Black holes and boson stars with new scalar hair

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Collaborations and References

Work done in collaboration with

- Yves Brihaye - Université de Mons, Belgium
- Felipe Console - Universidade de São Paulo, Brazil

Literature

- Y. Brihaye, B. Hartmann, Class. Quant. Grav. **38** (2021) (Letters)
- Y. Brihaye, F. Console, B. Hartmann, Symmetry **13** (2020)
- Y. Brihaye, B. Hartmann, Class. Quant. Grav. **39** (2022)
- Y. Brihaye, B. Hartmann, Phys. Rev. D **105** 104063 (2022)

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Outline

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Why scalar fields?

- **collective phenomena** often described *effectively* by scalar fields, e.g. Ginzburg-Landau model of superconductivity [1]
- There exists a fundamental scalar field in nature [2] : **Brout-Englert-Higgs field** [3] with mass 125 GeV/c^2
- Important in **cosmology**
  - primordial epoch of rapid expansion ("inflation") described by scalar field ("inflaton") [4]
  - dark energy and dark matter models
- appear in (nearly) all **extensions of General Relativity**, e.g. Kaluza-Klein theory, String Theory, Supergravity, ...

[1] Ginzburg, Landau, 1950
[2] CMS & ATLAS collaborations @ CERN, 2012
[3] Brout, Englert; Higgs; Guralnik, Hagen, Kibble, 1964
[4] Starobinsky, 1980; Guth, 1981; Linde, 1982

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Scalar fields in flat space-time : Derrick's theorem

For flat space-time **Derrick's theorem** [5] states that static, finite energy solitons in purely scalar field theories are only possible in $d = 1$, but not in $d \geq 2$

- Energy of "standard" static scalar field $\phi = \phi(\vec{r})$, $\vec{r} \in \mathbb{R}^d$ with $V(\phi) \geq 0$

$$E = \int \left(\frac{1}{2}(\vec{\nabla}\phi)^2 + V(\phi) \right) \, d^d x \equiv E_g + E_p$$

- Rescaling $\vec{r} \to \lambda^{-1}\vec{r}$, $\lambda \in \mathbb{R}$ leads to

$$E = E_g + E_p \to \lambda^{2-d} E_g + \lambda^{-d} E_p$$

[5] Derrick, 1964

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Scalar fields in flat space-time : Derrick's theorem

But: $\lambda = 1$ should be a stationary point of this scaling

$$\frac{dE}{d\lambda}(\lambda = 1) \stackrel{!}{=} 0$$

From above

$$\frac{dE}{d\lambda}(\lambda = 1) = (2 - d) \cdot E_g - d \cdot E_p \stackrel{!}{=} 0$$

- For $d = 1$ possible: $E_g = E_p \rightarrow$ e.g. **integrable models with kink solutions**, ...
- For $d = 2$ possible only if $E_p = 0$: E_g independent of λ
- For $d \geq 3$ not possible!!!!

One way around Derrick's theorem

- add gauge fields $\Rightarrow$ static case

$$E = \int \left(F_{kj}^2 + (D_k\phi)^2 + V(\phi)\right)\ d^dx \equiv E_F + E_g + E_p$$

$$D_k = \vec{\nabla}_k - ieA_k\,,\ F_{kj} = \vec{\nabla}_k A_j - \vec{\nabla}_j A_k$$

- With

$$A_j \to \lambda A_j \quad , \quad F_{kj} \to \lambda^2 F_{kj}$$

we have

$$\frac{dE}{d\lambda}(\lambda = 1) = (4-d)\cdot E_F + (2-d)\cdot E_g - d\cdot E_p \overset{!}{=} 0$$

Another way around Derrick's theorem

- **Complex scalar field + time-dependence** $\Rightarrow$ Q**-balls**:

$$S = \int \mathrm{d}^4x \, \sqrt{-g} \left(-\partial_\mu \Psi^* \partial^\mu \Psi - U(|\Psi|)\right)$$

- invariant under global U(1) symmetry $\Psi \rightarrow \exp(i\alpha)\Psi$, $\alpha \in \mathbb{R}$
  $\Rightarrow$ **globally conserved Noether charge**

$$Q_N = -i \int \mathrm{d}^3x \, \sqrt{-g} \left(\Psi^* \partial^t \Psi - \Psi \partial^t \Psi^*\right)$$

- minimizing the energy under the constraint that Q_N conserved gives

$$\Psi \sim \exp(i\omega t)\psi(\vec{r}) \quad , \quad \Psi \in \mathbb{C} \quad , \quad \psi \in \mathbb{R} \quad , \quad \omega \in \mathbb{R}$$

NOTE: Q_N **interpreted as number of scalar bosons making up non-topological soliton**

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Static scalar field

Consider **massless, static** “test” scalar field $\phi(r) \in \mathbb{R}$ in a spherically symmetric, static, asymptotically flat space-time with metric

$$\mathrm{d}s^2 = -N\sigma^2\mathrm{d}t^2 + N^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)$$

Klein-Gordon equation

$$\partial_\mu\left(\sqrt{-g}\partial^\mu\phi(r)\right) = 0 \quad \Rightarrow \quad \frac{d\phi(r)}{dr} \sim \frac{1}{N\sigma r^2}$$

if we require regularity of the metric functions at $r = 0$, the scalar field derivative (and with it the energy-momentum tensor) diverges as $r \to 0$

Complex scalar field + time-dependence

- **Boson stars: Q-balls in curved space-time** $\Psi = \exp(i\omega t)\psi(r)$
- energy-momentum tensor components ($U(\psi)$ scalar field potential)

$$-T_t^t = N\psi'^2 + \frac{\omega^2\psi^2}{N\sigma^2} + U(\psi)$$
$$T_r^r = N\psi'^2 + \frac{\omega^2\psi^2}{N\sigma^2} - U(\psi)$$
$$T_\theta^\theta = -N\psi'^2 + \frac{\omega^2\psi^2}{N\sigma^2} - U(\psi)$$

- Metric: static, spherically symmetric

$$\mathrm{d}s^2 = -N(r)(\sigma(r))^2\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

energy-momentum tensor static, but scalar field is not

Outline

Static scalar field

Consider a massless, static "test" scalar field $\phi(r) \in \mathbb{R}$ in a Schwarzschild black hole background with

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Klein-Gordon equation $\partial_\mu \left(\sqrt{-g}\partial^\mu \phi(r)\right) = 0$ leads to the solution

$$\phi(r) \sim \ln\left(\frac{2M}{r} - 1\right) \xrightarrow[r \to 2M]{} -\infty$$

with energy-momentum tensor

$$T_r^r = -T_0^0 = -T_\theta^\theta = -T_\varphi^\varphi \sim \frac{1}{r^4}\left(1 - \frac{2M}{r}\right)^{-1} \xrightarrow[r \to 2M]{} \infty$$

NOTE: scalar field has same symmetries as space-time

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Complex scalar field + time-dependence

Theorem[6]:

A **static, spherically symmetric, asymptotically flat** black hole space-time, with regular (non-extreme) horizon, which satisfies Einstein's equations with matter fields fulfilling the weak energy condition (WEC)[7] and its energy-momentum tensor satisfying the following condition

$$T_\theta^\theta \leq T_r^r$$

is **necessarily trivial**, this is T_ν^μ vanishes identically and the space-time corresponds to the Schwarzschild solution.

- Complex scalar field with harmonic time-dependence $\Psi = \exp(i\omega t)\psi(r)$ has $T_\theta^\theta \leq T_r^r$

[6] Peña & Sudarsky, 1997

[7] $\rho = -T_t^t \geq 0$, $T_i^i \geq T_t^t$, $i = 1, 2, 3$

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Adding gauge fields

Gauging the U(1) symmetry with $A_\mu \mathrm{d}x^\mu = v(r)\mathrm{d}t$, e gauge coupling, we get

$$-T_t^t = \frac{v'^2}{2\sigma^2} + N\psi'^2 + \frac{(\omega - ev)^2\psi^2}{N\sigma^2} + U(\psi) \quad (1)$$

$$T_r^r = -\frac{v'^2}{2\sigma^2} + N\psi'^2 + \frac{(\omega - ev)^2\psi^2}{N\sigma^2} - U(\psi) \quad (2)$$

$$T_\theta^\theta = \frac{v'^2}{2\sigma^2} - N\psi'^2 + \frac{(\omega - ev)^2\psi^2}{N\sigma^2} - U(\psi) \; . \quad (3)$$

For $N\psi'^2 \leq \frac{v'^2}{2\sigma^2}$ **we get** $T_\theta^\theta \geq T_r^r$

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Excluding backreaction: Charged Q-clouds in fixed Schwarzschild background [8]

[8] C. Herdeiro and E. Radu, Eur. Phys. J. C 80 (2020) 5, 390; Y. Brihaye and B. Hartmann, CQG Lett. **38** (2021)

Parameter choices

- Choose gauge such that $\omega = 0$
- Need $U(\psi) = \mu^2\psi^2 - \lambda\psi^4 + \nu\psi^6$ with $\lambda > 0$, $\nu > 0$
- rescale coordinates and fields such that $\mu = 1$, $\lambda = 1$
- Regularity and finite energy require the following boundary conditions

$$\psi'|_{r=r_h} = \frac{r_h}{2} \left.\frac{\mathrm{d}U}{\mathrm{d}\psi}\right|_{\psi=\psi_h} \ , \ \ v(r_h) = 0 \ , \ \ v(r \to \infty) \sim v_\infty - \frac{Q}{r} \to v_\infty$$

with

$$\psi(r \to \infty) \sim \frac{\exp(-\mu_{\mathrm{eff},\infty} r)}{r} \to 0 \ , \ \ \mu_{\mathrm{eff},\infty} = \sqrt{\mu^2 - e^2 v_\infty^2}$$

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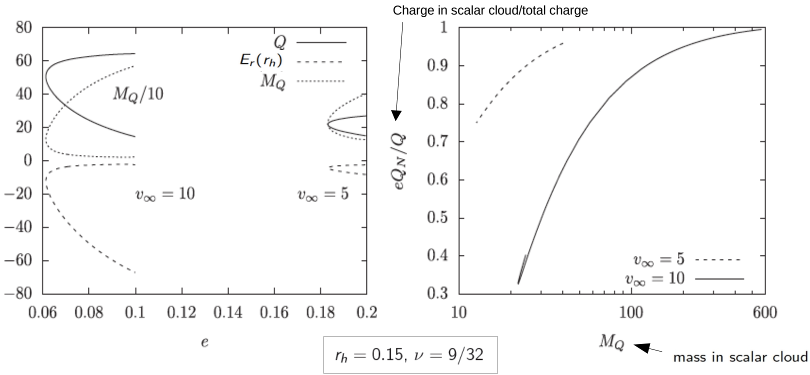
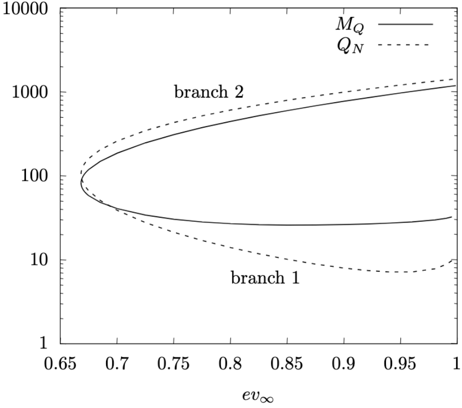
Charged Q-clouds: Fixing the potential difference

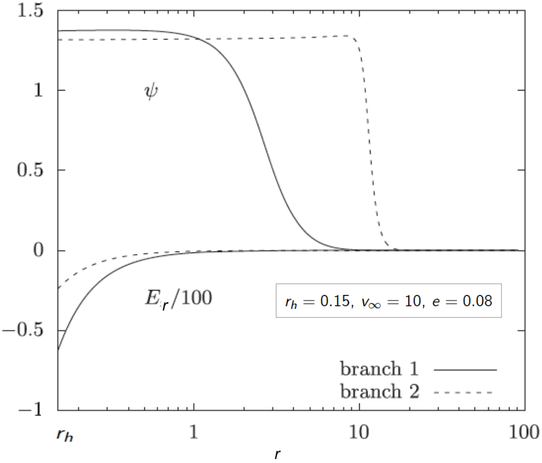


$r_h = 0.15,\ \nu = 9/32$

Charged Q-clouds: Two branches



Charged Q-clouds: Solutions on branches



Charged Q-clouds: Two branches

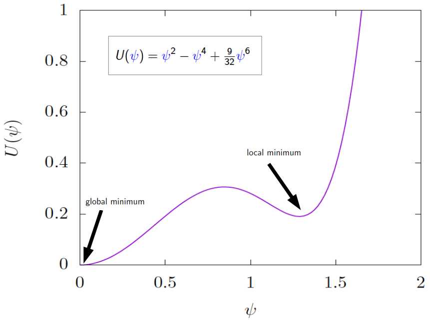
(a) scalar clouds more extended on second branch

(b) electric charge moved from horizon to cloud

(c) Formation of two distinct regions separated by *hard wall*

(c1) scalar field trapped in *false vacuum* inside

(c2) scalar field in *true vacuum* outside



Charged Q-clouds: *Hard wall* solution

- **Scalar field**

$$\psi(r) \sim \begin{cases} \tilde{\psi} & \text{for} \quad r \in [r_h : \tilde{r}] \\ 0 & \text{for} \quad r \in]\tilde{r} : \infty[\end{cases}$$

$\tilde{\psi} = const.$ local minimum of $U(\psi)$

- **Electric potential**

$$v(r) \sim \begin{cases} v_\infty - \frac{Q}{r} \exp(-\sqrt{2} e \tilde{\psi} r) & \text{for} \quad r \in [r_h : \tilde{r}] \\ v_\infty - \frac{Q}{r} & \text{for} \quad r \in]\tilde{r} : \infty[\end{cases}$$

For $r \in [r_h : \tilde{r}]$: **screened point charge**

Backreaction of Q-clouds

Minimal coupling to General Relativity

- Einstein equation $G_{\mu\nu} = 2\alpha T_{\mu\nu}$
- REMINDER: metric

$$ds^2 = -N(r)(\sigma(r))^2 dt^2 + \frac{1}{N(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- REMINDER: energy-momentum tensor components

$$-T_t^t = \frac{v'^2}{2\sigma^2} + N\psi'^2 + \frac{(\omega - ev)^2\psi^2}{N\sigma^2} + U(\psi)$$
$$T_r^r = -\frac{v'^2}{2\sigma^2} + N\psi'^2 + \frac{(\omega - ev)^2\psi^2}{N\sigma^2} - U(\psi)$$
$$T_\theta^\theta = \frac{v'^2}{2\sigma^2} - N\psi'^2 + \frac{(\omega - ev)^2\psi^2}{N\sigma^2} - U(\psi) \ .$$

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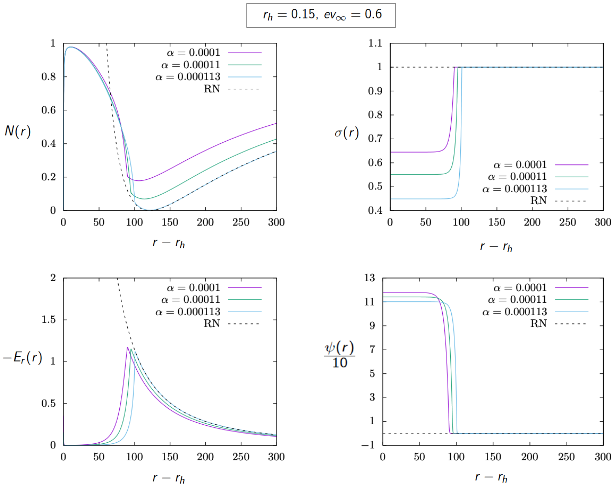
Increasing backreaction at fixed potential difference

Behaviour of solutions on second branch



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Limiting solution

- Local minimum forms at $r \sim \tilde{r}$
- Metric functions: $\sigma \equiv \sigma_0 = const.$ and

$$N(r) = 1 - \frac{2M}{r} + \left[\frac{\alpha Q^2}{\sigma_0^2 r^2} + \frac{\sqrt{2}\alpha Q^2 e\psi_0}{2r}\right]\exp(-2\sqrt{2}e\psi_0 r) - \frac{2\alpha}{3}U(\psi_0)r^2$$

- at $\alpha \sim \alpha_{\rm cr}$ with $\tilde{\sigma} \in]0:1[$

$$(\sigma_0, \psi_0) \approx \begin{cases} (\tilde{\sigma}, \tilde{\psi}) & \text{for} \quad r \in [r_h : \tilde{r}] \quad \Leftarrow \quad \textbf{sRNdS} \\ (1, 0) & \text{for} \quad r \in]\tilde{r} : \infty[\quad \Leftarrow \quad \textbf{RN} \end{cases}$$

sRNdS: screened Reissner-Nordström-de Sitter
RN: Reissner-Nordström

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Non-topological inflation

- For $r \gg 1$ and $r < \tilde{r}$

$$N(r) \approx 1 - \frac{2\alpha}{3} U(\psi_0) r^2 \quad \Leftarrow \quad \textbf{de Sitter}$$

  - cosmological constant $\Lambda = 2\alpha U(\psi_0)$
  - cosmological horizon $r_\Lambda = \sqrt{3/(2\alpha U(\psi_0))}$
- Similar phenomenon exists for boson stars [9]
- compare to **topological inflation** ('Monopoles as big as a universe') [10]

Non-topological inflation:
Inflation inside charged boson stars and static black hole scalar clouds

[9] Y. Brihaye, F. Console, B. Hartmann, Symmetry **13** (2020)

[10] Linde, 1994; Vilenkin, 1994

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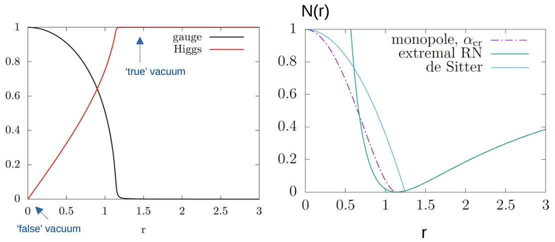
Topological inflation

SU(2) Yang-Mills-Higgs model, Higgs in adjoint representation,
ϕ^4-potential $V(\phi^a) = \frac{\lambda}{4}\left(\phi_a\phi^a - 1\right)^2$, $a = 1, 2, 3$
$\Rightarrow$ **magnetic monopole solutions** [11]



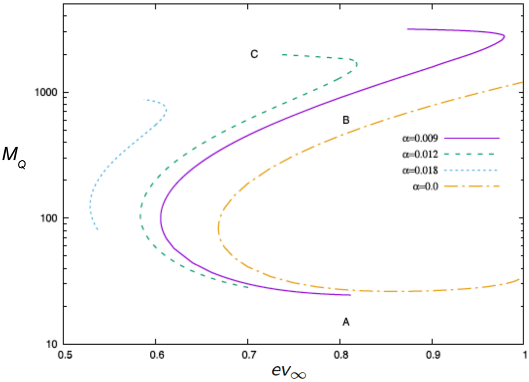
$\alpha \to \alpha_{cr}$: $r > r_s$ extremal RN, $r \ll 1$ de Sitter

[11] 'tHooft, 1974; Polyakov, 1974

Dependence on ev_∞

Three branches

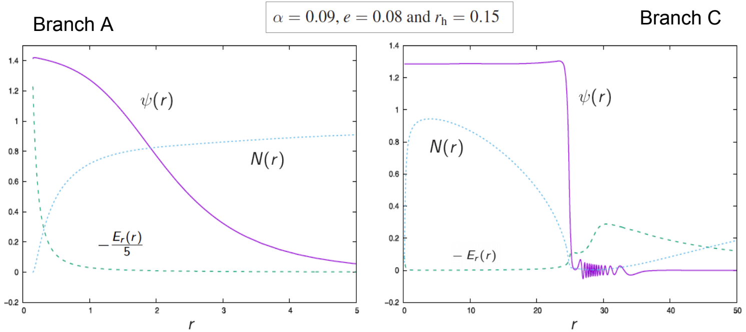
Y. Brihaye, B.Hartmann, Phys. Rev. D **105** 104063 (2022)



Branch A vs branch C

Y. Brihaye, B.Hartmann, Class. Quant. Grav. **39** (2022) & Phys. Rev. D **105** (2022)

$\alpha = 0.09$, $e = 0.08$ and $r_h = 0.15$



Black holes & boson stars with wavy scalar hair

Y. Brihaye, B.Hartmann, Phys. Rev. D **105** (2022)

- Similar phenomenon for boson stars
- (Mathematical) Reason for existence: scalar field equation

$$\frac{1}{r^2\sigma N}\left(r^2\sigma N\psi'\right)' = m_{\rm eff}^2\psi$$

with position-dependent 'mass'

$$m_{\rm eff}^2 = \frac{1}{N(r)} - \frac{e^2 v^2(r)}{N^2(r)\sigma^2(r)}$$

for $N(r)$ close to zero: $m_{\rm eff}^2 \ll 0$

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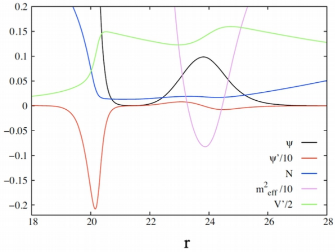
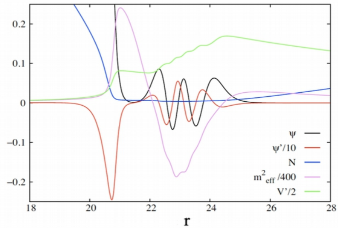
Black holes with standard vs wavy scalar hair

Y. Brihaye, B.Hartmann, Phys. Rev. D **105** (2022)

- Different branches exist that **differ in number of zeros** of scalar field
- charge Q and mass M increase (slightly) with number of zeros

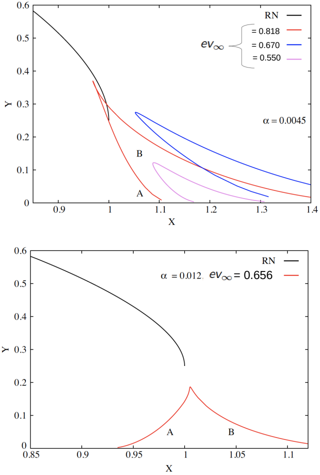




Thermodynamical quantities

Black holes with standard vs wavy scalar hair

Y. Brihaye, B.Hartmann, Phys. Rev. D **105** (2022)



- Dimensionless quantities

$$X = \frac{\sqrt{\alpha}}{M} \quad , \quad Y = \frac{A_H}{16\pi M^2}$$

- RN case:

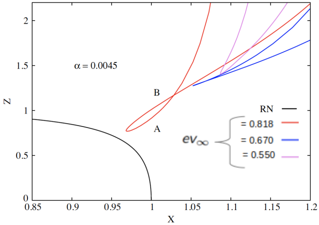
$$X = \frac{2\sqrt{W}}{1+W} \quad , \quad Y = \frac{1}{(1+W)^2}$$

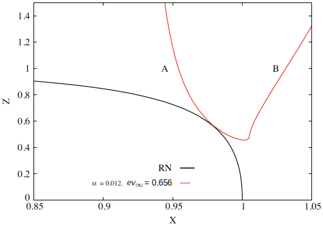
with

$$W = \frac{\alpha Q^2}{r_h}$$

Black holes with standard vs wavy scalar hair

Y. Brihaye, B.Hartmann, Phys. Rev. D **105** (2022)





- Dimensionless quantities

$$X = \frac{\sqrt{\alpha}}{M} \quad , \quad Z = 8\pi M T_H$$

- RN case:

$$X = \frac{2\sqrt{W}}{1+W} \quad , \quad Z = (1-W)(1+W)$$

with

$$W = \frac{\alpha Q^2}{r_h}$$

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Black holes and boson stars with new scalar hair

- **Wavy scalar hair** on black holes and boson stars exists in a **simple** gauged complex scalar field model minimally coupled to standard gravity
- appears outside
    - **inflating** boson star
    - **inflating scalar cloud** on static spherically symmetric BH
- Works also for potential of the form $V(\psi) = \mu^2\eta^2\left[1 - \exp\left(-\frac{\psi^2}{\eta^2}\right)\right]$,
  Example: $\alpha_{\rm cr} \sim \mathcal{O}(10^{-4})$ at $e = \mathcal{O}(10^{-3})$ in dimensionful units:
    - $\eta \sim \mathcal{O}(10^{-3} M_{\rm Planck})$
    - $e \sim \mathcal{O}(\mu/M_{\rm Planck})$
    - Hubble constant $H \sim \mathcal{O}(10^{-2}\mu)$

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THANK YOU

FOR LISTENING