL (2018)

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- Expand  $f(\varphi)$  in series around  $\varphi = 0$ :

$$f(\varphi) = f_0 + f_1 \varphi + f_2 \varphi^2 + f_3 \varphi^3 + f_4 \varphi^4 + O(\varphi^5)$$

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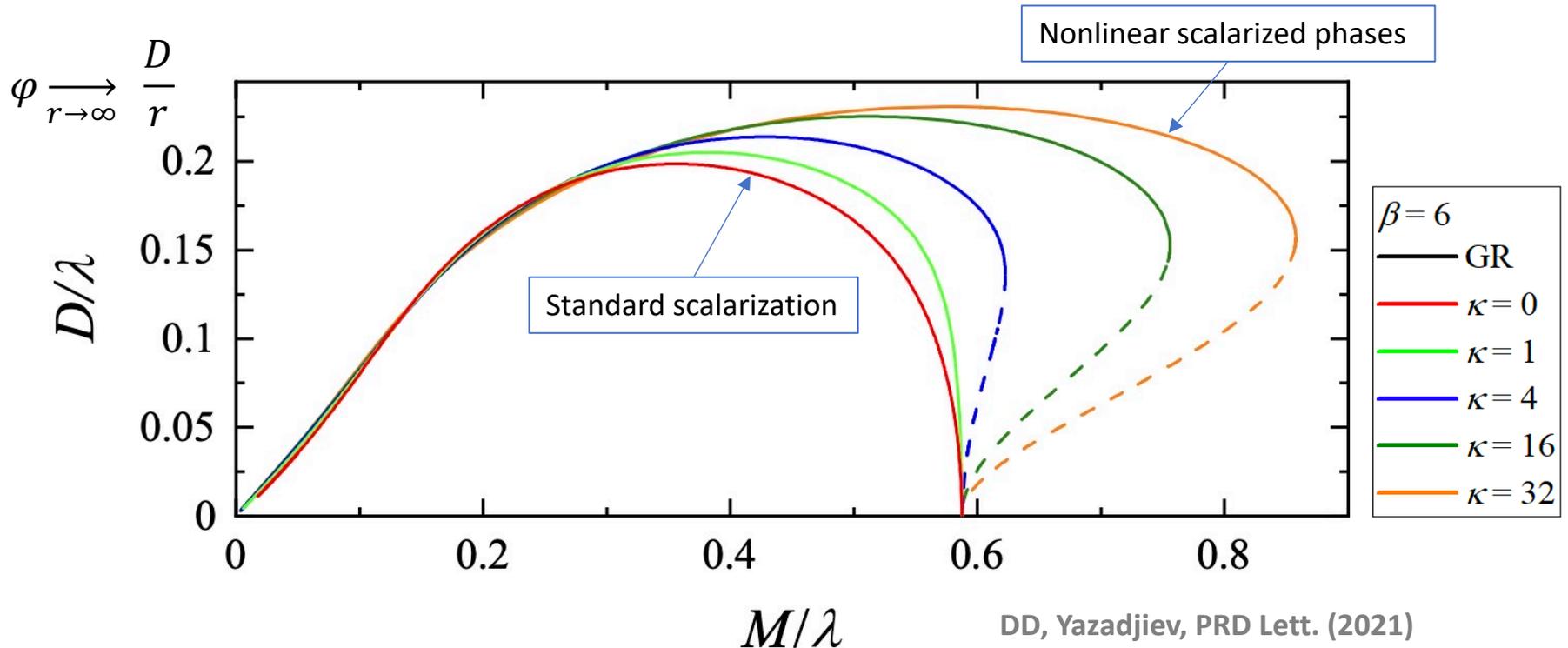
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For scalarization  $\frac{d^2 f}{d\varphi^2} \neq 0$

# (De)scalarization with a jump during merger

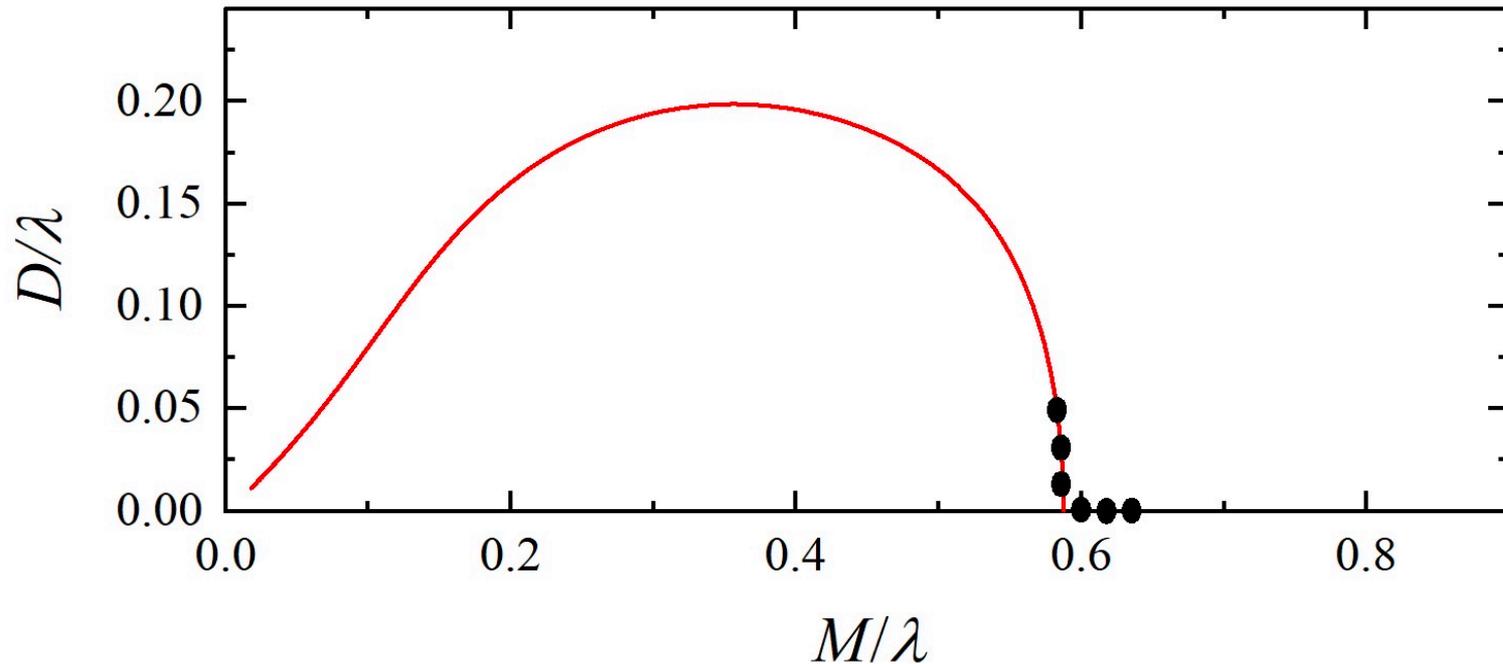
$$f(\varphi) = \frac{1}{2\beta} \left( 1 - e^{-\beta(\varphi^2 + \kappa\varphi^4)} \right)$$



- Transition from **stable scalarized to GR** happens with a **jump**
- For a similar effect for charged BH see Blázquez-Salcedo et al. PLB (2020)

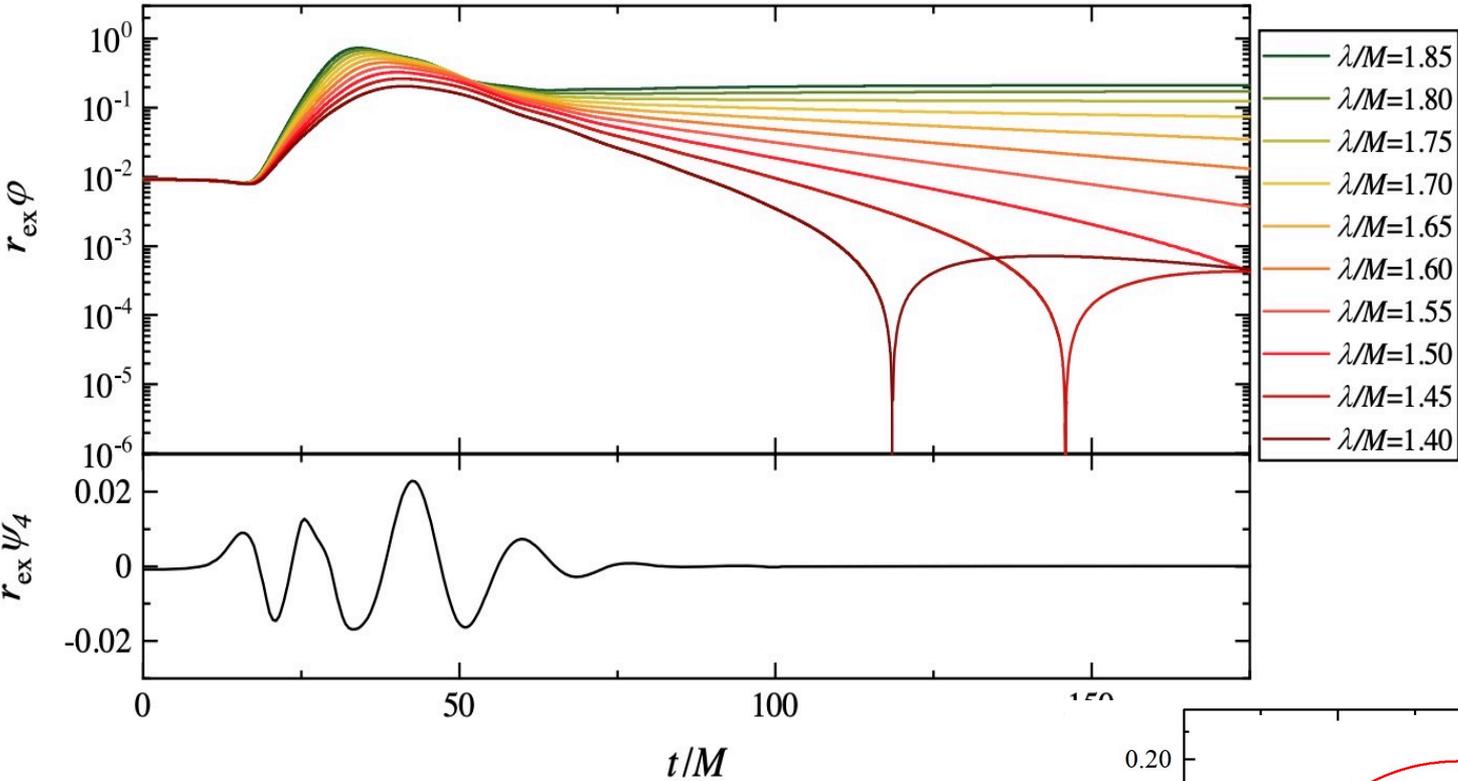
# (De)scalarization WITHOUT a jump during merger

$$f(\varphi) = \frac{1}{12} (1 - e^{-6\varphi^2}) \quad (\beta = 6, \kappa = 0)$$

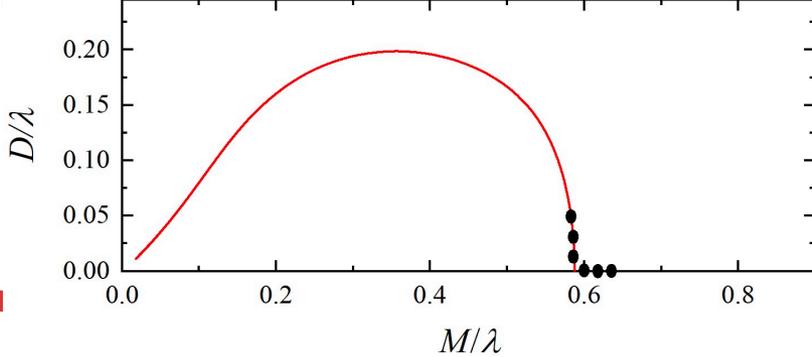


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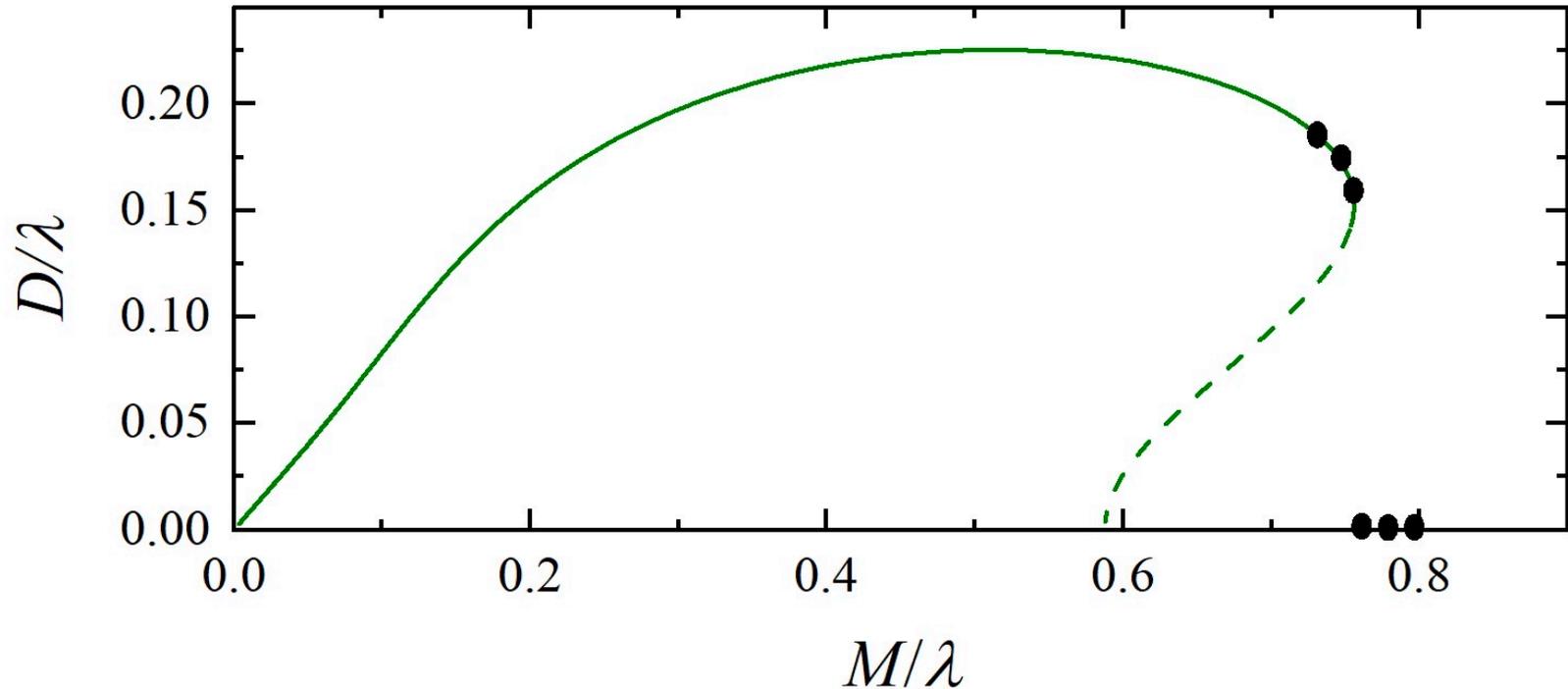


DD, Vano-Vinuales, Yazadjiev PRD (2022)



# (De)scalarization WITH a jump during merger

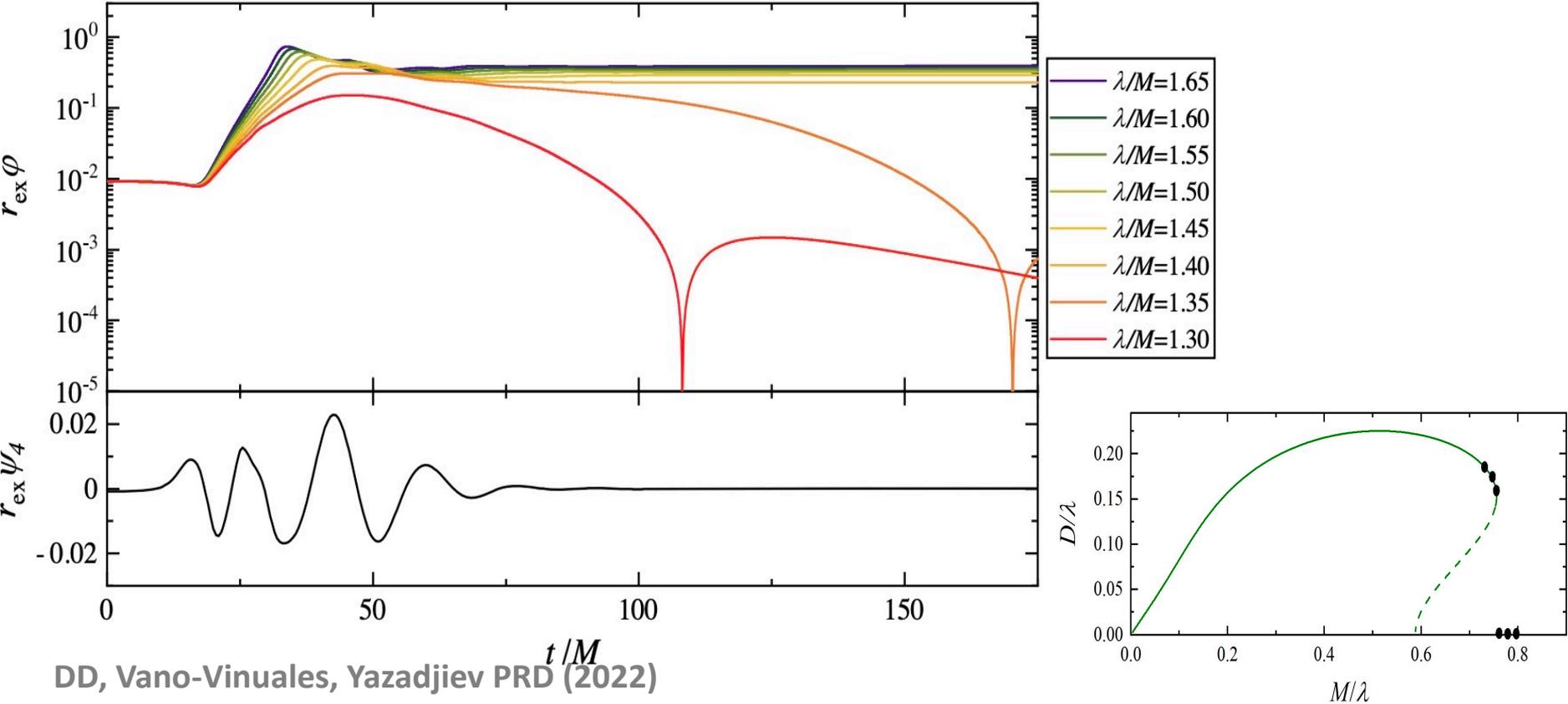
$$f(\varphi) = \frac{1}{12} \left( 1 - e^{-6(\varphi^2 + 16\varphi^4)} \right) \quad (\beta = 6, \kappa = 16)$$



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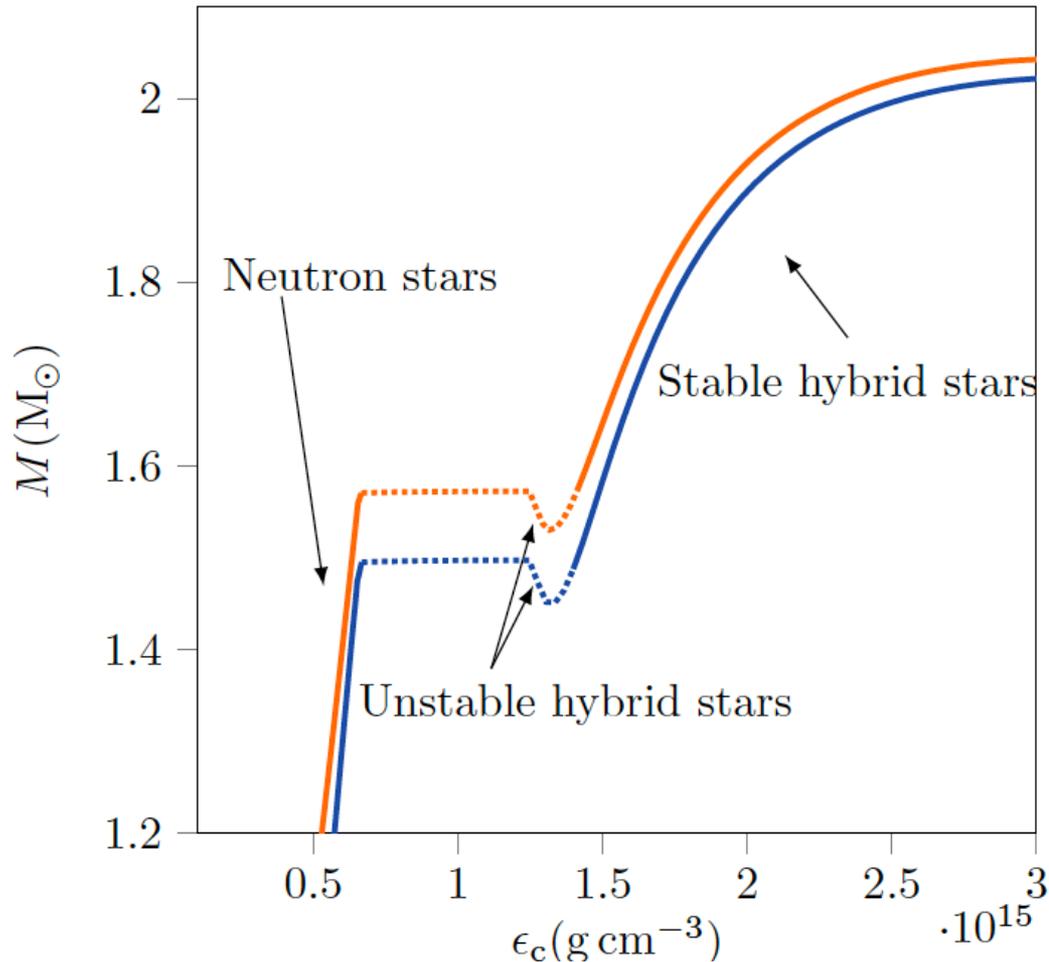
- **Similarities** with the **matter phase transitions** during neutron star binary mergers Most et al. PRL (2019), Bauswein et al. PRL (2019), Weih et al. (2020).

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# Gravitational Phase Transitions

# Matter phase transitions in GR: Twin Stars

- Matter phase transitions from nuclear matter to deconfined quark matter



Espino, Paschalidis (2021)

# Scalarized neutron stars – DEF model

- **Scalarization of neutron stars** Damour&Esposito-Farese PRL (1993) due to a **nonzero trace** of the energy momentum tensor. **Energetically more favorable** over the GR solutions.

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{g} [R - \underbrace{2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi}_{\text{Kinetic term}} - \cancel{4V(\varphi)}] + S_m[\psi_m, \underbrace{A^2(\varphi)}_{\text{Coupling term}} g_{\mu\nu}]$$

- **Coupling function** – polynomial expansion in  $\varphi$

$$\alpha(\varphi) = \frac{d \ln(A(\varphi))}{d\varphi} = \alpha_0 + \beta_0 \varphi$$

- Scalar field equation:  $\square\varphi = -4\pi G_* \alpha(\varphi) T$

(reminder in sGB gravity  $\square\varphi = -\frac{\lambda^2}{4} \frac{df(\varphi)}{d\varphi} R_{GB}^2$ )

# Scalarized neutron stars – DEF model

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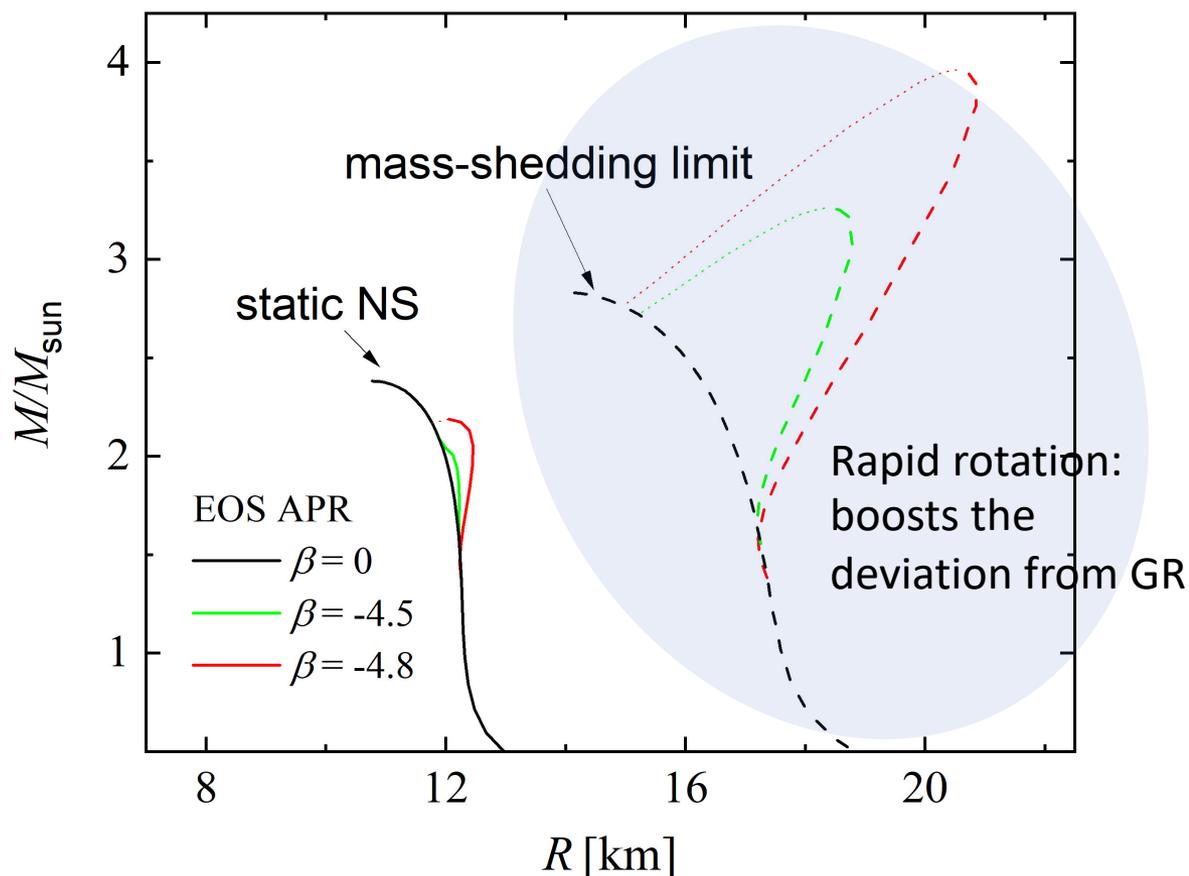
$$\alpha(\varphi) = \alpha_0 + \beta \varphi$$

- **Brans-Dicke theory** –  $\varphi = 0$  NOT a solutions, **ruled out** by weak field observations

# Scalarized neutron stars – DEF model

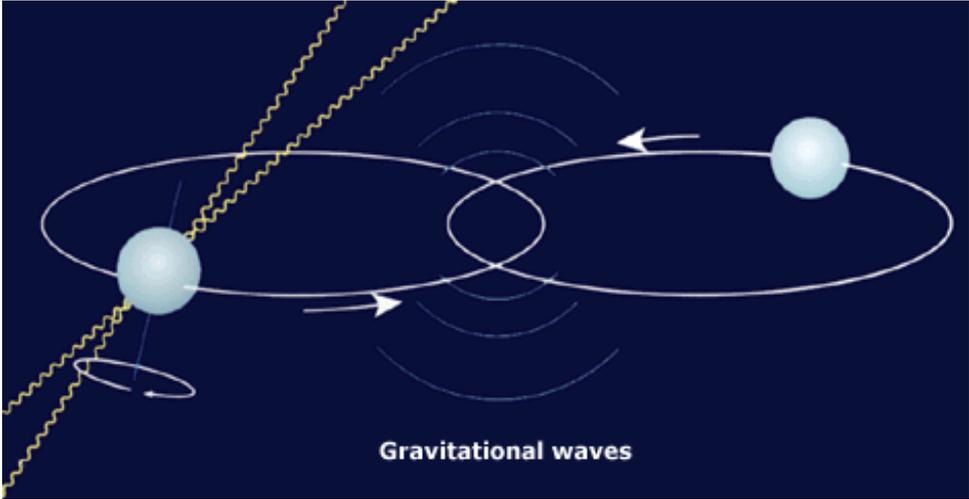
$$\alpha(\varphi) = \cancel{\alpha_0} + \beta_0 \varphi \quad (\text{reminder } \mu_{\text{eff}}^2 = \left. \frac{d\alpha}{d\varphi} \right|_{\varphi=0} 4\pi G_\star T < 0)$$

- Original DEF model Damour&Esposito-Farese (1993)

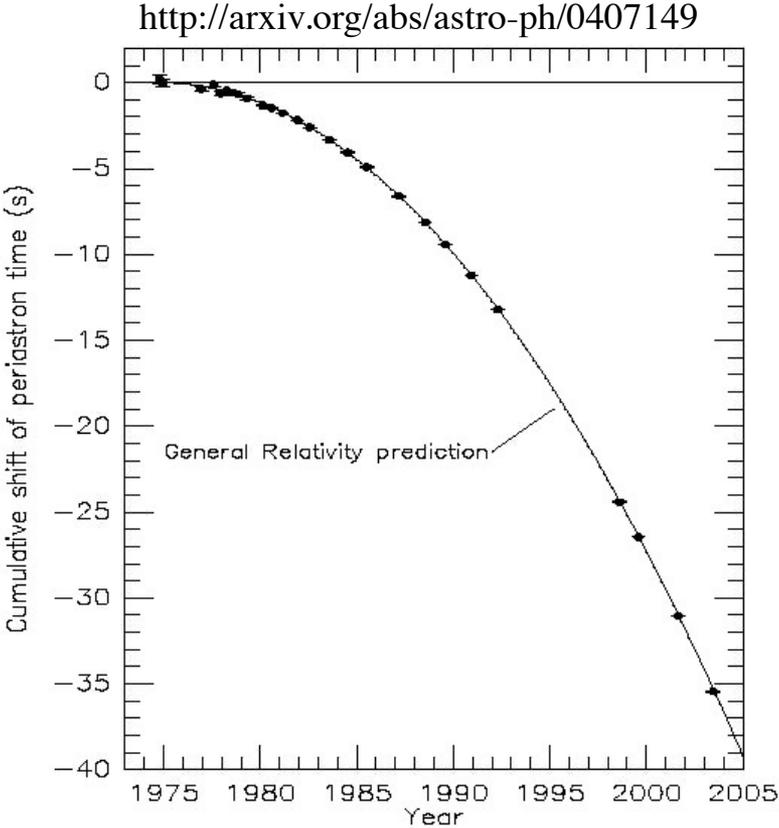


DD, Yazadjiev, Stergioulas, Kokkotas (2013,2014)

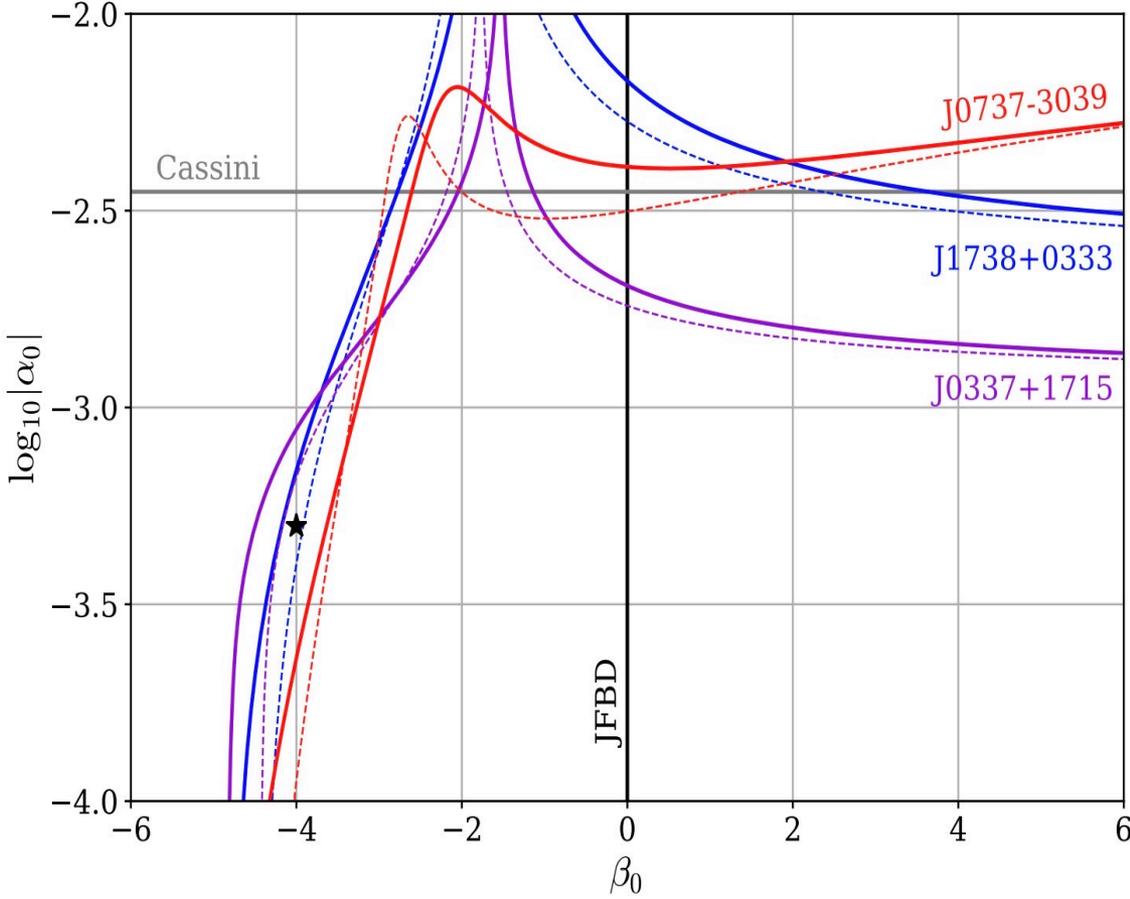
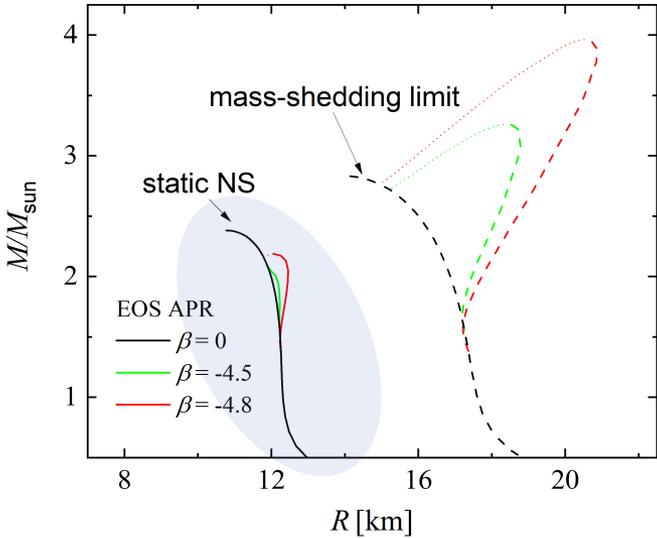
# Observational constraints – binary pulsars



$$\frac{\dot{P}_{b,corrected}}{\dot{P}_{b,GR}} = 1.0013 \pm 0.0021$$



# Original DEF model – Ruled out!



Kramer et al (2021), Zhao et al. (2022)

# Evading the constraints – massive scalar field

## Scalar field potential

$$V(\varphi) = \frac{1}{2} m_\varphi^2 \varphi^2 + \lambda \varphi^4$$

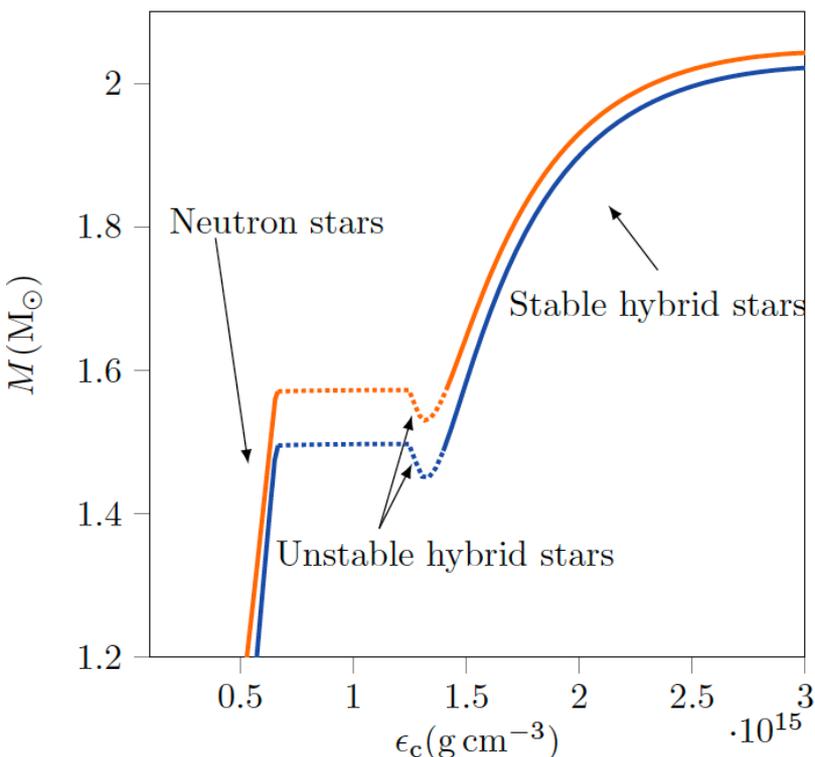
Scalar field mass

Self-interaction term

- Introduces an **effective range of the scalar field** connected to its **Compton wavelength**  $\lambda_\varphi = \frac{2\pi}{m_\varphi}$
- For  $r \gg \lambda_\varphi$  the scalar field drops exponentially.
- For  $m_\varphi \gg 10^{-16}$  eV : **not constraints on  $\beta_0$**  Ramazanoglu,Pretorius(2016), Yazadjiev,DD(2016), Rosca-Mead et al. (2020)

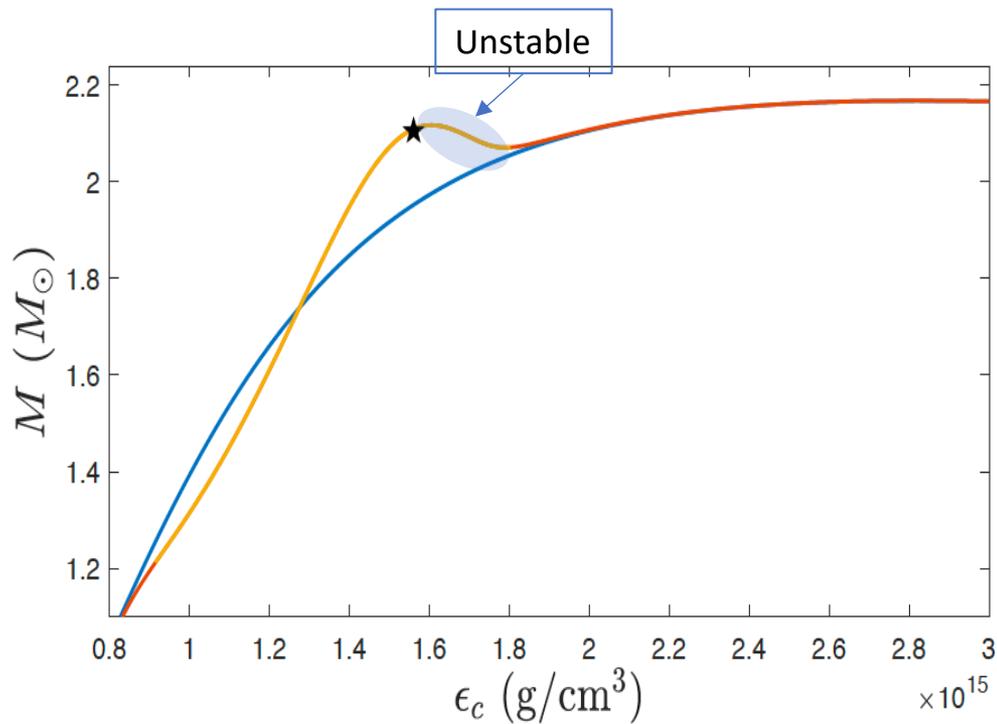
# Twin Stars vs. Scalarization

## Twin stars



Espino, Paschalidis (2021)

## Gravitationally induced phase transition



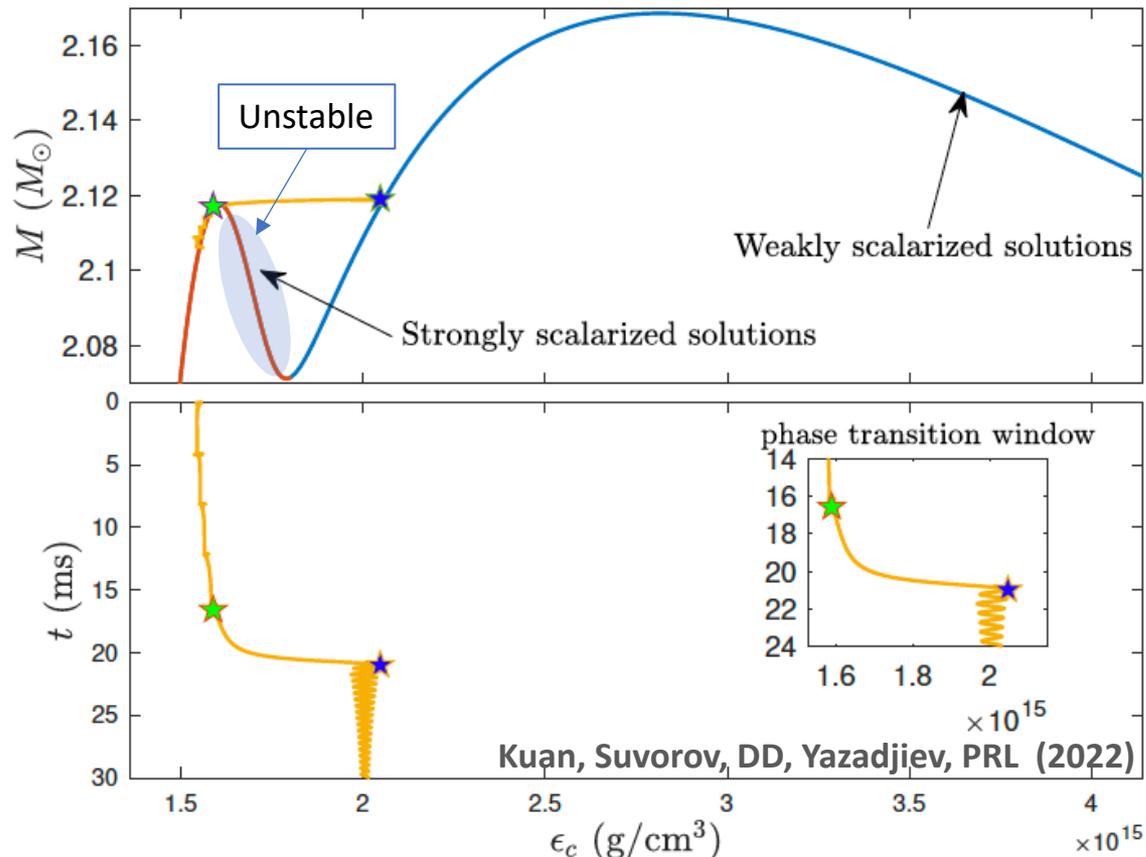
Kuan, Suvorov, DD, Yazadjiev, PRL (2022)

# Gravitational phase transition

- DEF model with a massive scalar field

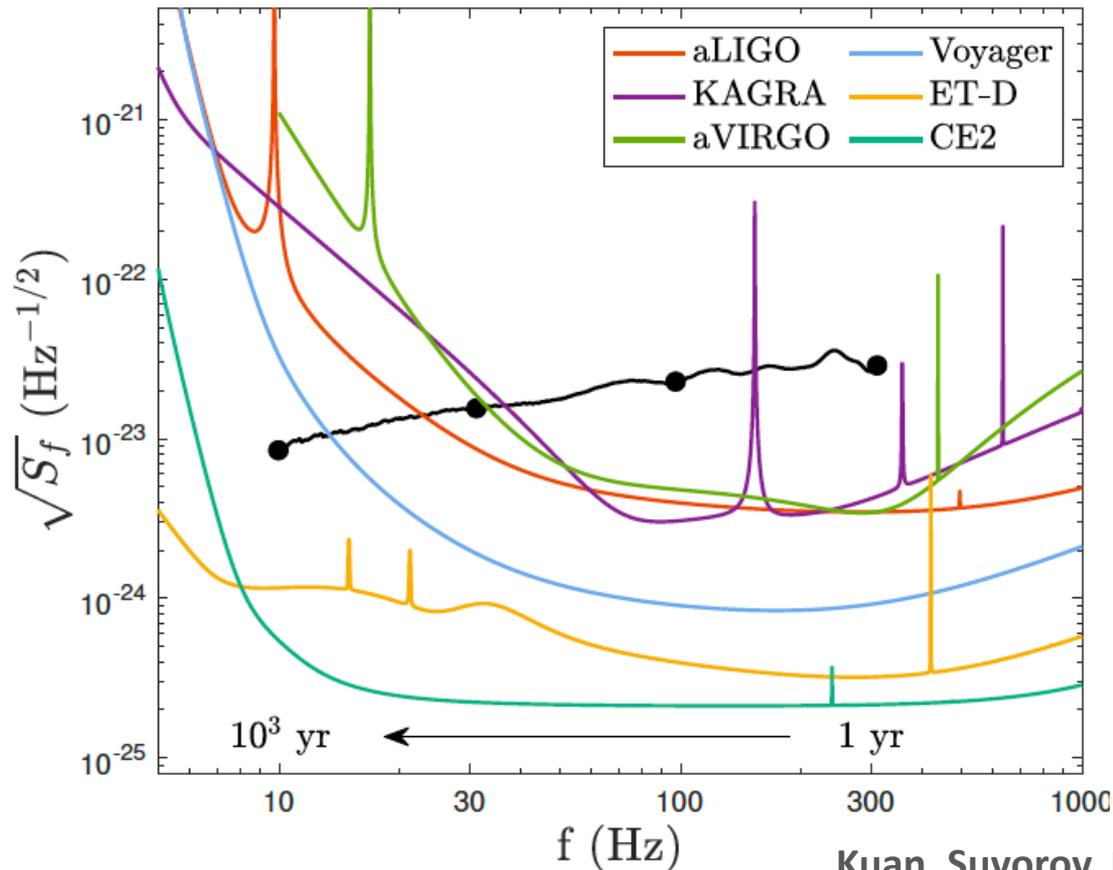
$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{g} [R - 2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 4V(\varphi)] + S_m[\psi_m, A^2(\varphi)g_{\mu\nu}]$$

- $V(\varphi) \neq 0$  in order to **avoid binary pulsar constraints** Zhao et al. (2022)



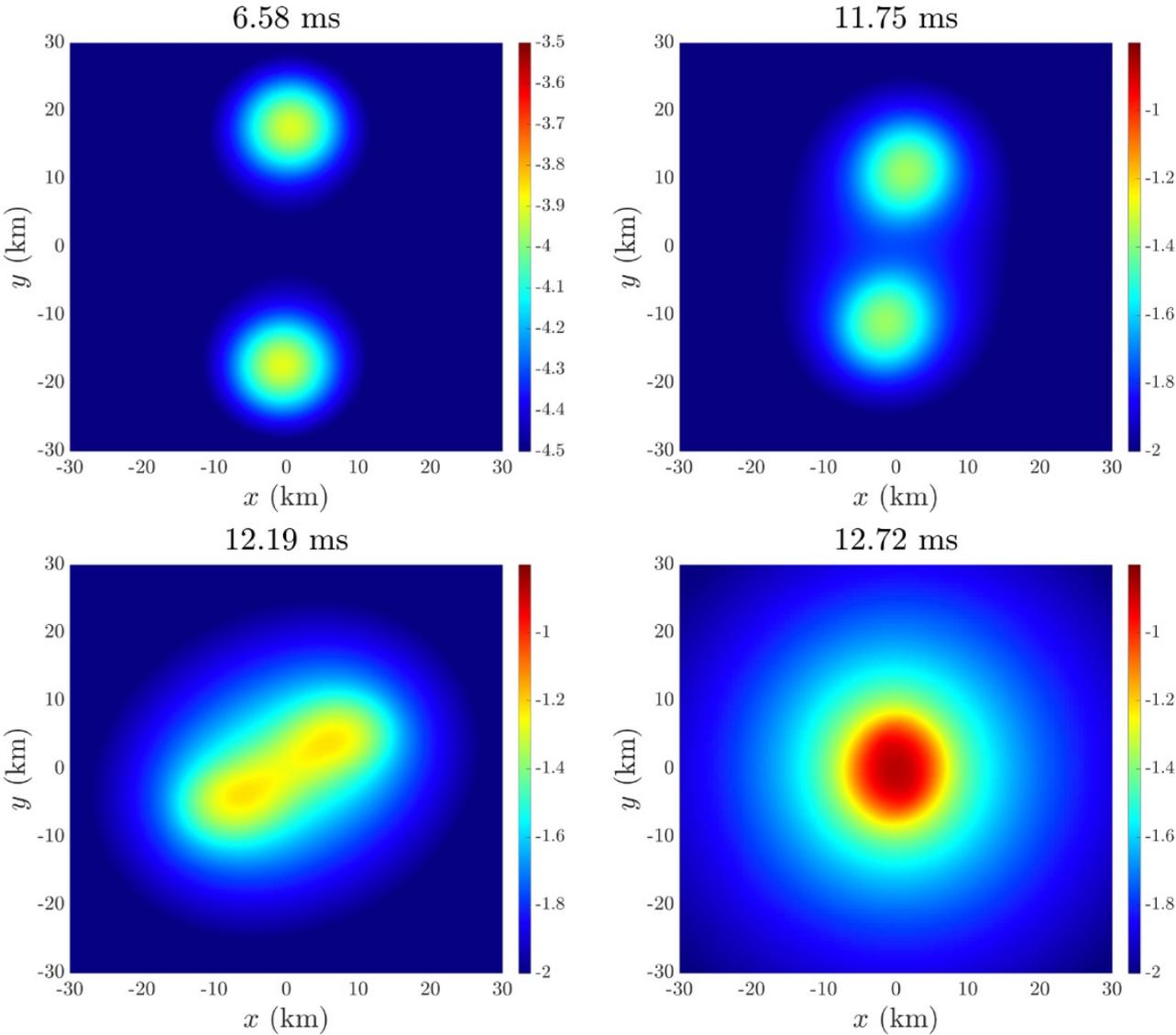
# Effective power-spectral density

- Spherically symmetric perturbations  $\Rightarrow$  emission of **breathing modes**
- Observational period 2 months, distance 10kpc



Kuan, Suvorov, DD, Yazadjiev, PRL (2022)

# Binary neutron star mergers

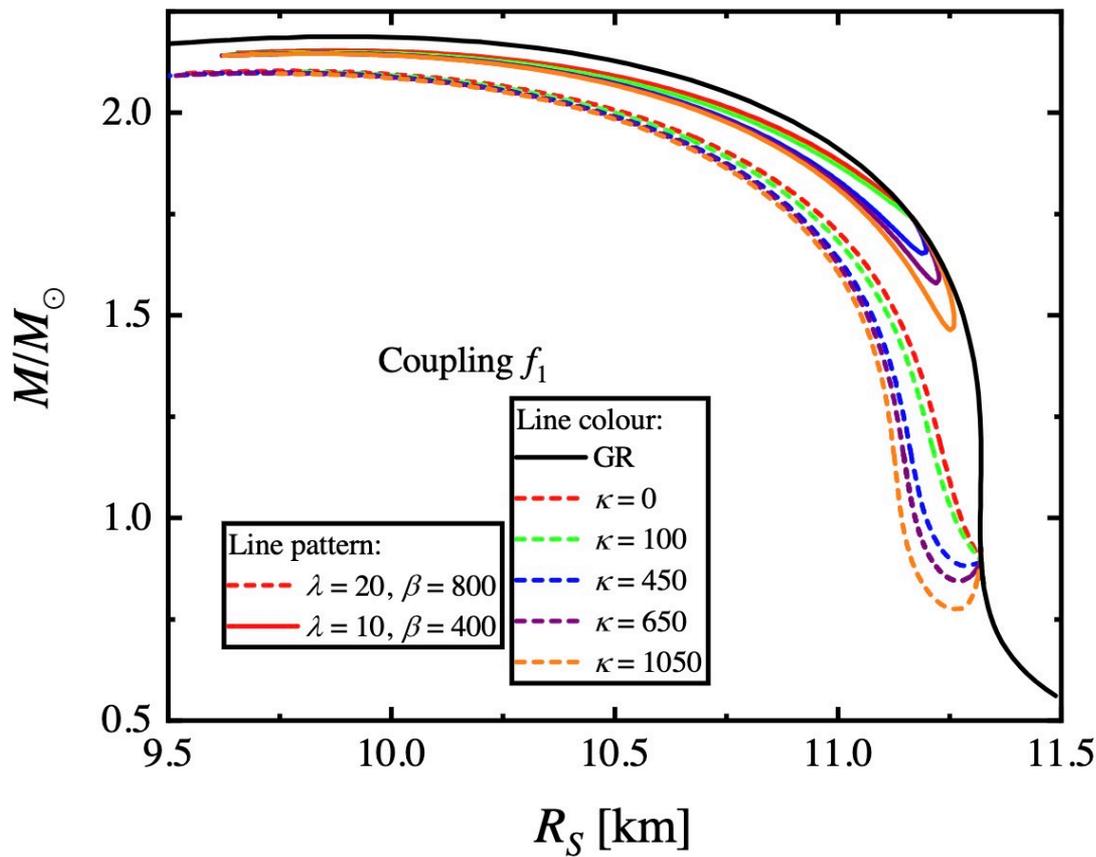


Kuan, Lam, DD, Yazadjiev, Shibata, Kiuchi (2023)

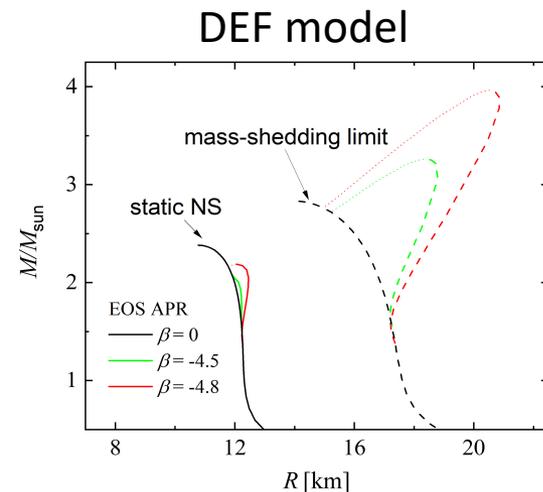
# NS scalarization in Gauss-Bonnet gravity

- Scalar field triggered by the curvature itself through  $R_{GB}^2$

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - 2\nabla_\mu \varphi \nabla^\mu \varphi + \lambda^2 f(\varphi) \mathcal{R}_{GB}^2 \right] + S_{\text{matter}}(g_{\mu\nu}, \chi)$$



$$f_1(\varphi) = \frac{1}{2\beta} \left( 1 - e^{-\beta(\varphi^2 + \kappa\varphi^4)} \right)$$



DD, Yazadjiev JCAP (2019), Staykov et al (in prep.)

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# EMRIs - inverse chirp signal

# Supermassive black holes beyond GR

## Kerr black holes with scalar hair

- A minimally coupled **complex massive scalar field**  $\Phi$

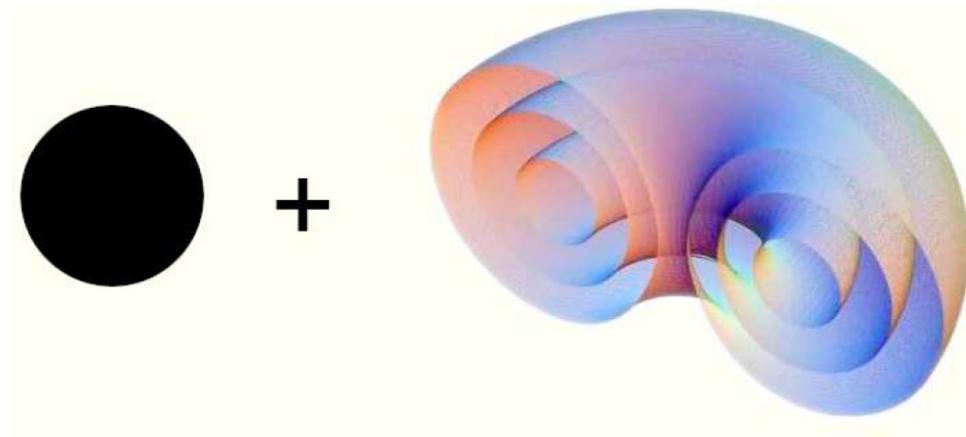
$$S = \int \left[ \frac{R}{2} - g^{\mu\nu} \partial_\mu \Phi^* \partial_\nu \Phi - 2U(\Phi) \right] \sqrt{-g} d^4x, \quad \text{with} \quad U = \frac{1}{2} \mu^2 |\Phi|^2$$

- Scalar field **NOT** stationary and axisymmetric (similar to boson star)

$$\Phi = \phi(r, \theta) e^{i(\omega t + m\varphi)}$$

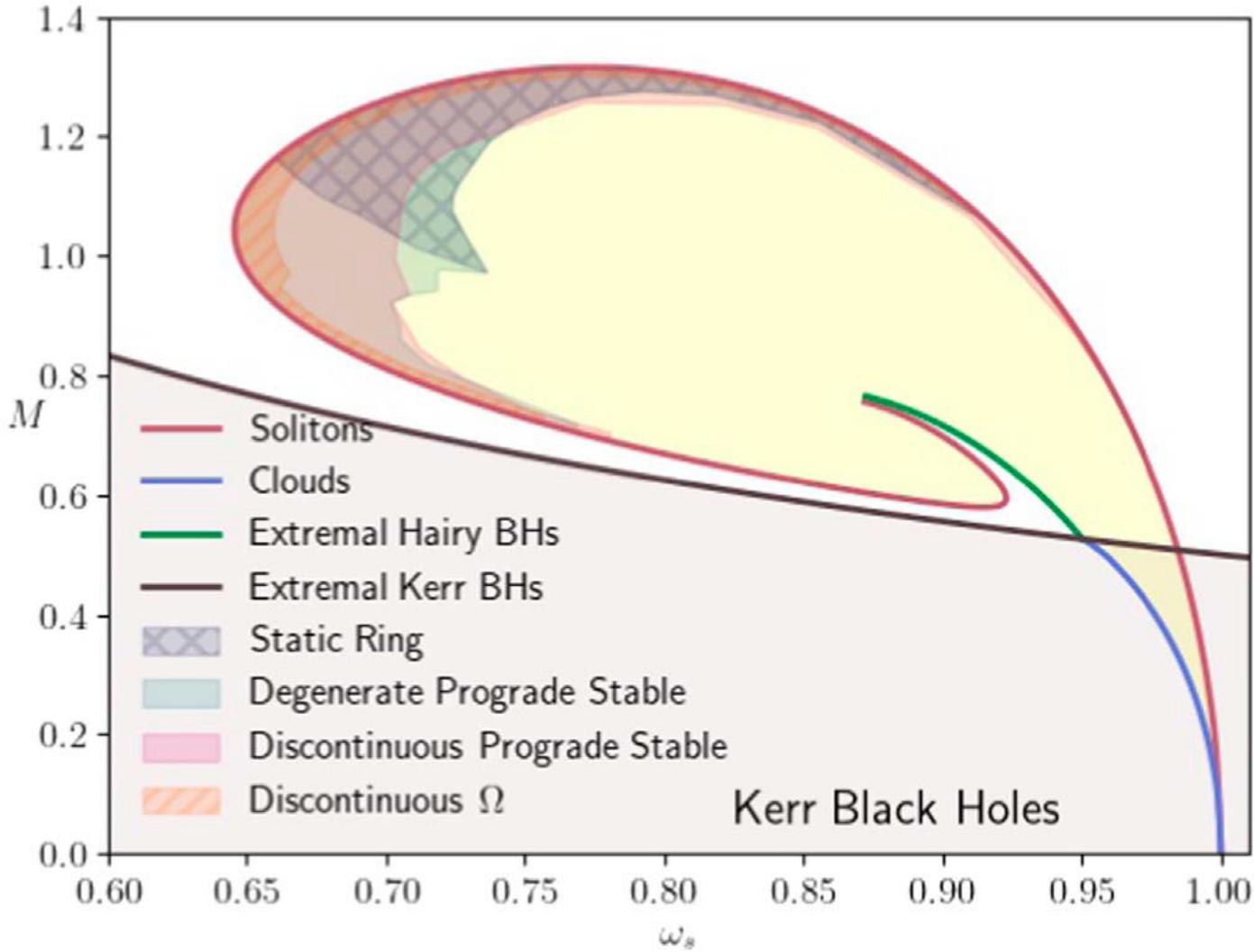
- The **Noether charge**  $\rightarrow$  number of particles.

- The scalar field forms a **torus**  
(similar to rotating boson stars)



Herdeiro, Radu PRL (2015)

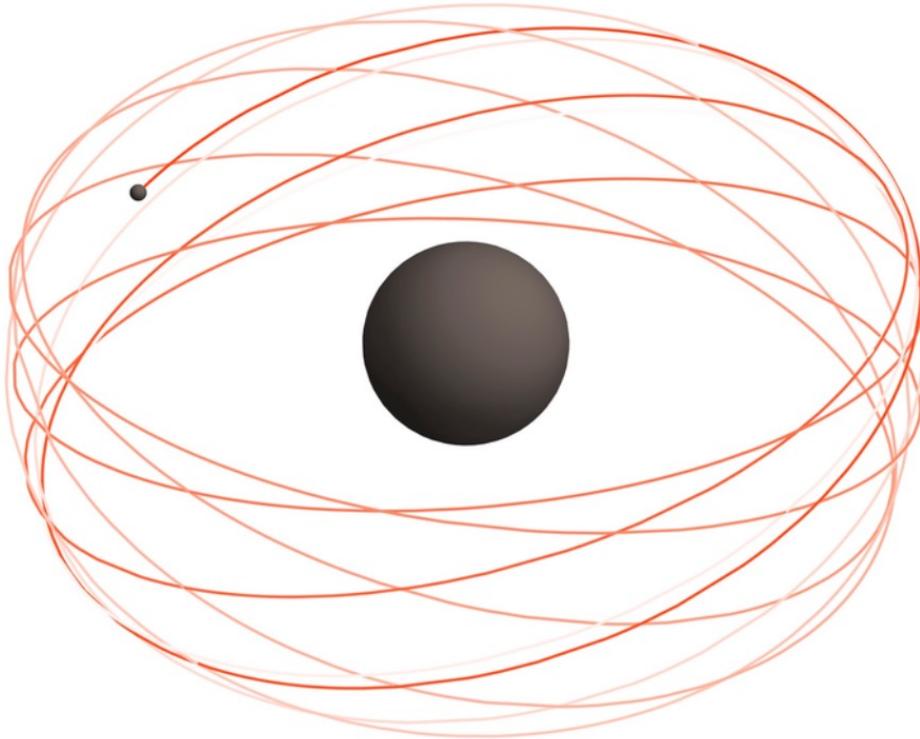
# Circular orbits structure



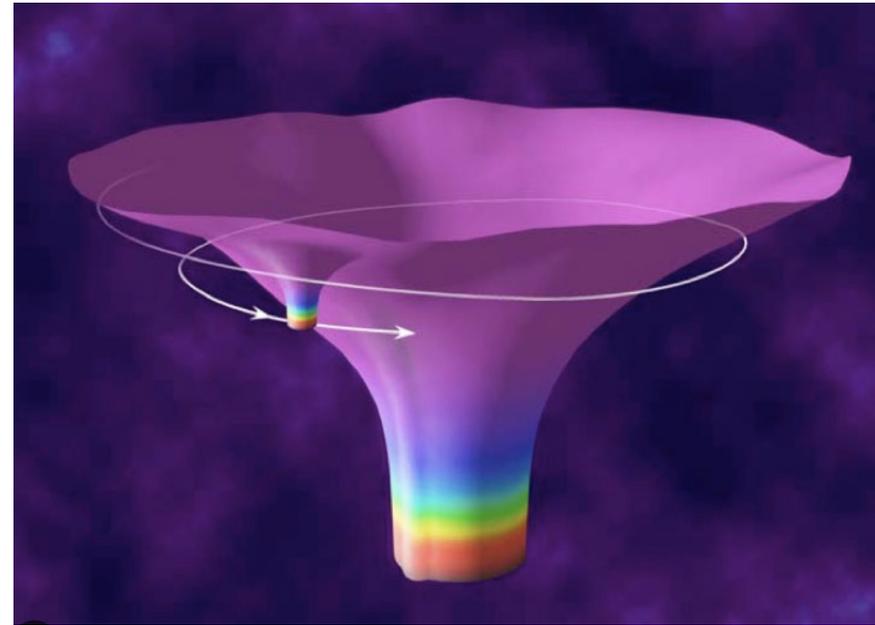
Collodel, DD, Yazadjiev PRD (2021, 2022)

# Extreme mass-ratio inspiral

- A small object (e.g. a black hole) orbiting a massive black
- Can be observed with LISA
- A perfect way to “feel” the geometry of spacetime

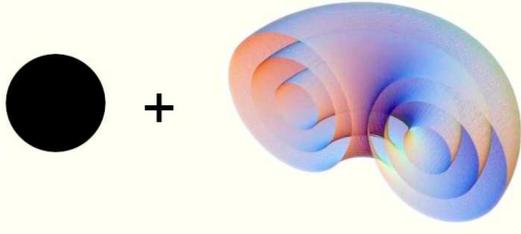


CREDIT: N. FRANCHINI

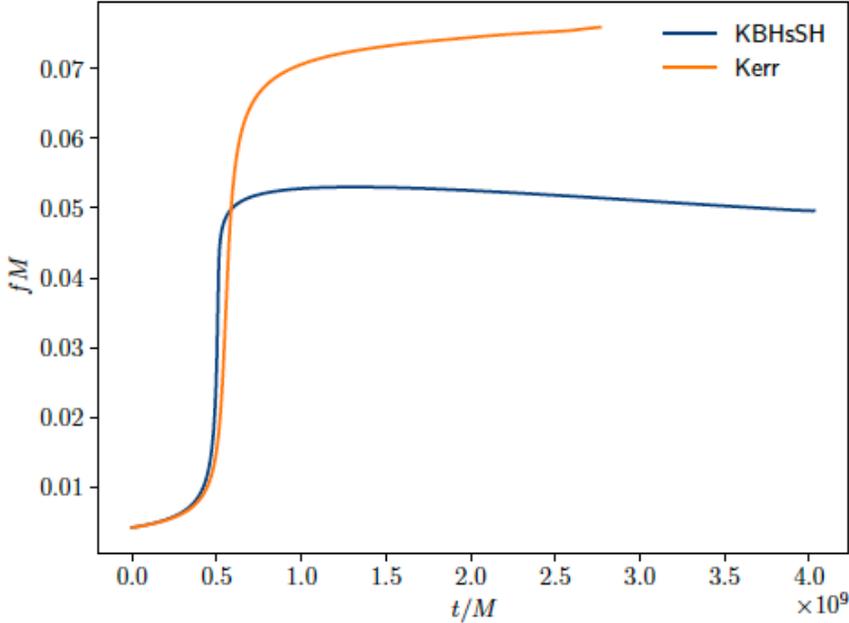
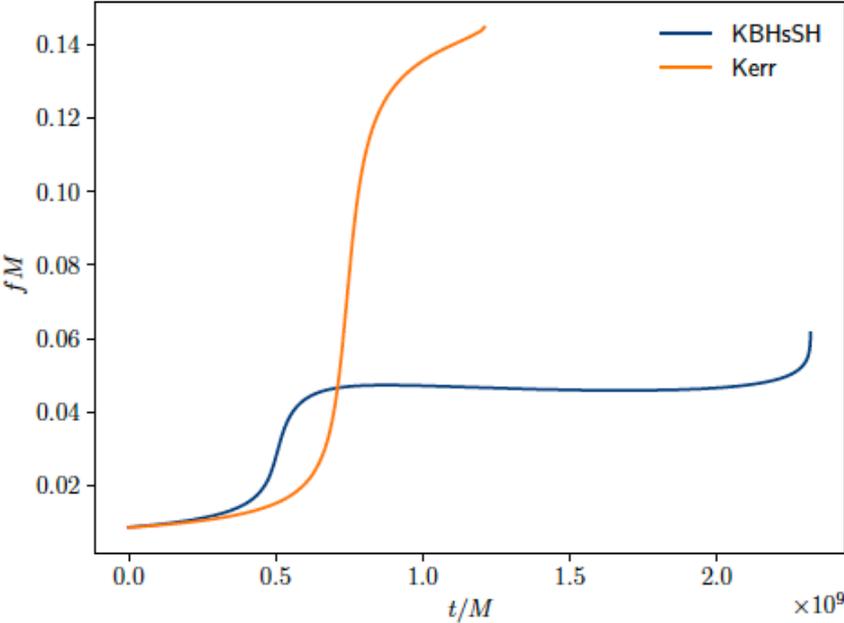


# Extreme mass-ratio inspiral

## Kerr BH + Rotating Boson Star



Herdeiro&Radu, RPL (2014)



Collodel, DD, Yazadjiev PRD(2021)

# Conclusions

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- GWs are among the ultimate tools to test beyond-GR physics
- **Quantitative vs. Qualitative** – tracing the smoking guns
- **Jumps (Gravitational phase transitions)** in the equilibrium properties  $\Rightarrow$  specifics in the GW signal
- **Final goal:** understand which exotics are physically motivated and constrain them via GWs.

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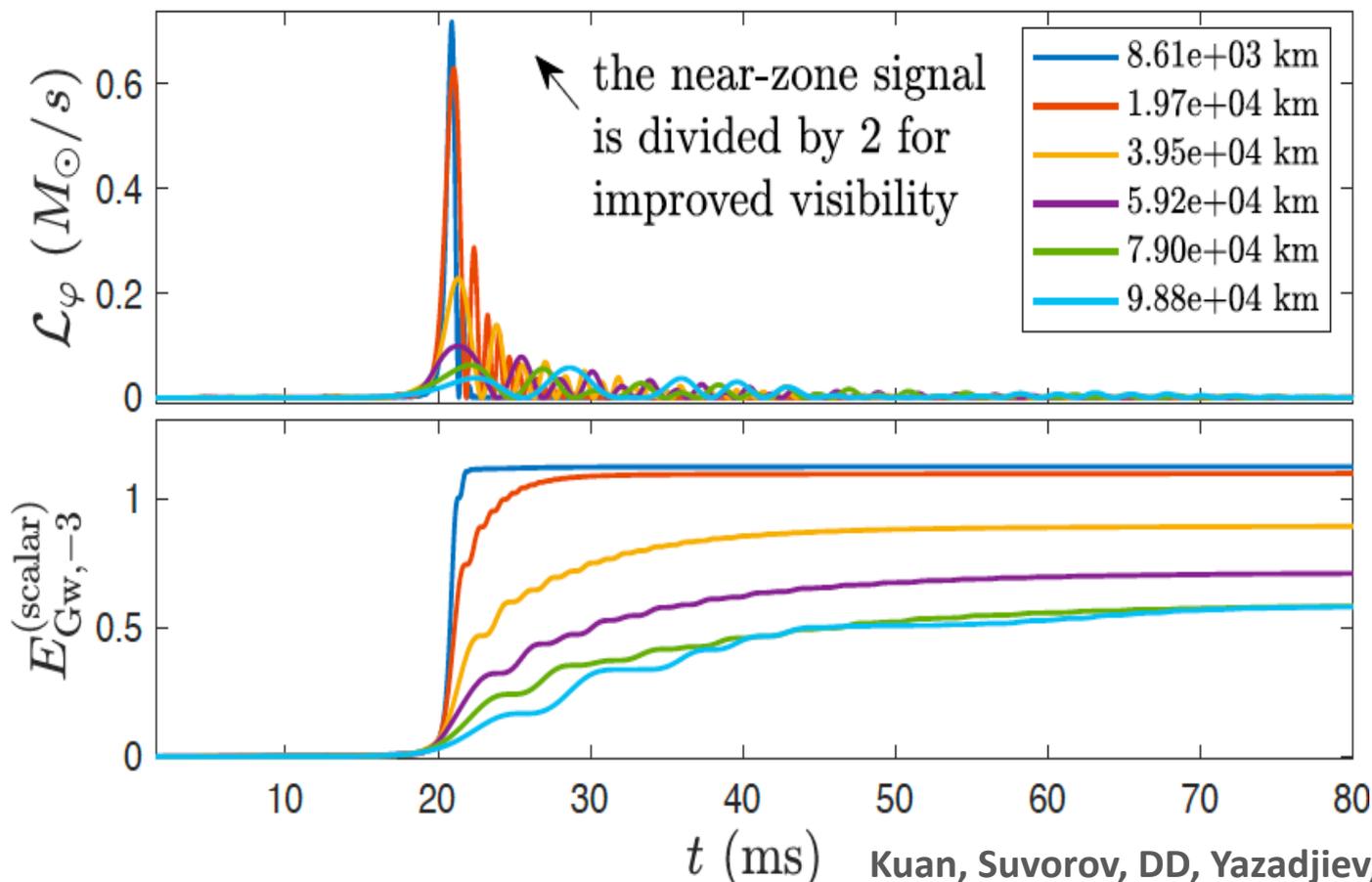
**THANK YOU!**

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# Scalar radiation

- **Massive scalar field:** Modes with distinct frequencies propagate at **different subluminal velocities**
- A dispersively stretched burst



Kuan, Suvorov, DD, Yazadjiev, PRL (2022)