From the Flamm-Einstein-Rosen bridge to the modern renaissance of traversable wormholes

Francisco S. N. Lobo

Instituto de Astrofísica e Ciências do Espaço (IA),

Faculdade de Ciências da Universidade de Lisboa

Online seminar [GR-QC-Cosmo-Astro]

5th October 2023

UIDB/04434/2020, UIDP/04434/2020,

CERN/FIS-PAR/0037/2019

Outline of the talk

- History:
  - The Flamm paraboloid;
  - The Einstein-Rosen bridge;
  - Geons and Spacetime foam
- Renaissance and Basics:
  - Embedding diagrams;
  - Exotic matter;
  - Violation of the Energy Conditions;
  - Minimizing the above violations, etc
- Wormholes in modified gravity:
  - f(R) gravity;
  - Curvature-matter couplings;
  - Braneworlds,
  - Hybrid metric-Palatini gravity
- Closed timelike curves and associated paradoxes.
- Conclusions

1. The Flamm-Einstein-Rosen bridge

- Wormhole physics can originally be tentatively traced back to Flamm in 1916, where his aim was to render the conclusions of the Schwarzschild solution in a clearer manner.

- Einstein and Rosen (ER), in 1935, were attempting to build a geometrical model of a physical elementary "particle" that is finite and singularity-free.

- Based their discussion in terms of a "bridge" across a double-sheeted physical space:
  - "… these solutions involve the mathematical representation of physical space by a space of two identical sheets, a particle being represented by a "bridge" connecting these sheets." ("The particle problem in general relativity", Phys.Rev. 1935)

- ER discussed two specific types of bridges, neutral and quasicharged. However, these can easily be generalized.

- Note that at the time ER were writing, the notions of "coordinate singularity" and "physical singularity" had not been cleanly separated.
  - To many physicists the event horizon *was* the singularity.

1.1. The neutral Einstein-Rosen bridge

- The neutral Einstein-Rosen bridge is an observation that a suitable coordinate change seems to make the Schwarzschild (coordinate) singularity disappear.

- In modern language: ER discovered that some coordinate systems naturally cover only two asymptotically flat regions of the maximally extended Schwarzschild spacetime.

- Consider the ordinary Schwarzschild geometry:

$$ds^2 = -\left(1 - 2M/r\right)dt^2 + \frac{dr^2}{1 - 2M/r} + r^2\, d\Omega^2$$

- Consider a coordinate change $u^2 = r - 2M$, this can be represented into the ER form:

$$ds^2 = -\frac{u^2}{u^2 + 2M}dt^2 + 4\left(u^2 + 2M\right)du^2 + \left(u^2 + 2M\right)^2\, d\Omega^2$$

with $u \in \left(-\infty, +\infty\right)$. This coordinate change discards the region containing the curvature singularity $r \in [0, 2M)$

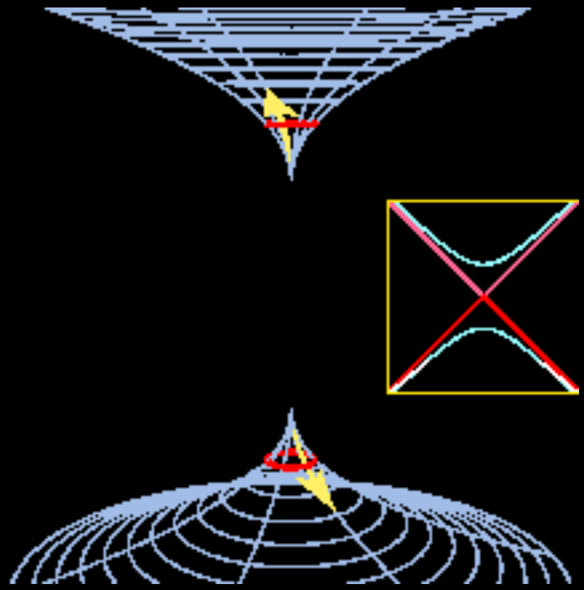
The region near $u = 0$ is interpreted as a “bridge” connecting the asymptotically flat region near $u = +\infty$ with the asymptotically flat region near $u = -\infty$.

- **To justify the “bridge” appellation, consider a spherical surface, with constant u.**
  - **The area of the surface is: $A(u) = 4\pi(2M+u^2)^2$.**
  - **The area has a minimum at u=0, with $A(0)= 4\pi(2M)^2$.**
- **One defines the narrowest part of the geometry as the “throat”, while the nearby region is denoted the bridge (or in modern terminology a “wormhole”).**

- **To justify the "bridge" appellation, consider a spherical surface, with constant u.**
  - **The area of the surface is: $A(u) = 4\pi(2M+u^2)^2$.**
  - **The area has a minimum at u=0, with $A(0)= 4\pi(2M)^2$.**
- **One defines the narrowest part of the geometry as the "throat", while the nearby region is denoted the bridge (or in modern terminology a "wormhole").**

- The net result is that the neutral "Einstein-Rosen" bridge (aka as the "Schwarzschild wormhole") is identical to a part of the maximally extended Schwarzschild geometry.

- Non-traversable: The throat will pinch off before an observer may traverse the throat.



1.2. The quasicharged Einstein-Rosen bridge

- Start with the Reissner-Nordstrom geometry:

$$ds^2 = -\left(1 - 2M/r + Q^2/r^2\right)dt^2 + \frac{dr^2}{1 - 2M/r + Q^2/r^2} + r^2\, d\Omega^2$$

- To make the bridge construction work, ER found that a completely *ad hoc* mutiliation of the theory was necessary – had to reverse the sign of the electromagnetic stress-energy tensor (implies a negative energy-density).
- Considering M=0, with a coordinate change $u^2 = r^2 - \varepsilon^2/2$, results:

$$ds^2 = -\frac{u^2}{u^2 + \varepsilon^2/2}dt^2 + du^2 + \left(u^2 + \varepsilon^2/2\right)^2\, d\Omega^2$$

Very peculiar geometry!

Represents a massless, quasicharged object, with a negative energy density.

Horizon at r = ε, u = 0.

It is this object that ER wished to interpret as an “electron”.

1.3. The generalized Einstein-Rosen bridge

- Merely consider an arbitrary spherically symmetric geometry, with an event horizon (Visser):

$$ds^2 = -e^{\Phi(r)}\left(1 - b(r)/r\right)dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2\, d\Omega^2$$

- The horizon occurs at $r = r_H$, defined by the equation $b(r_H) = r_H$.
- Introduce a coordinate change $u^2 = r - r_H$, which results in:

$$ds^2 = -e^{\Phi\left(r_H + u^2\right)} \frac{r_H + u^2 - b\left(r_H + u^2\right)}{r_H + u^2} dt^2 + \frac{r_H + u^2}{r_H + u^2 - b\left(r_H + u^2\right)} du^2 + \left(r_H + u^2\right)^2 d\Omega^2$$

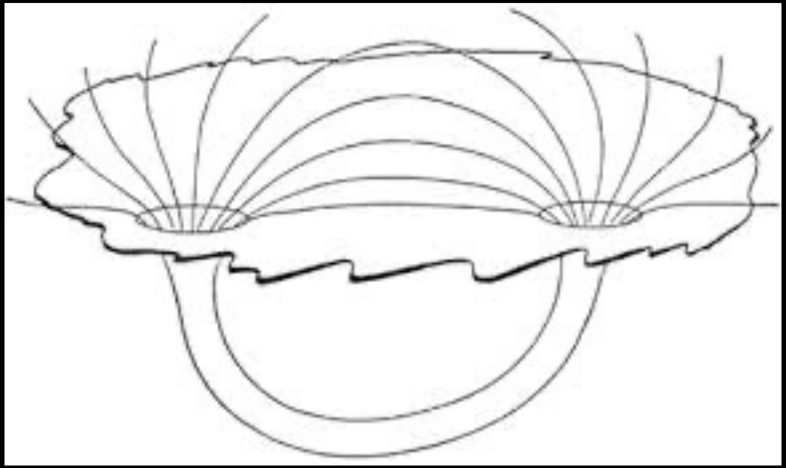
- The region near $u = 0$ is the bridge connecting the asymptotically flat regions.
- Near the bridge/horizon one has $r \approx r_H$, and $u \approx 0$, so that:

$$ds^2 = -e^{\Phi(r_H)} \frac{u^2\left[1 - b'\left(r_H\right)\right]}{r_H} dt^2 + 4\frac{r_H + u^2}{1 - b'\left(r_H\right)} du^2 + \left(r_H + u^2\right)^2 d\Omega^2$$

- Metric qualitatively analogous to the previous neutral and quasicharged bridges.
- **Key ingredient** of the bridge construction is the existence of an event horizon.
- **IMPORTANT**: The ER bridge is a coordinate artifact arising from choosing a coordinate patch, which is defined to double-cover the asymptotically flat region exterior to the BH event horizon

2. *Geons* and *Spacetime foam*

- After the pioneering work by Einstein and Rosen, in 1935, the field lay dormant for a good 20 years.
- Interest in systems of this nature was rekindled in 1955 by John Wheeler, who was beginning to be interested in topological issues in General Relativity. Citing his paper on "Geons":
  - "One can consider a metric which on the whole is nearly flat except in two widely separated regions, where a double connectedness comes into evidence as symbolized in Fig…"



- The “geon” concept was coined by Wheeler to denote a “gravitational-electromagnetic entity”.
- Geons are hypothesized unstable but long lived solutions to the coupled Einstein-Maxwell field equations. (In modern language the geon may best be thought of as a hypothetical “unstable gravitational-electromagnetic quasisoliton”).

- Furthermore, Wheeler’s concept can be used as the basis for building a model of nonzero charge with an everywhere “zero charge density”!
  - “The general divergence free electromagnetic disturbance holding sway in the space around one of these “tunnel mouths” will send forth lines of force into the space, and appear to have a charge. However, an equal number of lines of force must enter into the region of disturbance from the tunnel. Consequently the other mouth of the tunnel must manifest an equal and opposite charge.”

- Two routes now available:
  - Investigate the classical dynamics of these tunnel configurations, assuming their possible existence;
  - Investigation of the quantum gravitational processes that might give rise to such configurations (this led Wheeler to the tenacious concept of “spacetime foam”).

First route: “Classical Physics as Geometry” (The “*already* unified field theory”)

- Misner and Wheeler (Annals Phys. 1957) presented a *tour de force* wherein Riemannian geometry of manifolds of nontrivial topology was investigated with an ambitious view to explaining *all* of physics.

- This extremely ambitious idea was one of the very first uses of abstract topology, homology, cohomology, and differential geometry in physics.

- Their point of view is best summarised by their phrase:

  “Physics is geometry”.

  More specifically, the existing well-established “already unified classical theory” allows one to describe in terms of empty curved space:

  - Gravitation without gravitation;
  - Electromagnetism without electromagnetism;
  - Charge without charge;
  - Mass without mass (around the mouth of the “wormhole” lies a concentration of eletromagnetic energy that gives mass to this region of space).

First route: “Classical Physics as Geometry” (The “*already* unified field theory”)

- This is the first paper (Misner&Wheeler, Annals Phys. 1957) that introduces the term “wormhole” to the scientific community:
  - “… there is a net flux of lines of force through what topologists would call a handle of the multiply-connected space and what physicists might perhaps be excused for more vividly terming a *wormhole*.”

- Ultimately, the aim was to use the source-free Maxwell equations, coupled to Einstein gravity, with the seasoning of nontrivial topology, to build models for classical electrical charges and all other particle-like entities in classical physics.

- It is now known that this classical conception of “charge without charge”, etc, cannot in fact work as originally conceived – classically the tunnels will collapse to form black holes, and the otherwise interesting topology is hidden behind event horizons.

Second route: *Spacetime foam* (Quantum fluctuations in the spacetime metric)

- Wheeler: the overwhelming importance of the Planck scale in gravitational physics. At distances below the Planck length quantum fluctuations are so large, that linearized theory breaks down irretrievably and the quantum physics of full nonlinear Einstein gravity must be faced.
  - "…. On the atomic scale the metric appears flat, as does the ocean to an aviator far above. The closer the approach, the greater the degree of irregularity. Finally, at distances of the order of l_P, the fluctuations in the typical metric component, $g_{\mu\nu}$, become of the same order as the metric components themselves."

- Once the metric fluctuations become nonlinear and strongly interacting, the analogy with the surface of the ocean suggests that the metric fluctuations might "break" in a manner analogous to the breaking of ocean waves.

- This might be expected to endow spacetime with a "foamlike" structure. Wheeler:
  - "… space resonates between one foamlike structure and another".

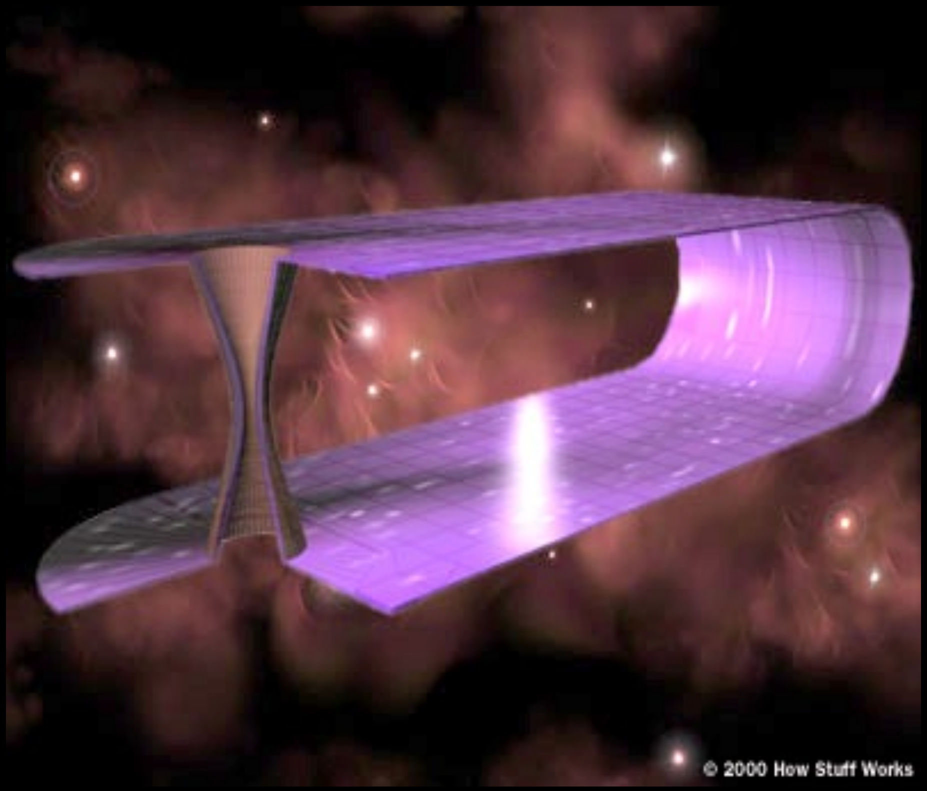
- More succinctly, one often encounters the phrase "spacetime foam" which refers to Wheeler's suggestion that the geometry, and topology, of space might be constantly fluctuating. (Topology change???)

3. Interregnum and Renaissance

- 30 year gap between Wheeler's (and Misner's) 1955/1957 work and the 1988 Morris-Thorne renaissance of wormhole physics.
- Much was accomplished during this period, but little effort was devoted to Lorentzian wormholes.
- Considerable effort was invested in topics as attempting to understand the "spacetime foam" picture and the "geon" concept.
- **During this period Lorentzian wormholes seem to have been considered a curiosity and relegated to a back stage. Isolated pieces of work do appear, such as the Homer Ellis' drainhole concept and Kirill Bronnikov's tunnel-like solutions, in the early 1970s.**
- However, it is only in 1988 that a full-fledged renaissance of wormhole physics took place … and is still in full flight.

4. Traversable wormholes

- Approach: What do the laws of physics permit?
- Wormholes: Useful primarily as a theoretician's probe of the foundations of general relativity.
- Very interesting physics involved!
- Pedagogical purposes: Useful for teaching General Relativity (Morris &Thorne, AJP, 56, 395, 1988)



4.1. Approach in solving the Einstein field equations (EFE)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

- General Relativity (GR) has been an extremely successful theory, with a well established experimental footing, at least for weak gravitational fields.
- It's predictions range from the existence of black holes, gravitational radiation (now confirmed) to the cosmological models.
- All these solutions have been obtained by first considering plausible distributions of matter, i.e., a plausible stress-energy tensor, and through the EFE, the spacetime metric of the geometry is determined.
- **Reverse philosophy**: solve the EFE in the reverse direction, namely, one first considers an interesting metric, then finds the matter source responsible for the respective geometry.

4.1.1 Spacetime metric:

$$ds^2 = -e^{2\Phi(r)}\, dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2)$$

- b(r): shape function – determines the shape of the wormhole
- Φ(r): redshift function – related to the gravitational redshift
- Wormhole throat, minimum radius: $r \,\square\, b(r) \,\square\, r_0$
- Impose traversability conditions (Total time in a traversal, acceleration and tidal accelerations felt by a traveller) !!

Field equations:

$$\begin{aligned}
\rho(r) &= \frac{1}{8\pi}\,\frac{b'}{r^2}\,, \\
\tau(r) &= \frac{1}{8\pi}\left[\frac{b}{r^3} - 2\left(1-\frac{b}{r}\right)\frac{\Phi'}{r}\right], \\
p(r) &= \frac{1}{8\pi}\left(1-\frac{b}{r}\right)\left[\Phi'' + (\Phi')^2 - \frac{b'r-b}{2r^2(1-b/r)}\Phi' - \frac{b'r-b}{2r^3(1-b/r)} + \frac{\Phi'}{r}\right].
\end{aligned}$$

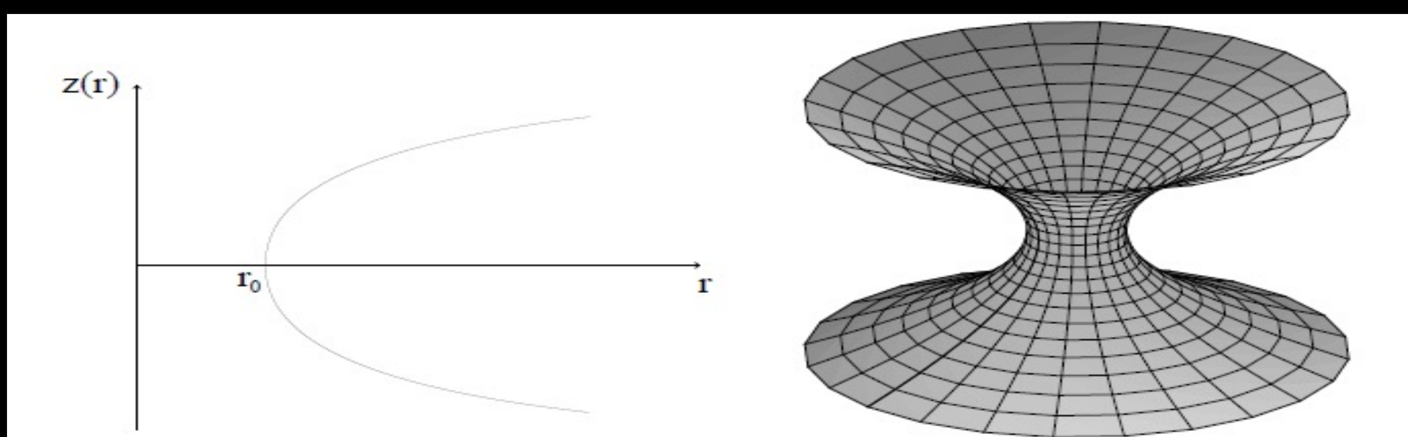
4.1.2 Mathematics of embedding:

- Due to spherical symmetry, consider an equatorial plane θ□π□.
  Fixed time slice: t=const
- **To visualise this slice:** Embed this metric into 3-d Euclidean space in cylindrical coordinates (r, ϕ, z), with surface equation z=z(r)
- Equation for the embedding surface:

$$\frac{dz}{dr} = \pm \left(\frac{r}{b(r)} - 1 \right)^{-1/2} .$$

- **Fundamental ingredient: Flaring-out condition!!**

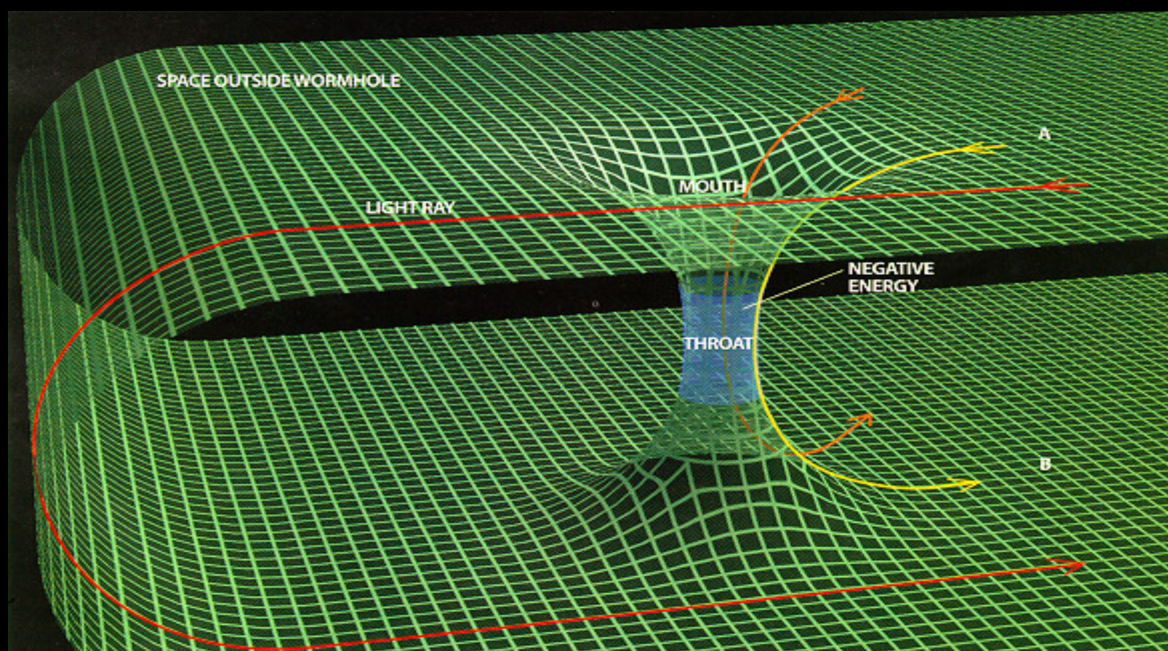
$$\frac{d^2 r}{dz^2} = \frac{b - b' r}{2b^2} > 0 \, .$$



4.1.3 Exotic matter

- **The flaring-out condition entails the violation of the NULL ENERGY CONDITION (NEC) at the throat:**
  (negative energy densities not essential)

$$\rho \,\square\, p_r \,\square\, 0 \;!!$$



4.1.3 Exotic matter

• **The flaring-out condition entails the violation of the NULL ENERGY CONDITION (NEC) at the throat:**
(negative energy densities not essential)

$$\rho + p_r < 0 \,!!$$

- **Note that the null energy condition arises when one refers back to the Raychaudhuri equation: the positivity condition of the expansion term** $R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ **appears;**

- **In GR, through the EFE the positivity condition reflects the null energy condition:** $T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$

4.1.3 Exotic matter

- **The flaring-out condition entails the violation of the NULL ENERGY CONDITION (NEC) at the throat:**
(negative energy densities not essential)

$$\rho \square p_r \square 0 \,!!$$

- **In fact, it violates all of the pointwise energy conditions:**

$$1.\ \mathrm{NEC}: \rho \square p_r \geq 0$$

$$2.\ \mathrm{WEC}: \rho \geq 0,\ \rho \square p_r \geq 0$$

$$3.\ \mathrm{SEC}: \rho \square p_r \geq 0,\ \rho \square p_r \square 2 p_t \geq 0$$

$$4.\ \mathrm{DEC}: \rho \geq |p_r|$$

- **And the averaged energy conditions!**
- **(Quantum inequalities)**

4.1.3 Exotic matter

- **The flaring-out condition entails the violation of the NULL ENERGY CONDITION (NEC) at the throat:** (negative energy densities not essential)

$$\rho \; \square \; p_r \; \square \; 0 \; !!$$

As the violation of the energy conditions is a problematic issue, it is useful to minimize this violation.

Several approaches:

- ❑ **Rotating solutions;**
- ❑ **Evolving wormhole spacetimes;**
- ❑ **Thin-shell wormholes: cut-and-paste procedure;**
- ❑ **Modified theories of gravity.**

4.2.1. Rotating wormhole solutions (very messy!!)

Metric:

$$ds^2 = -N^2 dt^2 + e^{\mu} dr^2 + r^2 K^2 \left[d\theta^2 + \sin^2\theta (d\varphi - \omega dt)^2 \right]$$

EFE at the throat:

$$\begin{aligned}
8\pi T_{\hat{t}\hat{t}} &= -\frac{(K_\theta \sin\theta)_\theta}{r^2 K^3 \sin\theta} - \frac{\omega_\theta^2 \sin^2\theta}{4N^2} + e^{-\mu}\,\mu_r \frac{(rK)_r}{rK} + \frac{K^2 + K_\theta^2}{r^2 K^4}\,, \\
8\pi T_{\hat{r}\hat{r}} &= \frac{(K_\theta \sin\theta)_\theta}{r^2 K^3 \sin\theta} - \frac{\omega_\theta^2 \sin^2\theta}{4N^2} + \frac{(N_\theta \sin\theta)_\theta}{N r^2 K^2 \sin\theta} - \frac{K^2 + K_\theta^2}{r^2 K^4}\,, \\
8\pi T_{\hat{\theta}\hat{\theta}} &= \frac{N_\theta (K \sin\theta)_\theta}{N r^2 K^3 \sin\theta} + \frac{\omega_\theta^2 \sin^2\theta}{4N^2} - \frac{\mu_r\, e^{-\mu} (NrK)_r}{2NrK}\,, \\
8\pi T_{\hat{\phi}\hat{\phi}} &= -\frac{\mu_r\, e^{-\mu}\,(NKr)_r}{2NKr} - \frac{3\sin^2\theta\,\omega_\theta^2}{4N^2} + \frac{N_{\theta\theta}}{N r^2 K^2} - \frac{N_\theta K_\theta}{N r^2 K^3}\,, \\
8\pi T_{\hat{t}\hat{\phi}} &= \frac{1}{4N^2 K^2 r} \Big(6NK\,\omega_\theta \cos\theta + 2NK \sin\theta\,\omega_{\theta\theta} \\
&\quad - \mu_r e^{-\mu} r^2 N K^3 \sin\theta\,\omega_r + 4N\,\omega_\theta \sin\theta\, K_\theta - 2K \sin\theta\, N_\theta\, \omega_\theta \Big)\,.
\end{aligned}$$

NEC violation
(exotic matter confined to certain regions):

$$8\pi\, T_{\hat{\mu}\hat{\nu}} k^{\hat{\mu}} k^{\hat{\nu}} = e^{-\mu} \mu_r \frac{(rK)_r}{rK} - \frac{{\omega_\theta}^2 \sin^2\theta}{2N^2} + \frac{(N_\theta \sin\theta)_\theta}{(rK)^2 N \sin\theta}\,.$$

4.2.2 Evolving wormholes in a cosmological background

Metric:

$$ds^2 = \Omega^2(t)\left[-e^{2\Phi(r)}\,dt^2 + \frac{dr^2}{1 - kr^2 - \frac{b(r)}{r}} + r^2\,\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right]$$

EFE:

$$\rho(r,t) = \frac{1}{8\pi}\frac{1}{\Omega^2}\left[3e^{-2\Phi}\left(\frac{\dot{\Omega}}{\Omega}\right)^2 + \left(3k + \frac{b'}{r^2}\right)\right],$$

$$\tau(r,t) = -\frac{1}{8\pi}\frac{1}{\Omega^2}\left\{e^{-2\Phi(r)}\left[\left(\frac{\dot{\Omega}}{\Omega}\right)^2 - 2\frac{\ddot{\Omega}}{\Omega}\right] - \left[k + \frac{b}{r^3} - 2\frac{\Phi'}{r}\left(1 - kr^2 - \frac{b}{r}\right)\right]\right\}$$

$$f(r,t) = -\frac{1}{8\pi}\left[2\frac{\dot{\Omega}}{\Omega^3}\,e^{-\Phi}\,\Phi'\left(1 - kr^2 - \frac{b}{r}\right)^{1/2}\right],$$

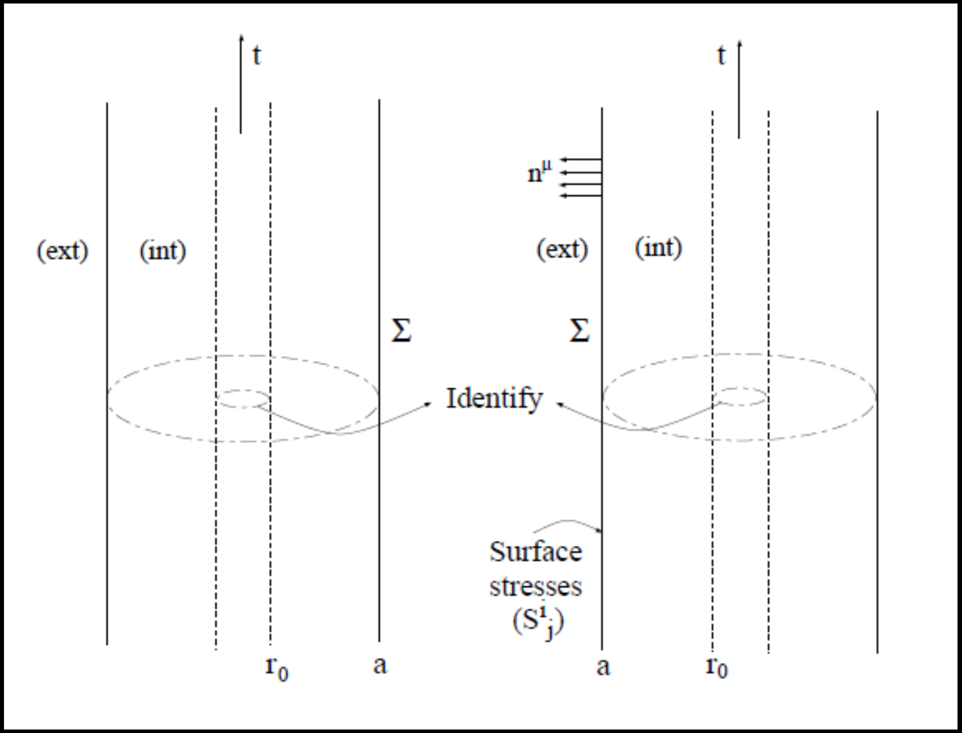
$$p(r,t) = \frac{1}{8\pi}\frac{1}{\Omega^2}\left\{e^{-2\Phi(r)}\left[\left(\frac{\dot{\Omega}}{\Omega}\right)^2 - 2\frac{\ddot{\Omega}}{\Omega}\right] + \left(1 - kr^2 - \frac{b}{r}\right)\times\right.$$
$$\left.\times\left[\Phi'' + (\Phi')^2 - \frac{2kr^3 + b'r - b}{2r(r - kr^3 - b)}\Phi' - \frac{2kr^3 + b'r - b}{2r^2(r - kr^3 - b)} + \frac{\Phi'}{r}\right]\right\}.$$

Possible to show the existence of NEC violation flashes in time!

4.2.3 Thin-shell wormholes: Cut-and-paste procedure

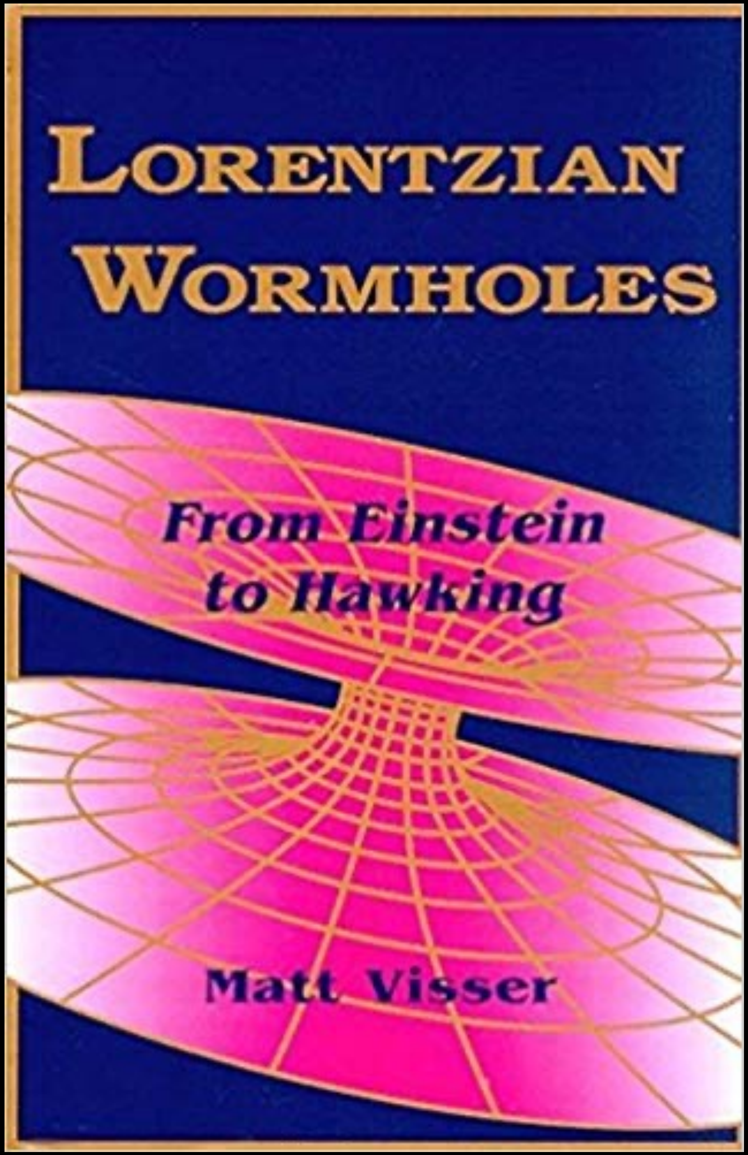
Thin-shells: matching conditions

Match an interior wormhole solution to a exterior vaccum spacetime:



Garcia, FL & Visser, PRD 2012; Martin-Moruno, Garcia, FL & Visser, JCAP 2012;
Bouhmadi-Lopez, FL, Martin-Moruno, JCAP 2014

Review until 1996



4.3. Modified theories of gravity:

Higher order actions may include various curvature invariants, such as:

$$R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, etc.$$

Consider f(R) gravity, for simplicity:

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa} f(R) + L_m(g^{\mu\nu}, \psi)\right]$$

Appealing feature: combines mathematical simplicity and a fair amount of generality!

4.3.1. f(R) gravity

Action:

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa} f(R) + L_m(g^{\mu\nu},\psi)\right]$$

Gravitational field equations (vary action with $g^{\mu\nu}$):

$$FR_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} - \nabla_\mu \nabla_\nu F + g_{\mu\nu} \nabla_\alpha \nabla^\alpha F = \kappa T^{(m)}_{\mu\nu}, \quad F = \frac{df}{dR}$$

Effective Einstein equation: $G_{\mu\nu} = \kappa T^{eff}_{\mu\nu}$ with $T^{eff}_{\mu\nu} = \bar{T}^{(m)}_{\mu\nu} + T^{(c)}_{\mu\nu}$

$$\bar{T}^{(m)}_{\mu\nu} = T^{(m)}_{\mu\nu}/F, \quad \text{and} \quad T^{(c)}_{\mu\nu} = \frac{1}{\kappa F}\left[\nabla_\mu \nabla_\nu F - \frac{1}{4} g_{\mu\nu}\left(RF + \nabla_\mu \nabla^\mu F + \kappa T\right)\right]$$

Conservation law:

$$\nabla^\mu T^{(c)}_{\mu\nu} = \frac{1}{F^2} T^{(m)}_{\mu\nu} \nabla^\mu F$$

Energy condition violations

- In modified gravity, it is the effective Stress-Energy Tensor (SET) that comes into play and the NEC violation imposes:

$$T^{eff}_{\mu\nu} k^{\mu} k^{\nu} < 0$$

- Impose that normal matter satisfies the NEC: $T_{\mu\nu} k^{\mu} k^{\nu} \geq 0$

- Thus, it is the higher order curvature terms that are responsible for supporting the wormhole!

(Oliveira, FL, PRD 2009; PRD 2010)

4.3. Generalized modified gravity:

Consider the generalized gravitational field equations for a large class of modified theories of gravity (Harko, FL, Mak, Sushkov, PRD 2013):

$$g_1(\Psi^i)(G_{\mu\nu} + H_{\mu\nu}) - g_2(\Psi^j)\,T_{\mu\nu} = \kappa^2\,T_{\mu\nu}\,,$$

$H_{\mu\nu}$ is an additional geometric term that includes the geometrical modifications inherent in the modified gravitational theory under consideration;

$g_i\left(\psi^j\right)$ are multiplicative factors that modify the geometrical sector of the field equations;

ψ^j denote generically curvature invariants or gravitational fields such as scalar fields;

$g_2\left(\psi^j\right)$ covers the coupling of the curvature invariants or the scalar fields with the matter stress-energy tensor.

4.3. Generalized modified gravity:

Useful to rewrite the field equation as an effective Einstein field equation (Harko, FL, Mak, Sushkov, PRD 2013):

$$T_{\mu\nu}^{\rm eff} \equiv \frac{1+\bar{g}_2(\Psi^j)}{g_1(\Psi^i)} T_{\mu\nu} - \bar{H}_{\mu\nu} \,,$$

Where $\bar{g}_2\left(\psi^j\right) = g_2\left(\psi^j\right)/\kappa^2$ and $\bar{H}_{\mu\nu} = H_{\mu\nu}/\kappa^2$

Now, the violation of the generalized NEC $T^{eff}_{\mu\nu} k^{\mu} k^{\nu} < 0$ implies:

$$\frac{1+\bar{g}_2(\Psi^j)}{g_1(\Psi^i)} T_{\mu\nu} k^\mu k^\nu < \bar{H}_{\mu\nu} k^\mu k^\nu \,.$$

Specific examples:

- **Braneworlds (FL, PRD 2007):**
  - The stress energy tensor confined on the brane, threading the wormhole, is imposed to satisfy the NEC.
  - The local high-energy bulk effects and nonlocal corrections from the Weyl curvature in the bulk induce a NEC violating signature on the brane!

- **Conformal Weyl gravity (FL, CQG 2008)**

$$I_W = -\alpha \int d^4x \sqrt{-g}\, C_{\mu\nu\alpha\beta}\, C^{\mu\nu\alpha\beta} \,,$$

- f(R) gravity (Oliveira, FL, PRD 2009)
  - the higher order curvature terms that are responsible for supporting the wormhole!

- Curvature-matter couplings in f(R) gravity (Garcia, FL, PRD 2010; CQG 2011)

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa} f_1(R) + \left[1 + \lambda f_2(R)\right] L_m \right\}$$

Specific examples:

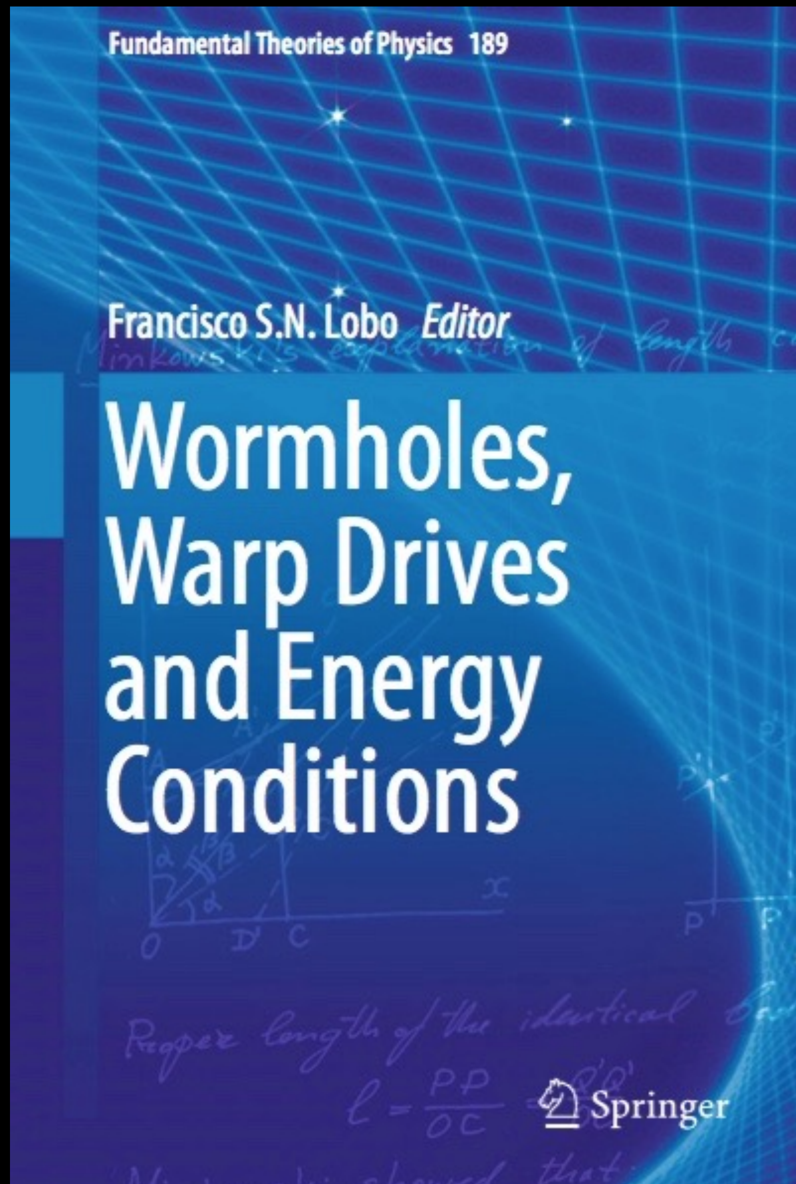
- **Brans-Dicke wormholes** (Oliveira & FL, PRD 2010; Garcia & FL, MPLA 2011)
- **Modified teleparralel gravity, f(T) gravity** (Boehmer, Harko & FL, PRD 2012)
- **Hybrid metric-Palatini gravity** (Harko, Koivisto, FL, Olmo, PRD 2012; Capozziello, Harko, Koivisto, FL, Olmo, PRD, 2012; JCAP 2013)

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,[R + f(\mathcal{R})] + S_m \ ,$$

   » Has the scalar-tensor representation:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[(1+\phi)R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + S_m \ .$$

Review until 2018



5. Closed timelike curves!!
(translation: "time travel")

- In fact, a great variety of solutions to the Einstein Field Equations (EFEs) containing CTC exist, but, two particularly notorious features seem to stand out:

1. Solutions with a tipping over of the light cones due to a rotation about a cylindrically symmetric axis;

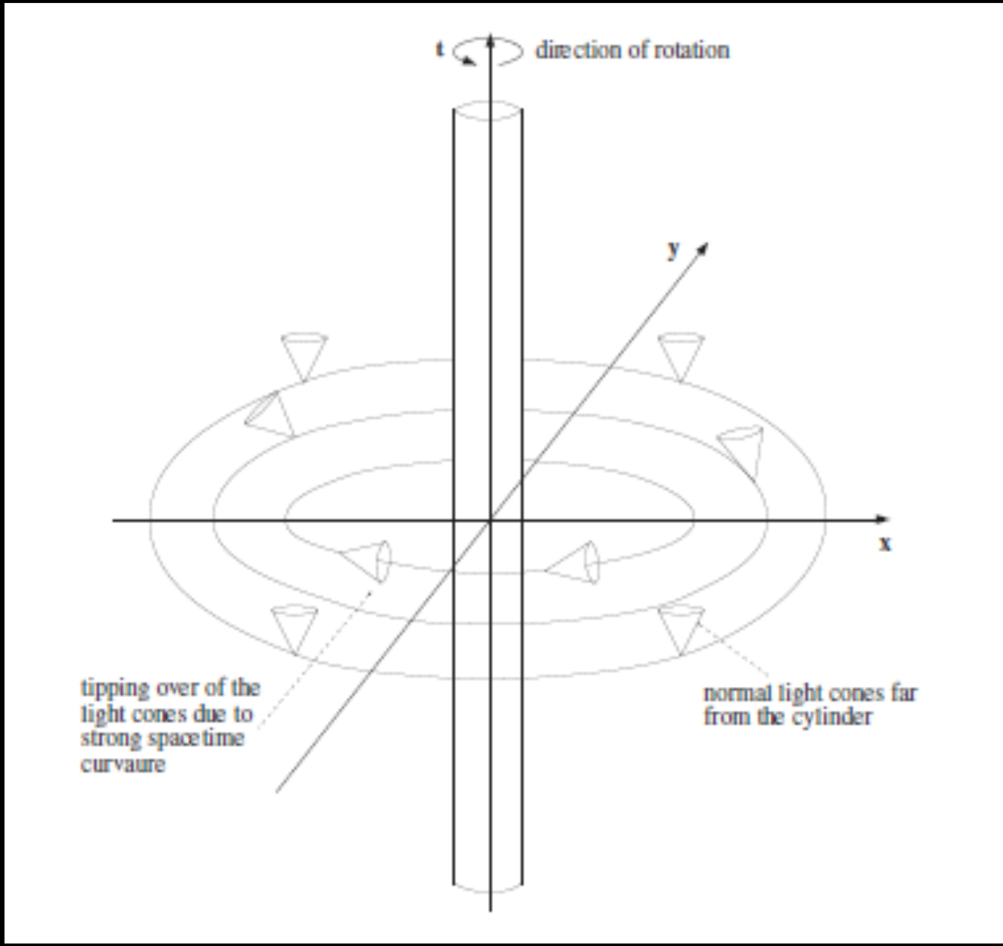
2. Solutions that violate the Energy Conditions of GR.

5.1. Rotating solutions

Van Stockum solution:

Rotating infinite dust cylinder

(tipping over of the light curves)



5.2. Wormholes induce closed timelike curves!!

Wormholes: Induce a timeshift between both mouths

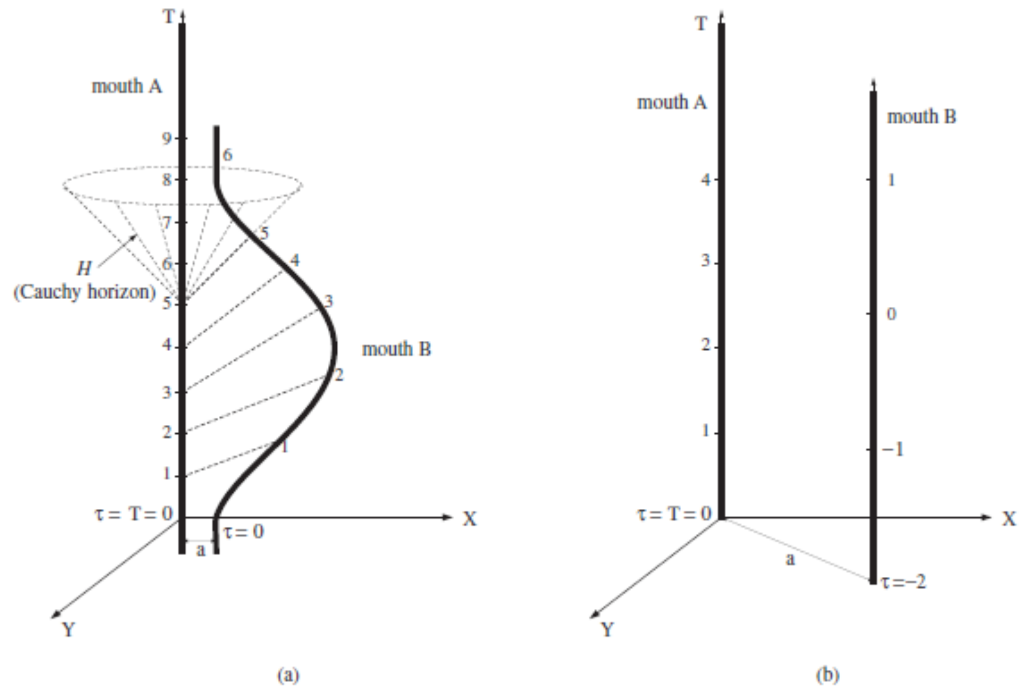


FIG. 12: Depicted are two examples of wormhole spacetimes with closed timelike curves. The wormholes tunnels are arbitrarily short, and its two mouths move along two world tubes depicted as thick lines in the figure. Proper time τ at the wormhole throat is marked off, and note that identical values are the same event as seen through the wormhole handle. In Figure (a), mouth A remains at rest, while mouth B accelerates from A at a high velocity, then returns to its starting point at rest. A time shift is induced between both mouths, due to the time dilation effects of special relativity. The light cone-like hypersurface H shown is a Cauchy horizon. Through every event to the future of H there exist CTCs, and on the other hand there are no CTCs to the past of H. In Figure (b), a time shift between both mouths is induced by placing mouth B in strong gravitational field. See text for details.

5.3. Paradoxes: Closed timelike curves!!

Opens Pandora's box and produces time travel paradoxes:

1. Causality violation (Grandfather paradox).

2. Causal loops.

Some conclusions

- Lorentzian wormholes are certainly speculative physics – there is zero(!) direct experimental evidence to support their existence

- Considerations on the energy conditions (much posterior work on energy conditions, averaged energy conditions, quantum inequalities, “volume integral quantifier”, etc)

- Gell-Mann: “Everything not forbidden is compulsory”
- However, traversable wormholes are primarily useful as a 'gedanken experiment' to explore the limitations of GR!

- Forces one to ponder seriously on the nature of time (manipulations of wormholes induce closed timelike curves)

- Fun physics!

Thank you
for your attention!