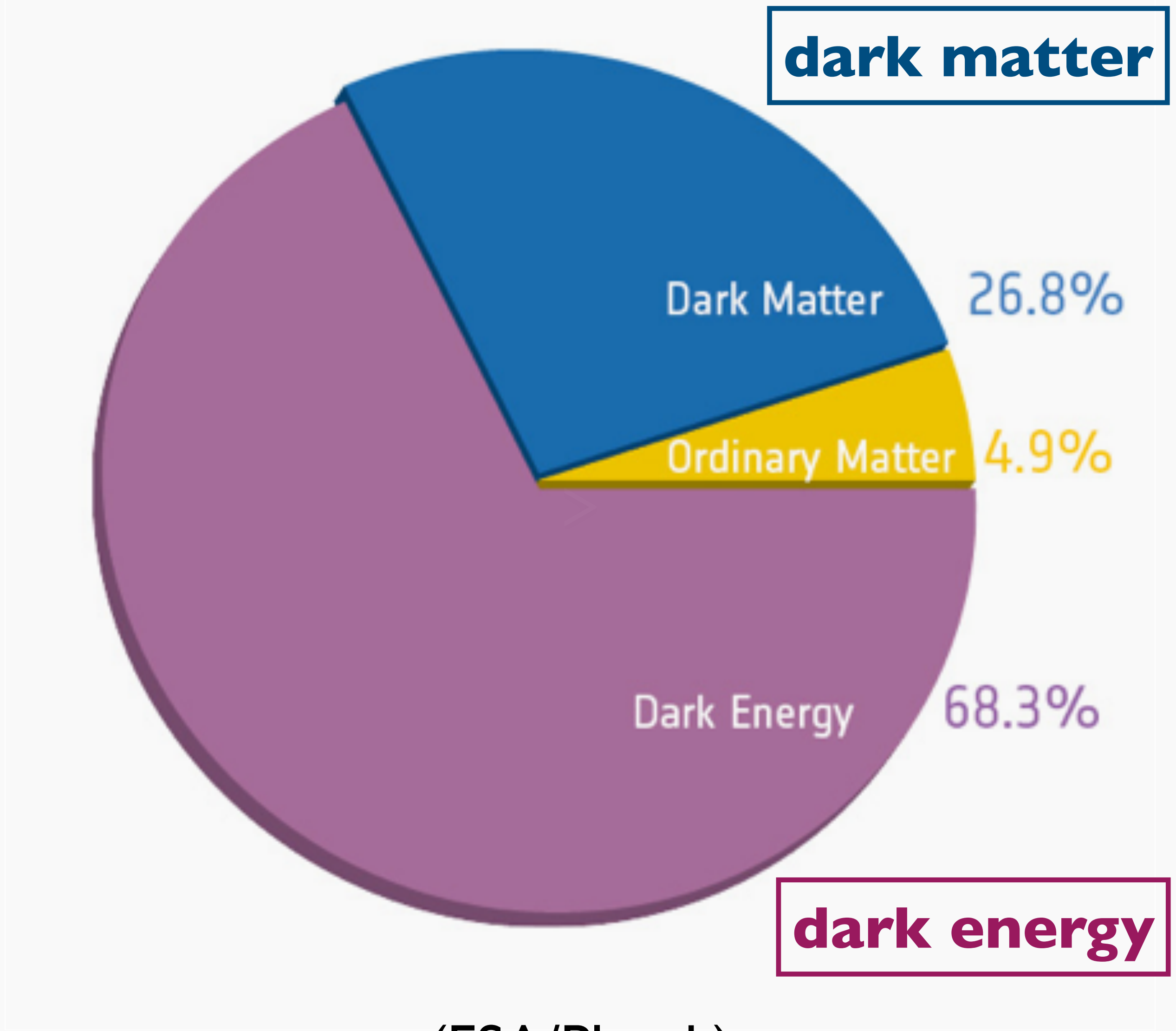
Probing the very small scale dark matter distribution with gravitational waves

Masamune Oguri
Center for Frontier Science, Chiba University

Outstanding question

- what is the nature of dark matter and dark energy?

today's talk

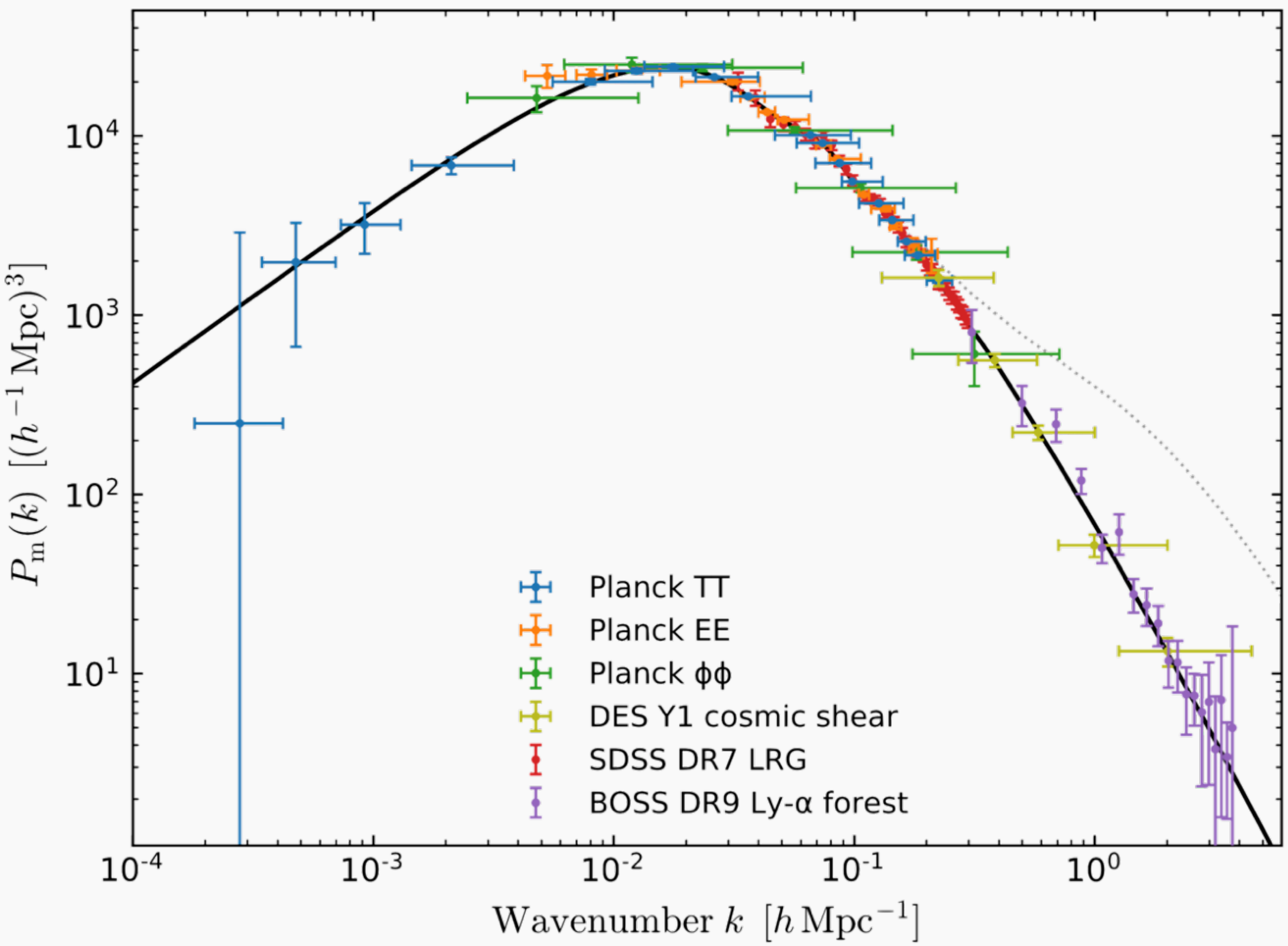


(ESA/Planck)

Dark matter distribution

Planck collaboration 2020

matter power spectrum

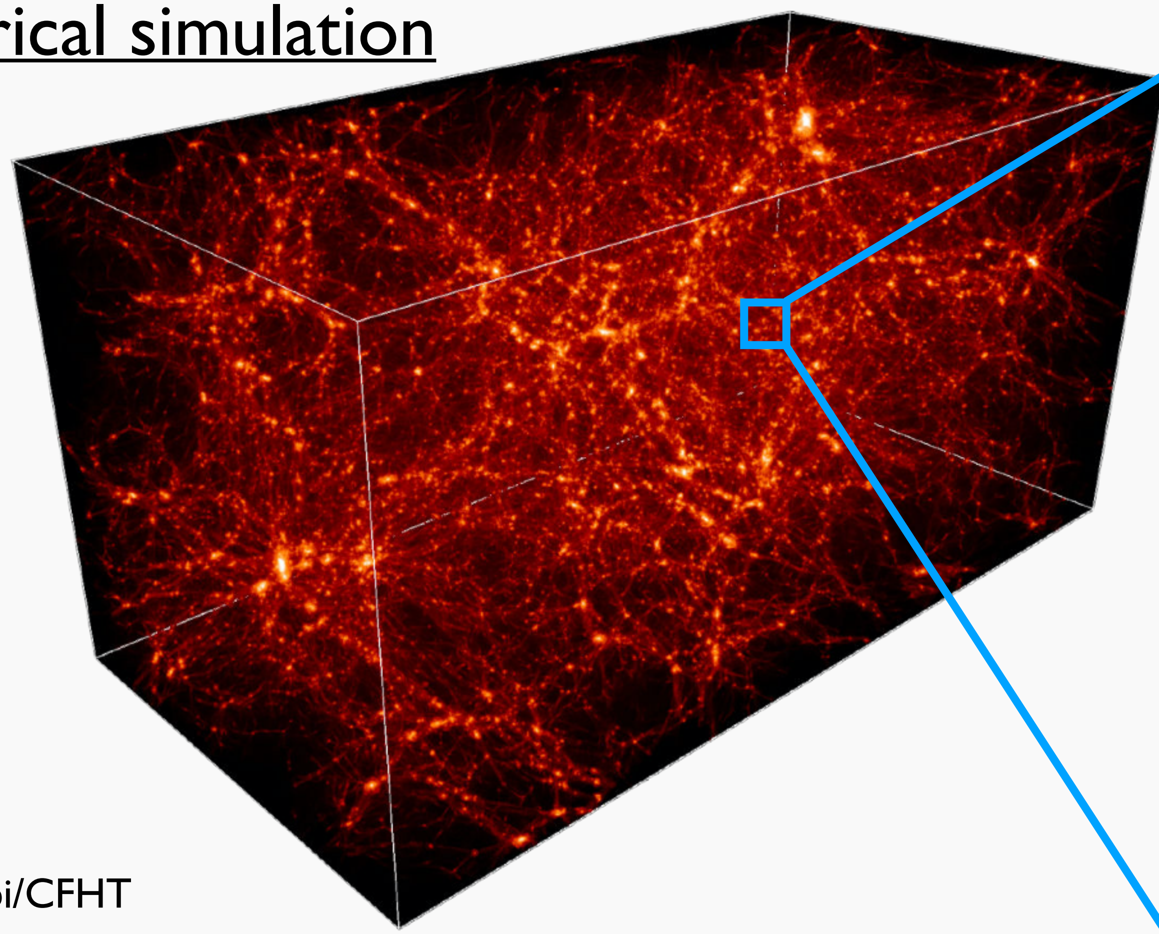


larger-scale **smaller-scale**

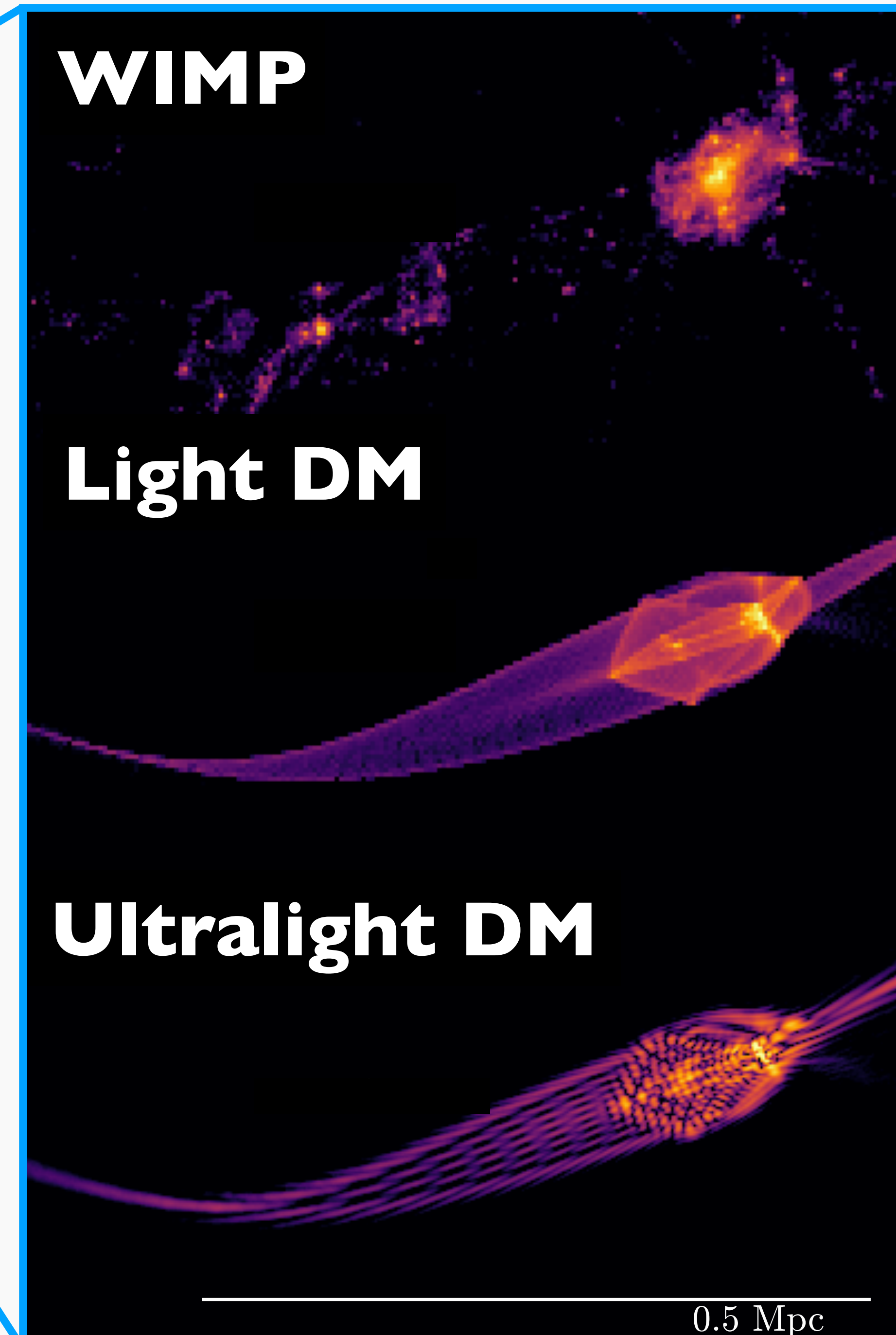
- **cold dark matter** can explain large-scale structure of the Universe ($\gtrsim$ 1 Mpc)

Dark matter (DM) at small-scale

Numerical simulation



S. Colombi/CFHT

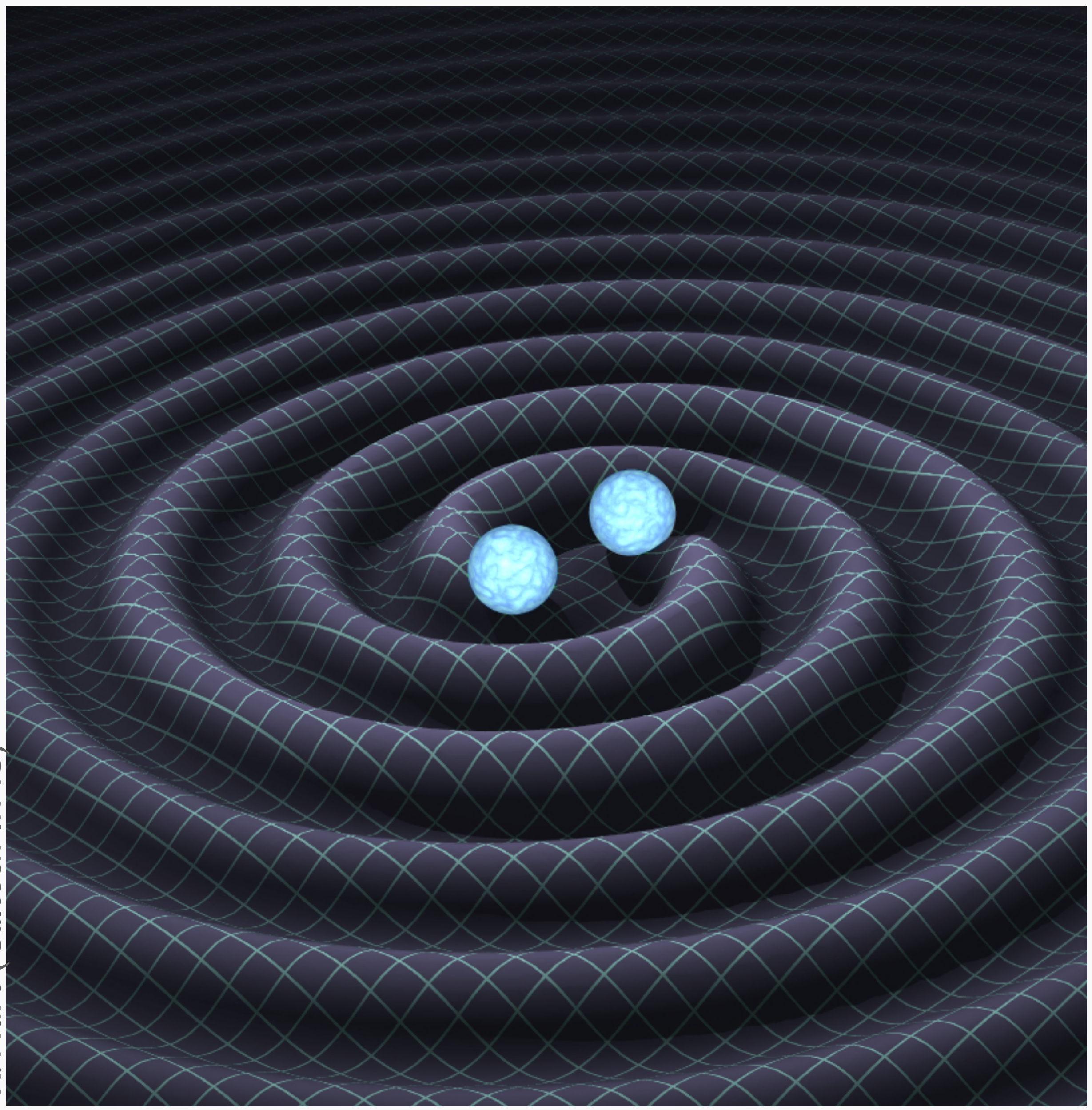


Mocz+2020

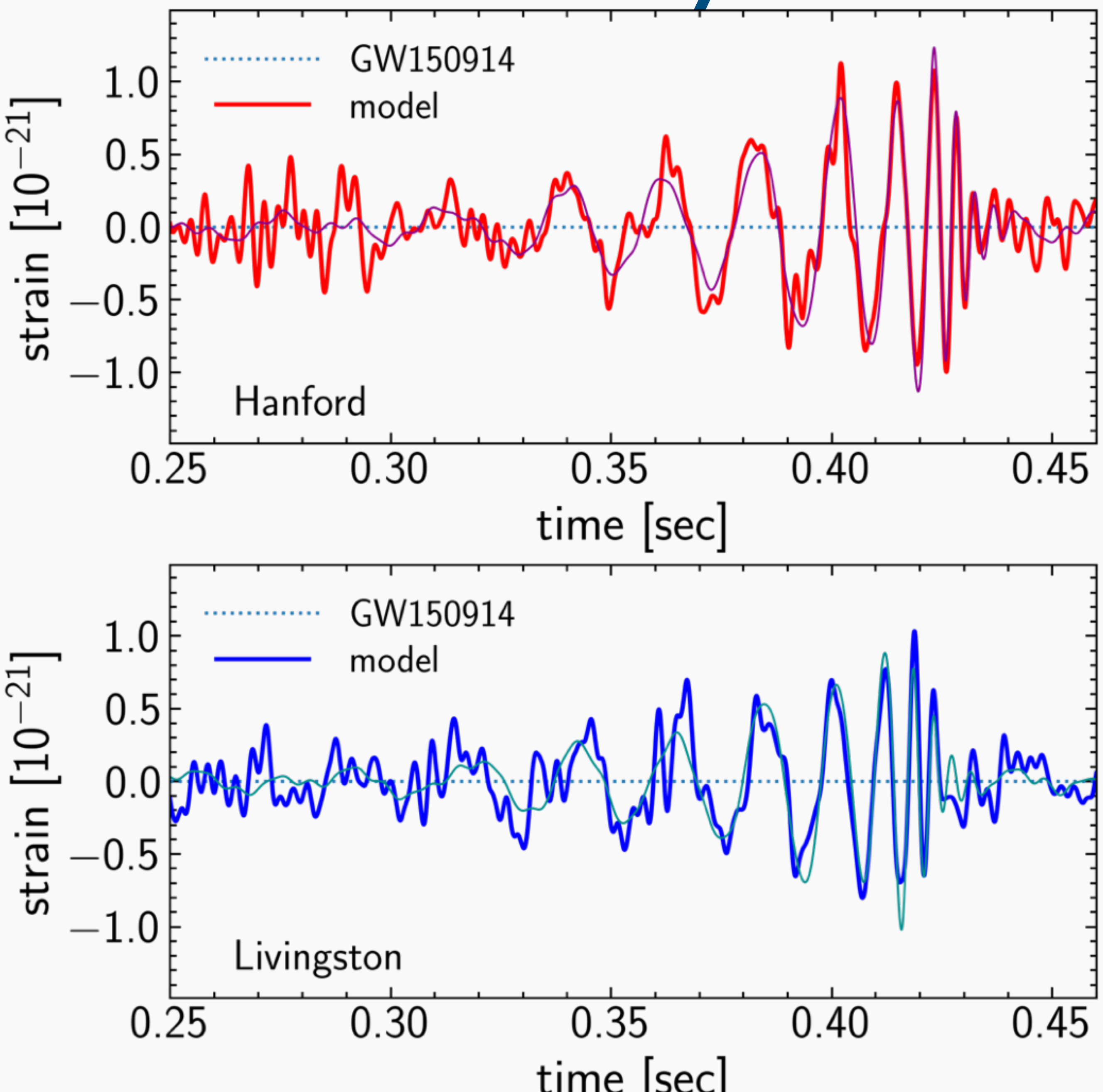
small-scale dark matter distribution is key for understanding dark matter

Gravitational waves

first discovery in 2015!



R. Hurt (Caltech-IPAC)

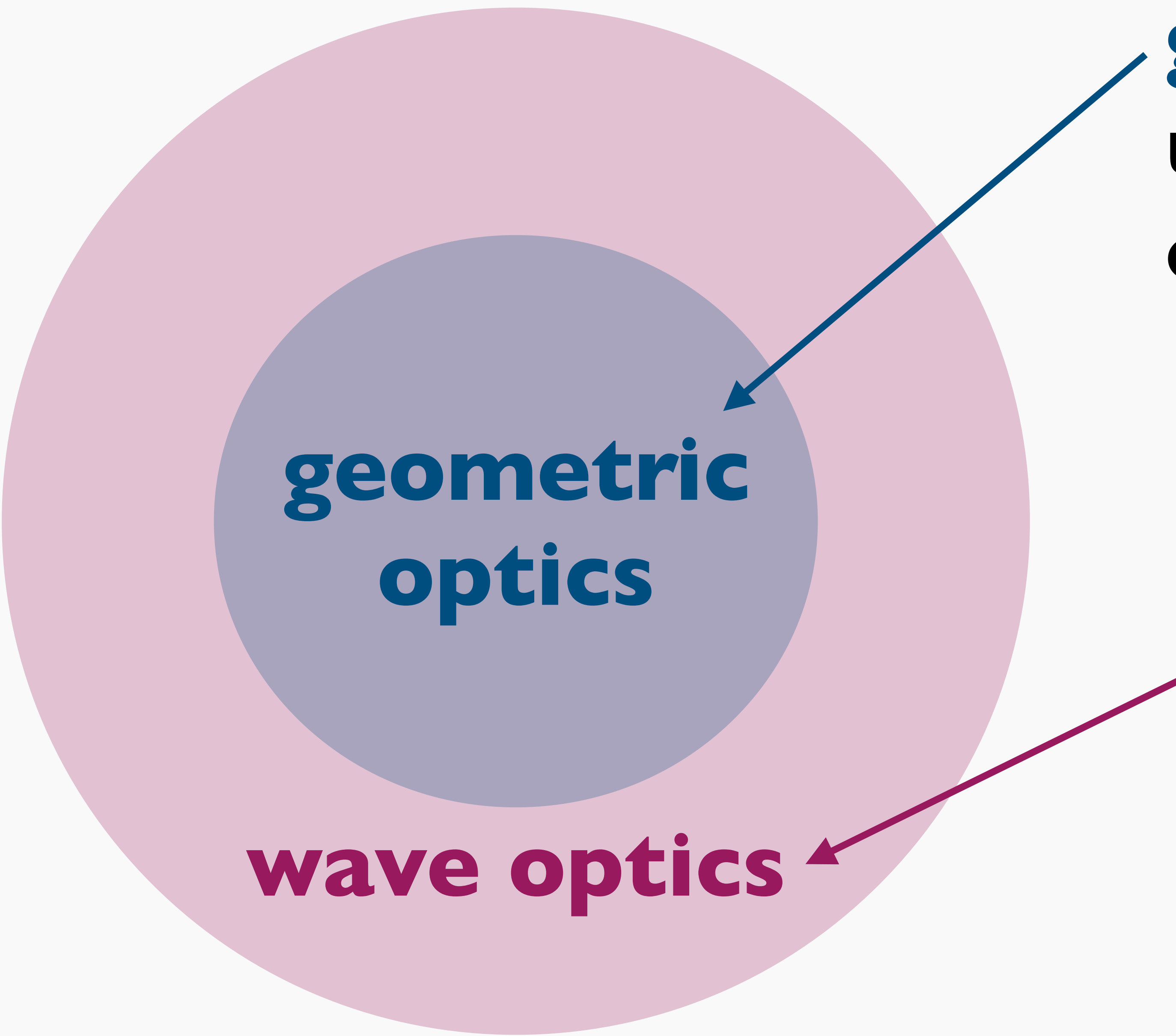


we argue that GWs probe very small-scale DM distribution!

Outline

- geometric optics vs. wave optics
- Born approximation
- amplitude and phase fluctuations of GWs

Geometric vs. wave optics



geometric optics is used for almost all analysis of gravitational lensing

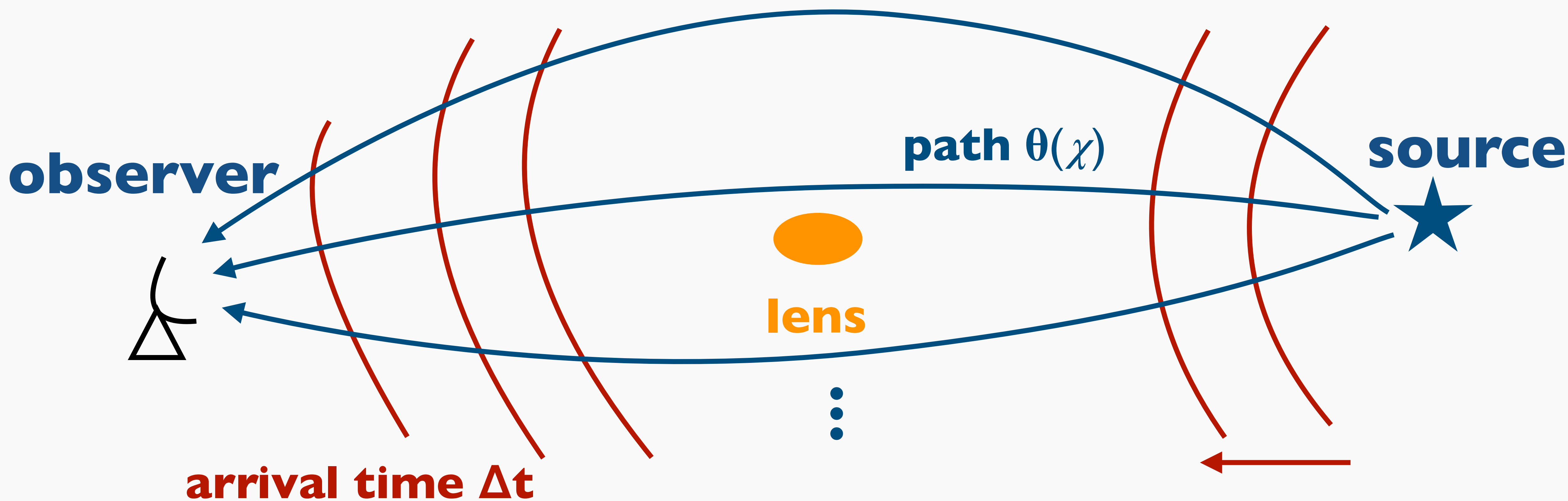
wave optics is more fundamental than geometric optics

Geometric vs. wave optics

wave optics

superposition of waves

$$\psi \propto \int \mathscr{D}\left[\boldsymbol{\theta}(\chi)\right]\, e^{2\pi i f \Delta t}$$



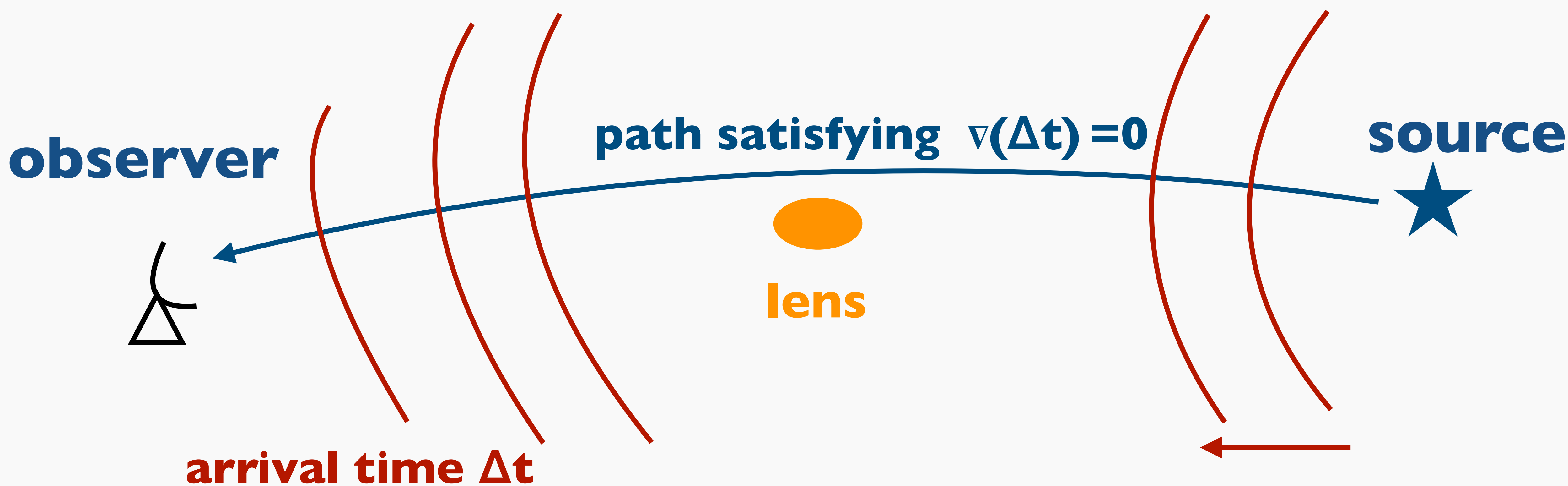
e.g., <u>MO</u> RPP **82**(2019)126901, for a review

Geometric vs. wave optics

wave optics

$$\psi \propto \int \mathscr{D}\left[\boldsymbol{\theta}(\chi)\right]\ e^{2\pi i f \Delta t}$$

superposition of waves

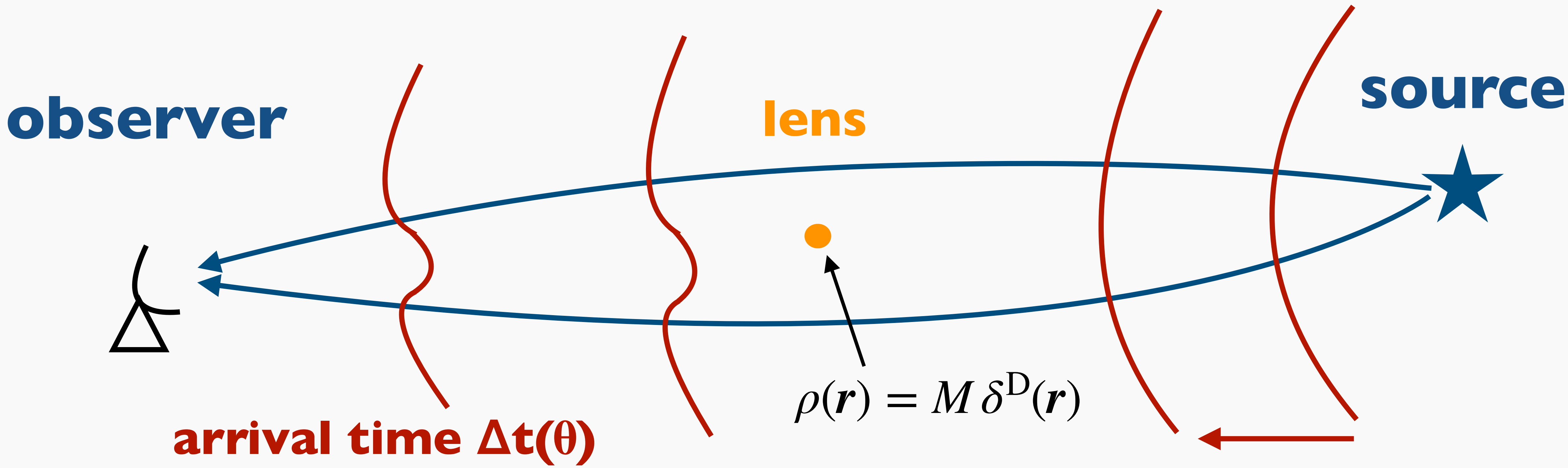


geometric optics

at high f limit, only stationary point of Δt is observed

$$\nabla(\Delta t) = 0$$ **(Fermat's principle)**

Simplest case: point mass lens



$$\psi \propto \int d\boldsymbol{\theta}\, e^{iw\Phi(\boldsymbol{\theta})}$$

O(1) function

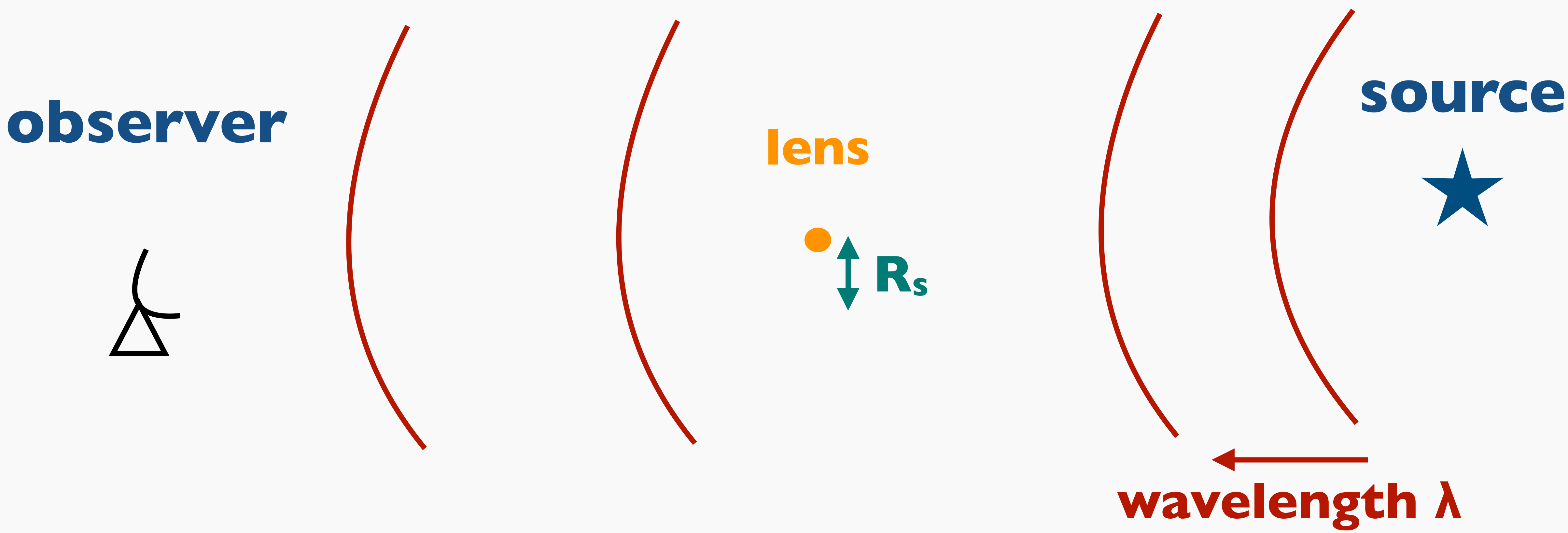
***analytic solution available**

$$w = 2\pi f \Delta t_{\mathrm{fid}} = 2\pi f (1+z) \frac{4GM}{c^3}$$

dimensionless parameter controlling wave optics effect

***geometric optics at w→∞**

Wave effect: diffraction

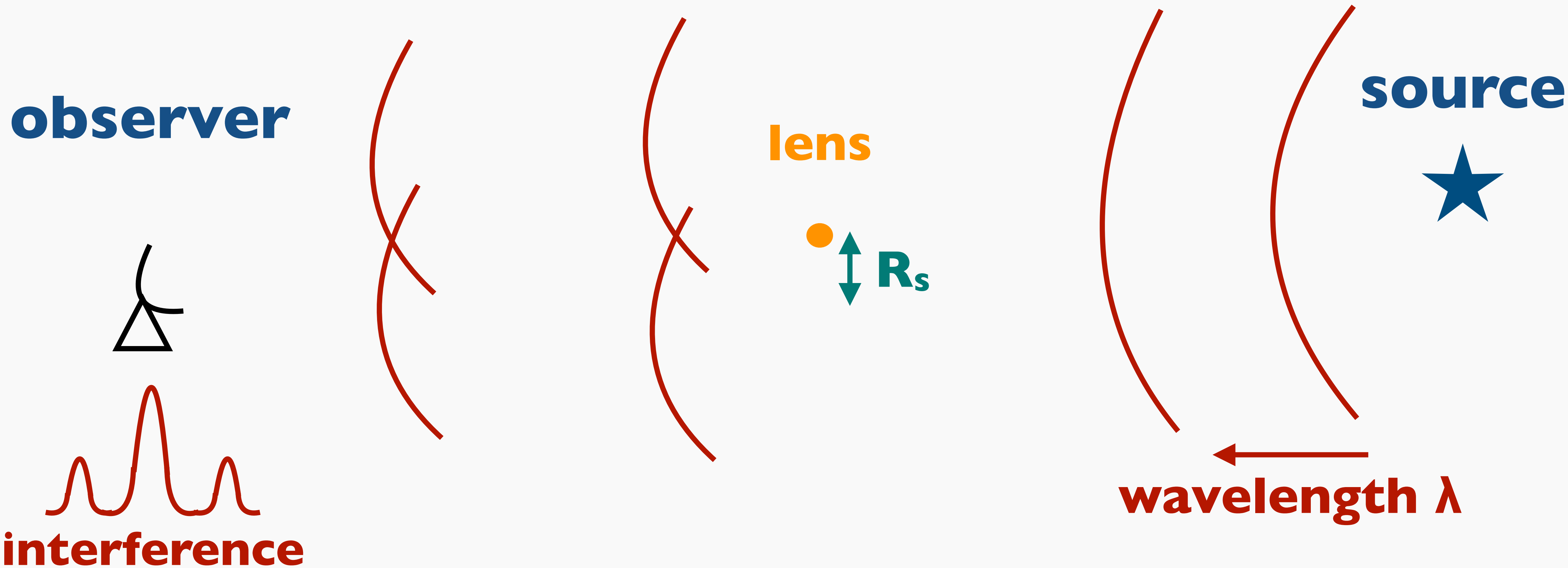


$$w = 2\pi f(1+z)\frac{4GM}{c^3} \ll 1 \quad \Rightarrow \quad \lambda \gg R_S$$

Schwarzschild radius

- no lensing effect (magnification μ~1)

Wave effect: interference

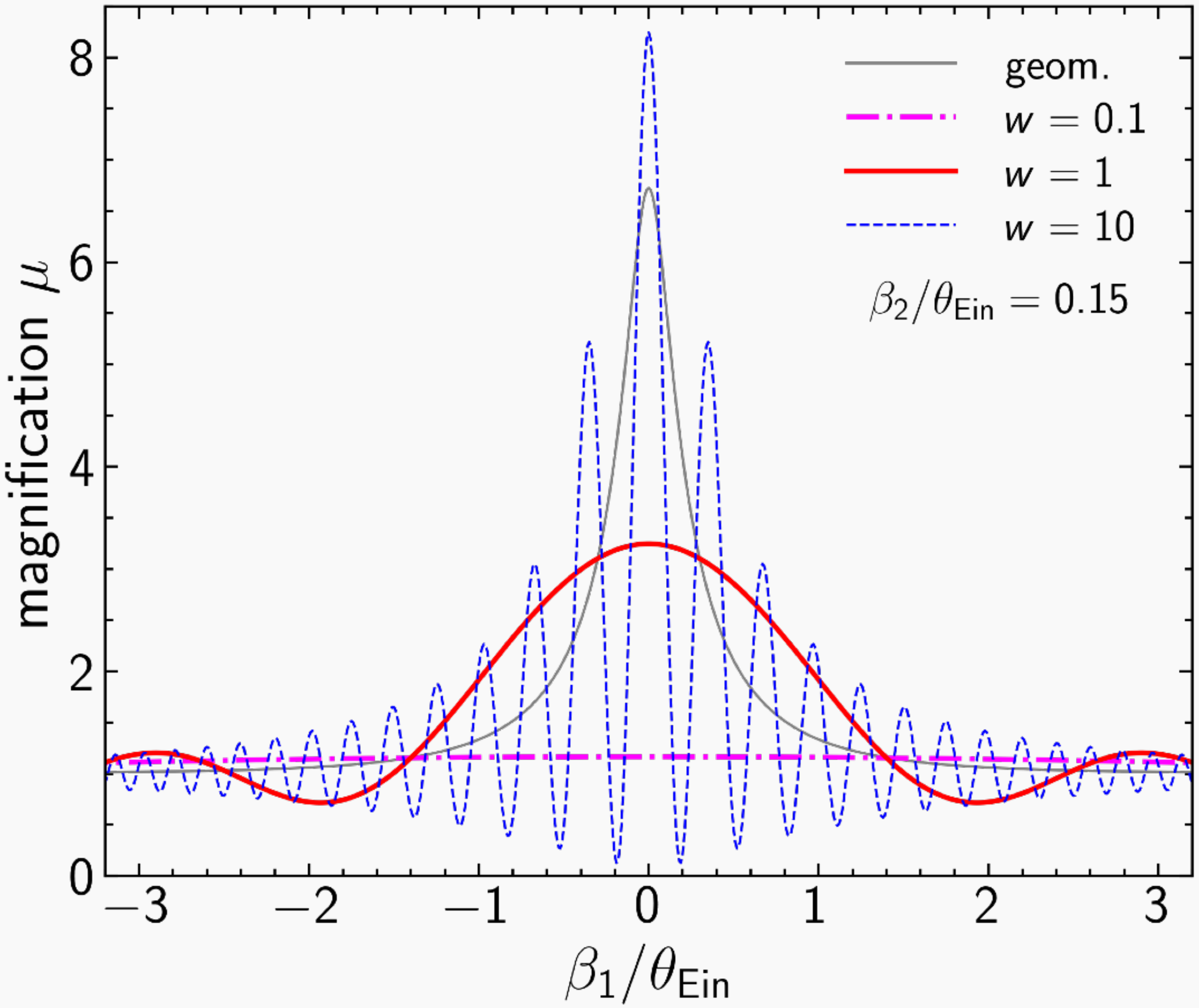


$$w = 2\pi f(1+z)\frac{4GM}{c^3} \sim 1 \quad \Rightarrow \quad \lambda \sim R_S$$

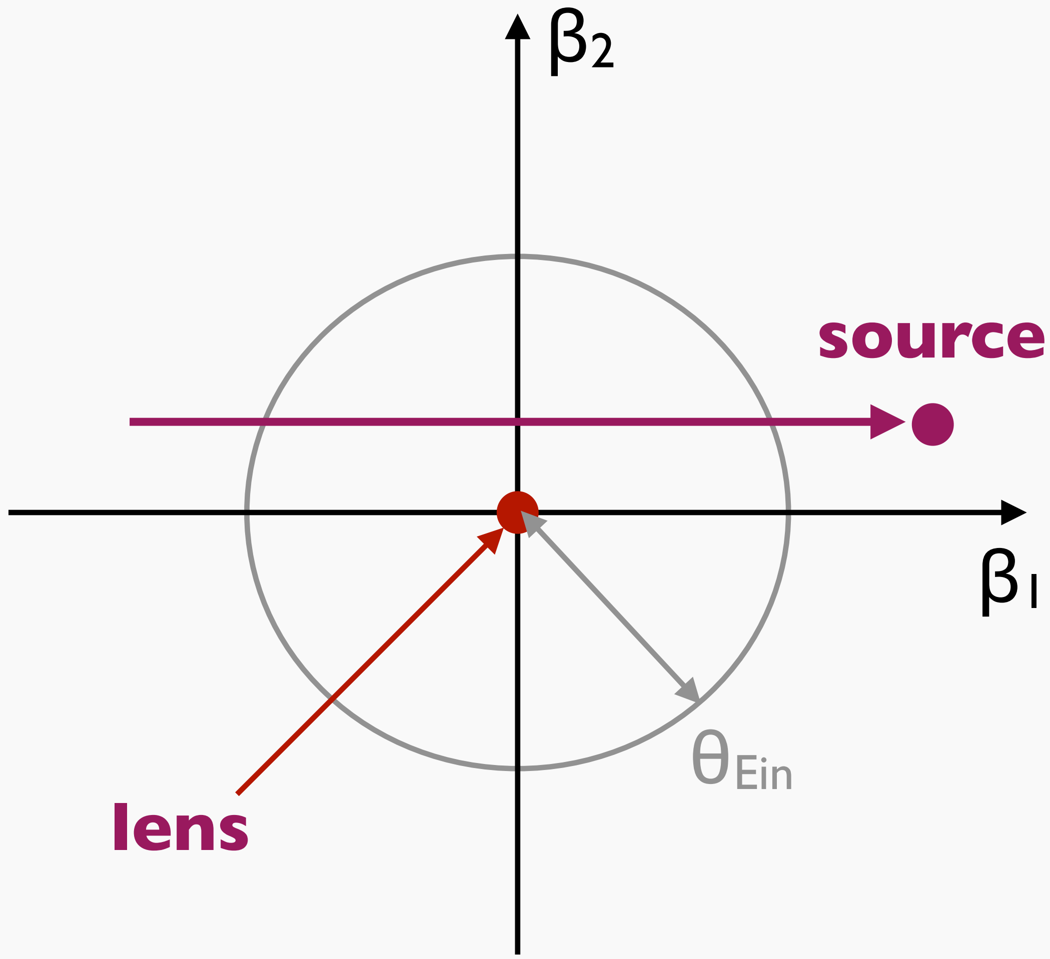
Schwarzschild radius

- multiple light ray paths interfere
  (magnification oscillates as a func. of position/frequency)

Example

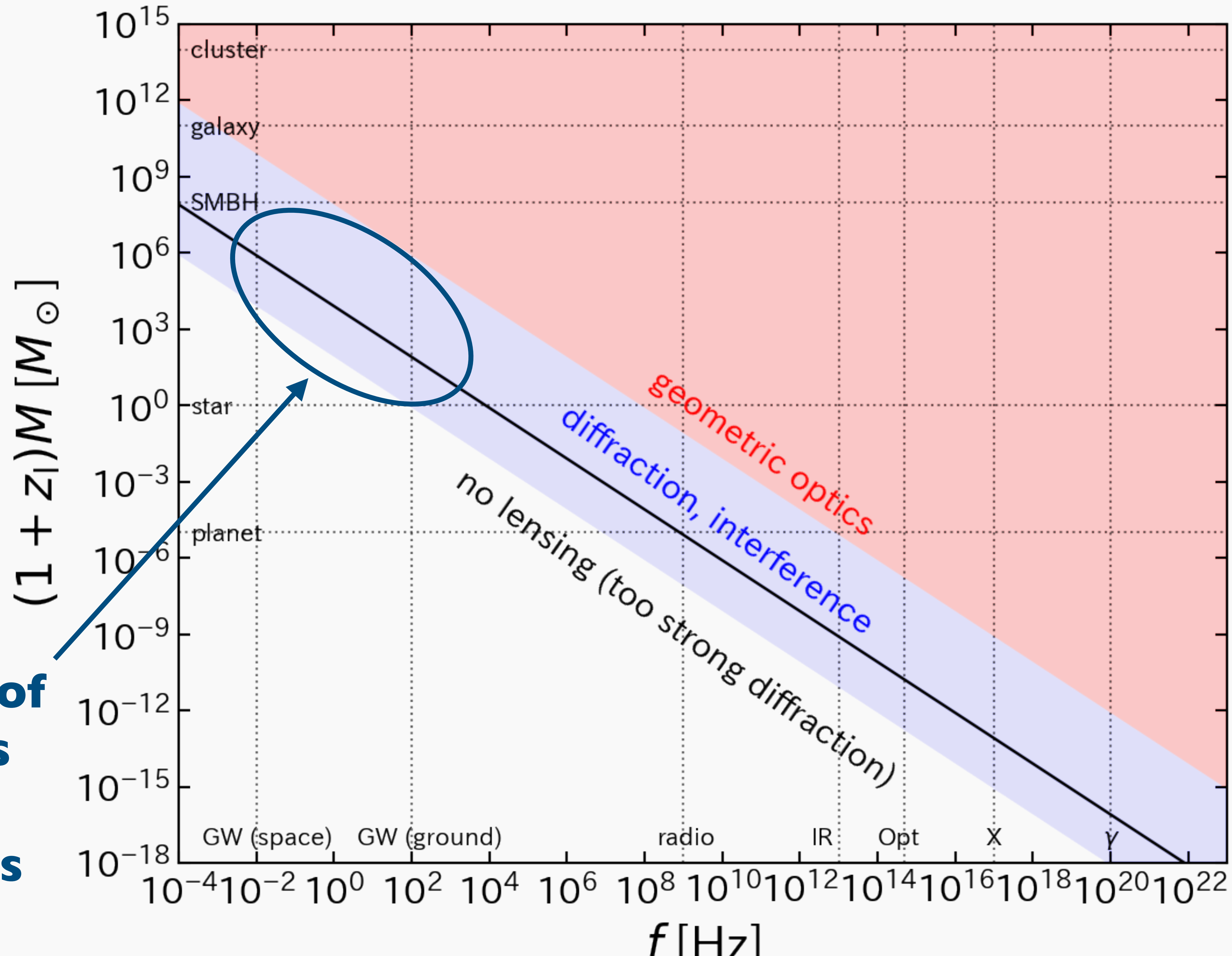


geom.
$w = 0.1$
$w = 1$
$w = 10$
$\beta_2/\theta_{\mathrm{Ein}} = 0.15$
magnification μ
$\beta_1/\theta_{\mathrm{Ein}}$



β_2
source
β_1
θ_{Ein}
lens

Can wave effects be observed?



mass range of wave effects for GW observations

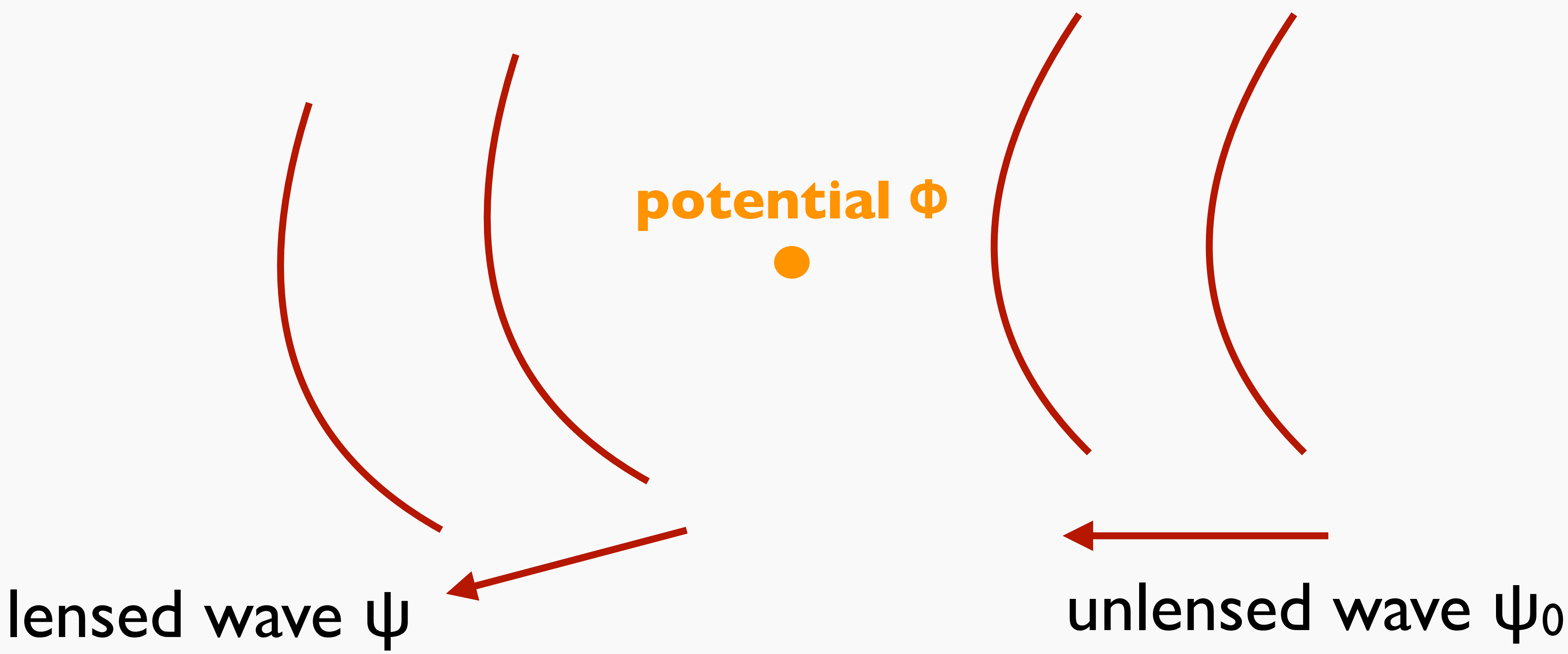
Difficulty

- highly oscillatory integral
  → computationally expensive and unstable

$$\psi \propto \int d\boldsymbol{\theta}\, e^{2\pi i f \Delta t(\boldsymbol{\theta})}$$

Integral equation

- wave propagation in gravitational potential Φ



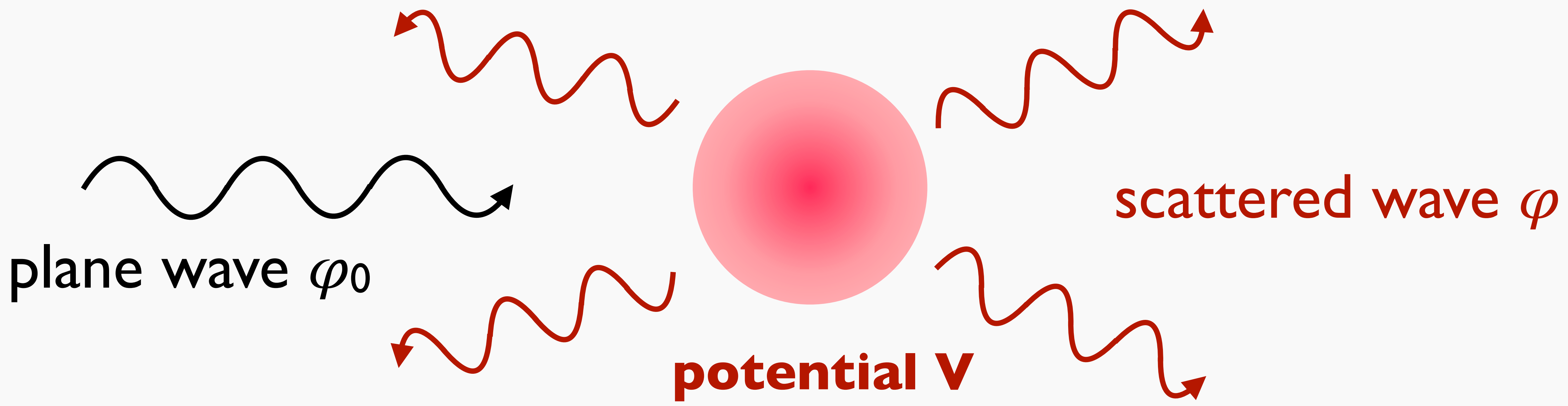
$$(\nabla^2 + 4\pi^2 f^2)\psi(\boldsymbol{x}) = 16\pi^2 f^2 \Phi(\boldsymbol{x})\psi(\boldsymbol{x})$$

$$(\nabla^2 + 4\pi^2 f^2)\psi_0(\boldsymbol{x}) = 0$$

- solution given by an integral equation

$$\psi(\boldsymbol{x}) = \psi_0(\boldsymbol{x}) - 4\pi f^2 \int d\boldsymbol{x}' \frac{e^{2\pi i f |\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|} \Phi(\boldsymbol{x}')\psi(\boldsymbol{x}')$$

Scattering problem in Q.M.



- solution given by an integral equation

$$\varphi(\boldsymbol{r}) = \varphi_0(\boldsymbol{r}) - \frac{\mu}{2\pi\hbar^2}\int d\boldsymbol{r}'\frac{e^{ik\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{|\boldsymbol{r}-\boldsymbol{r}'|}V(\boldsymbol{r}')\varphi(\boldsymbol{r}')$$

Born approximation $\varphi_0(\boldsymbol{r})$

Takahashi et al. A&A **438**(2005)L5

Born approximation

- solution given by an integral equation

$$\psi(\boldsymbol{x}) = \psi_0(\boldsymbol{x}) - 4\pi f^2 \int d\boldsymbol{x}' \frac{e^{2\pi i f |\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|} \Phi(\boldsymbol{x}')\psi(\boldsymbol{x}')$$

- first order approximation solution

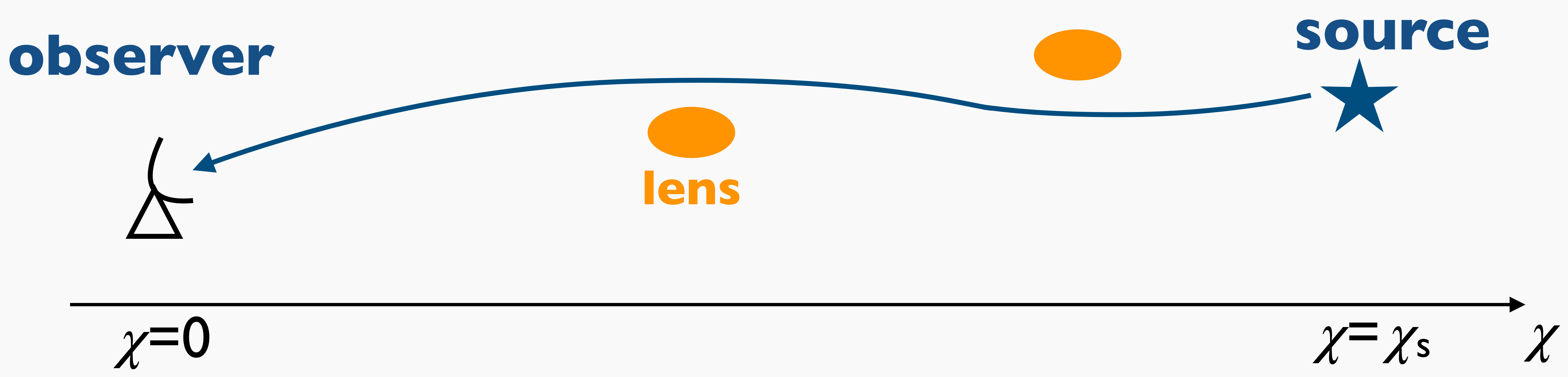
$$\psi(\boldsymbol{x}) = \psi_0(\boldsymbol{x}) - 4\pi f^2 \int d\boldsymbol{x}' \frac{e^{2\pi i f |\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|} \Phi(\boldsymbol{x}')\psi_0(\boldsymbol{x}')$$

(See e.g., Mizuno & Suyama 2023 for the validity of Born approximation)

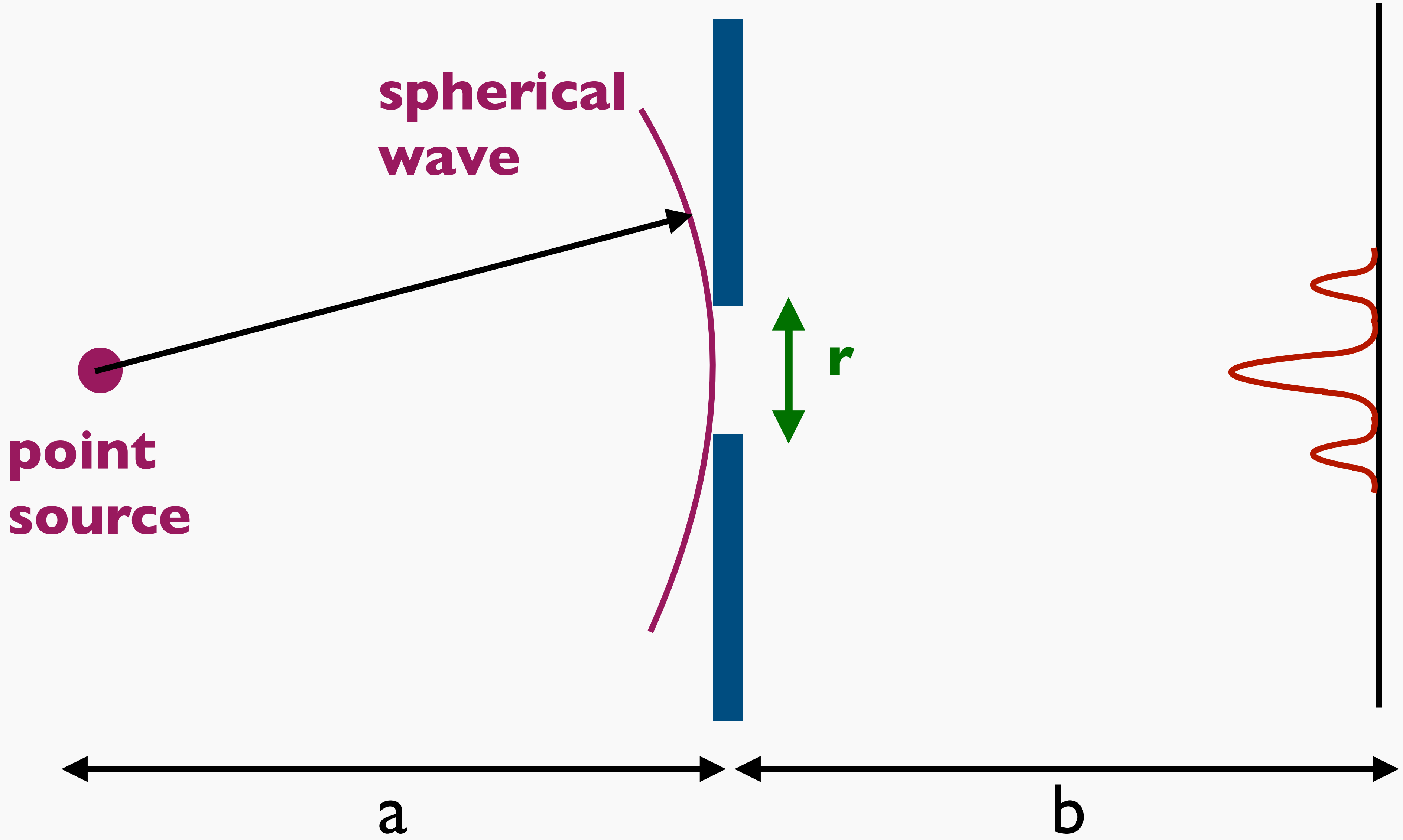
Solution in Born approximation

$$\frac{\psi}{\psi_0} = 1 - 4\pi f^2 \int d\chi \int d\boldsymbol{r} \frac{\chi_s}{\chi(\chi_s - \chi)} \Phi(\boldsymbol{x}) e^{2\pi i f \Delta t_g}$$

$$\Delta t_g = \frac{\chi_s}{2\chi(\chi_s - \chi)} \left| \boldsymbol{r} - \frac{\chi}{\chi_s}\boldsymbol{r}_s \right|^2 = \frac{\chi_s}{2\chi(\chi_s - \chi)} \left| \boldsymbol{r} - \boldsymbol{r}_\perp \right|^2$$



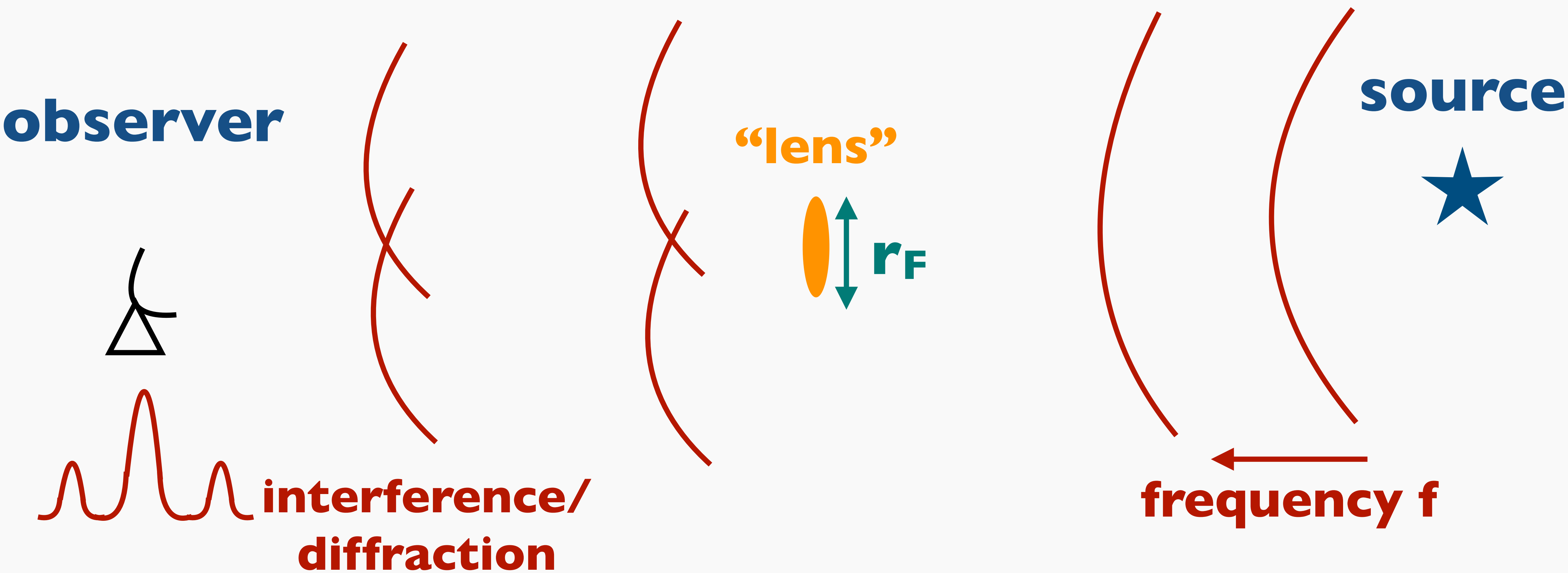
Fresnel diffraction in optics



$$r \lesssim \sqrt{\frac{ab}{a+b}\lambda}$$ → fringe pattern

Fresnel scale

- Fresnel scale r_F $$r_F \sim 1\,\mathrm{pc}\left(\frac{f}{1\,\mathrm{Hz}}\right)^{-1/2}\left(\frac{\chi(\chi_s-\chi)/\chi_s}{1\,\mathrm{Gpc}}\right)^{1/2}$$



➡ **probing small-scale DM distribution??**

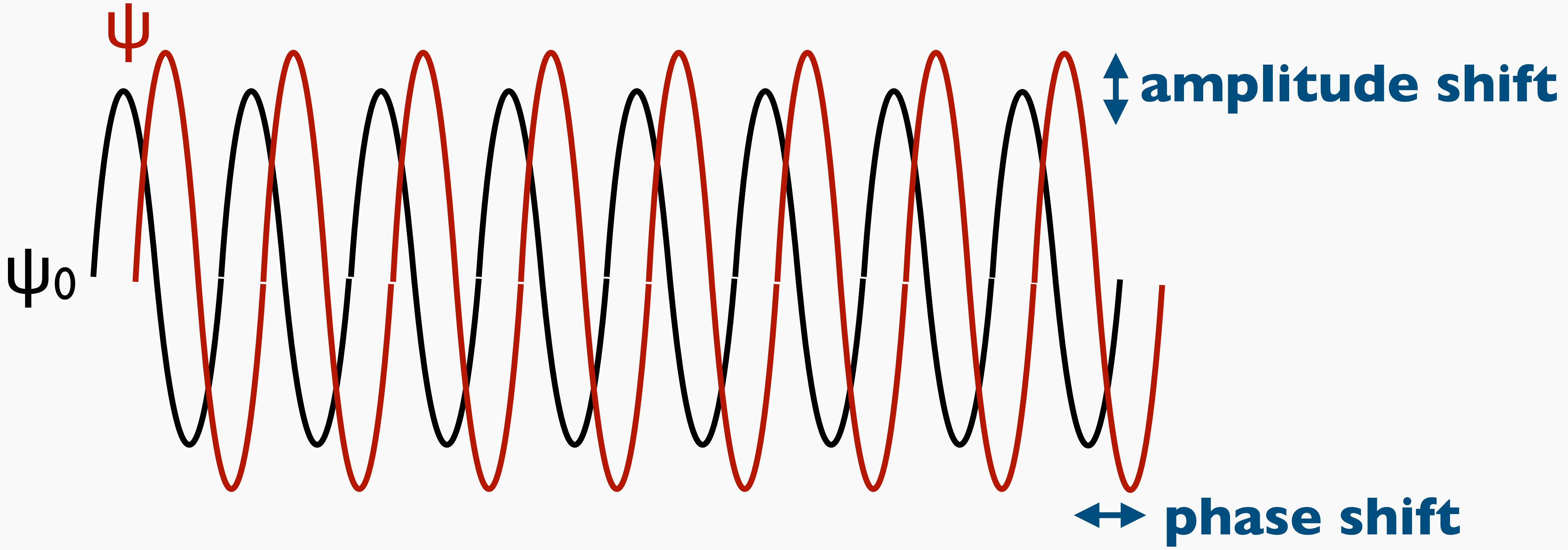
(Macquart 2004; Takahashi+2005; Takahashi 2006; …)

Observables: amplitude/phase shift

connection with deformation of observed waveform

$$\text{amplitude shift} = \mathrm{Re}\left(\frac{\psi}{\psi_0}\right)$$

$$\text{phase shift} = \mathrm{Im}\left(\frac{\psi}{\psi_0}\right)$$



Amplitude and phase fluctuations

- Born approximation to connect amplitude and phase fluctuations with matter power spectrum P(k)

$$\frac{\psi}{\psi_0} = \left[1 + K(f)\right] e^{iS(f)}$$

$$\langle K^2(f)\rangle = \int_0^{\chi_s} d\chi W^2(\chi) \int \frac{k\,dk}{2\pi} P(k) \left[\frac{\sin(r_F^2 k^2/2)}{r_F^2 k^2/2}\right]^2$$

amplitude

$$\langle S^2(f)\rangle = \int_0^{\chi_s} d\chi W^2(\chi) \int \frac{k\,dk}{2\pi} P(k) \left[\frac{\cos(r_F^2 k^2/2) - 1}{r_F^2 k^2/2}\right]^2$$

phase

$$W(\chi) = 4\pi G\bar{\rho} a^{-1} \frac{\chi(\chi_s - \chi)}{\chi_s}$$

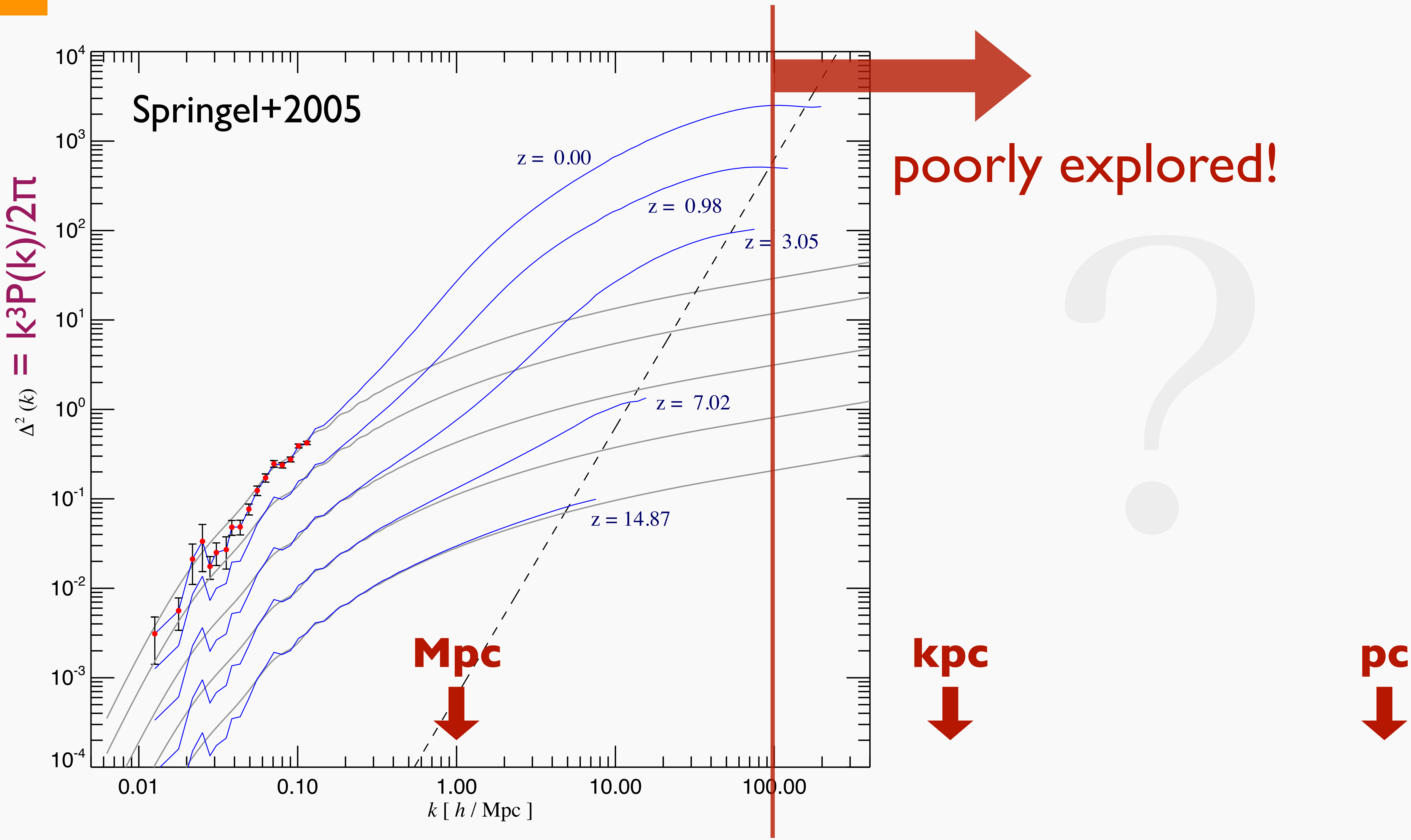
matter power spectrum

wave effect depending on Fresnel scale r_F

$$r_F = \sqrt{\frac{\chi(\chi_s - \chi)}{2\pi f \chi_s}}$$

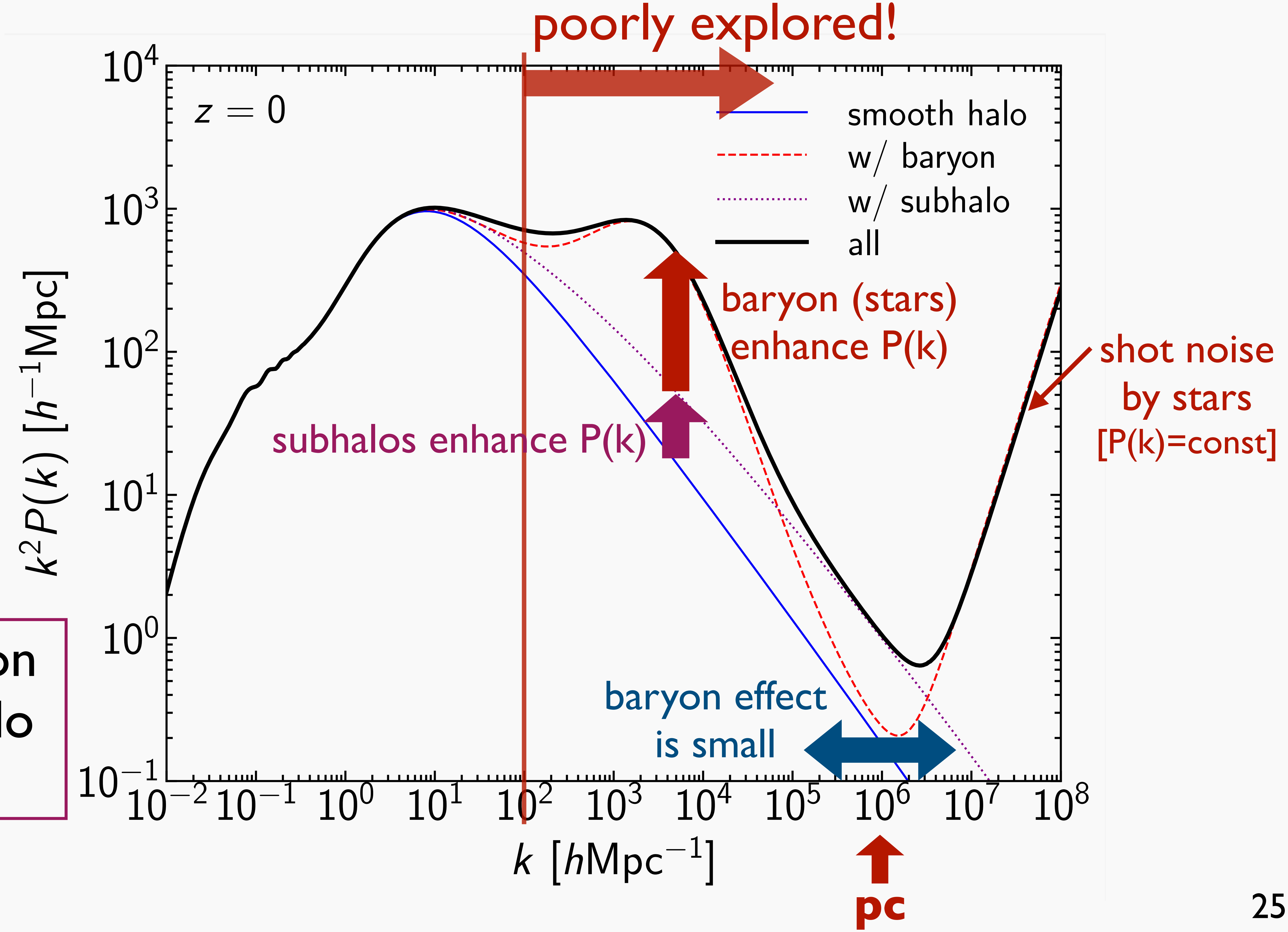
- can be measured using **frequency evolution**

P(k) at pc scale??



MO & Takahashi ApJ **901** (2020)58

P(k) at very high k

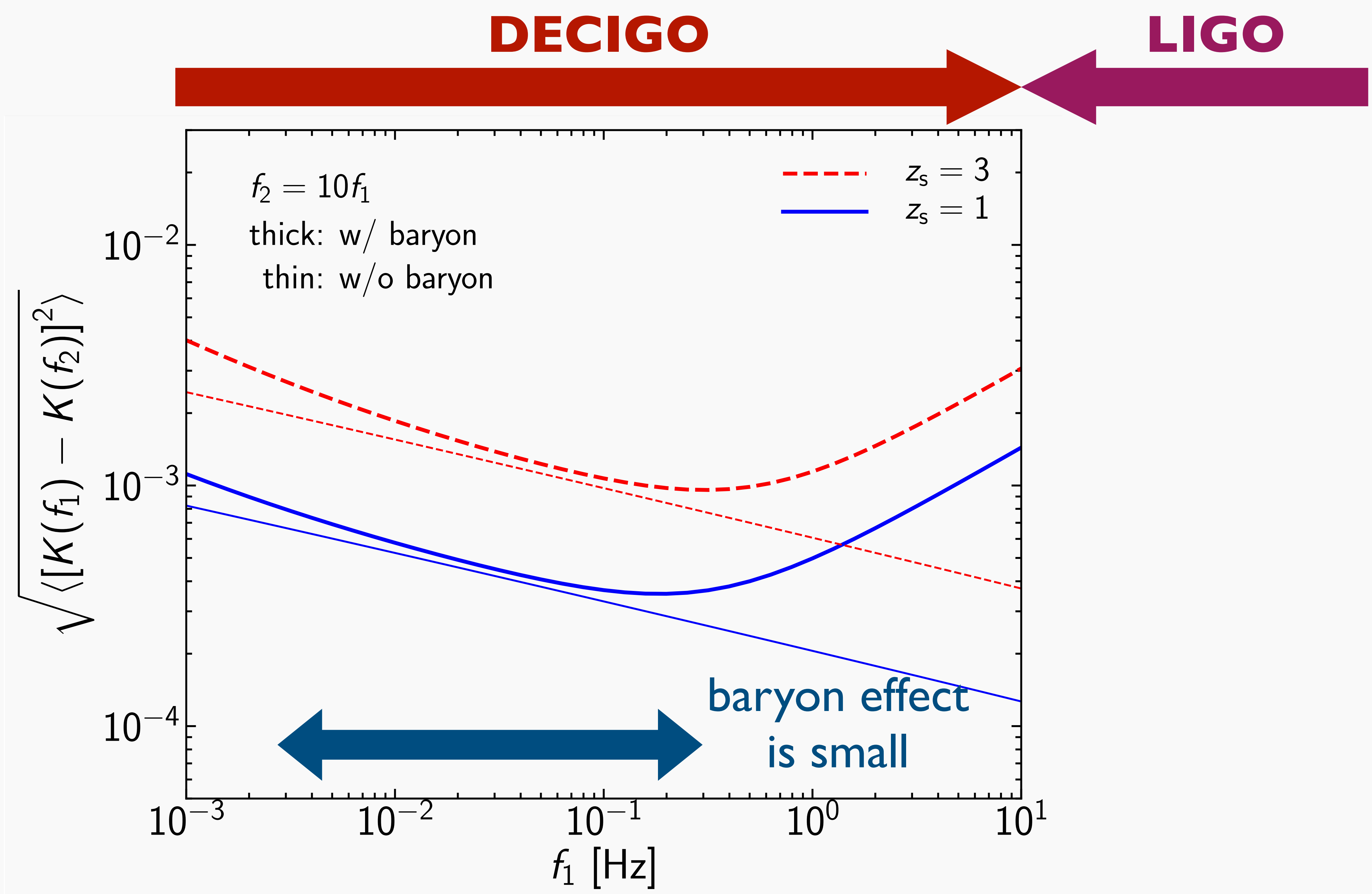


prediction using halo model

MO & Takahashi ApJ **901** (2020)58

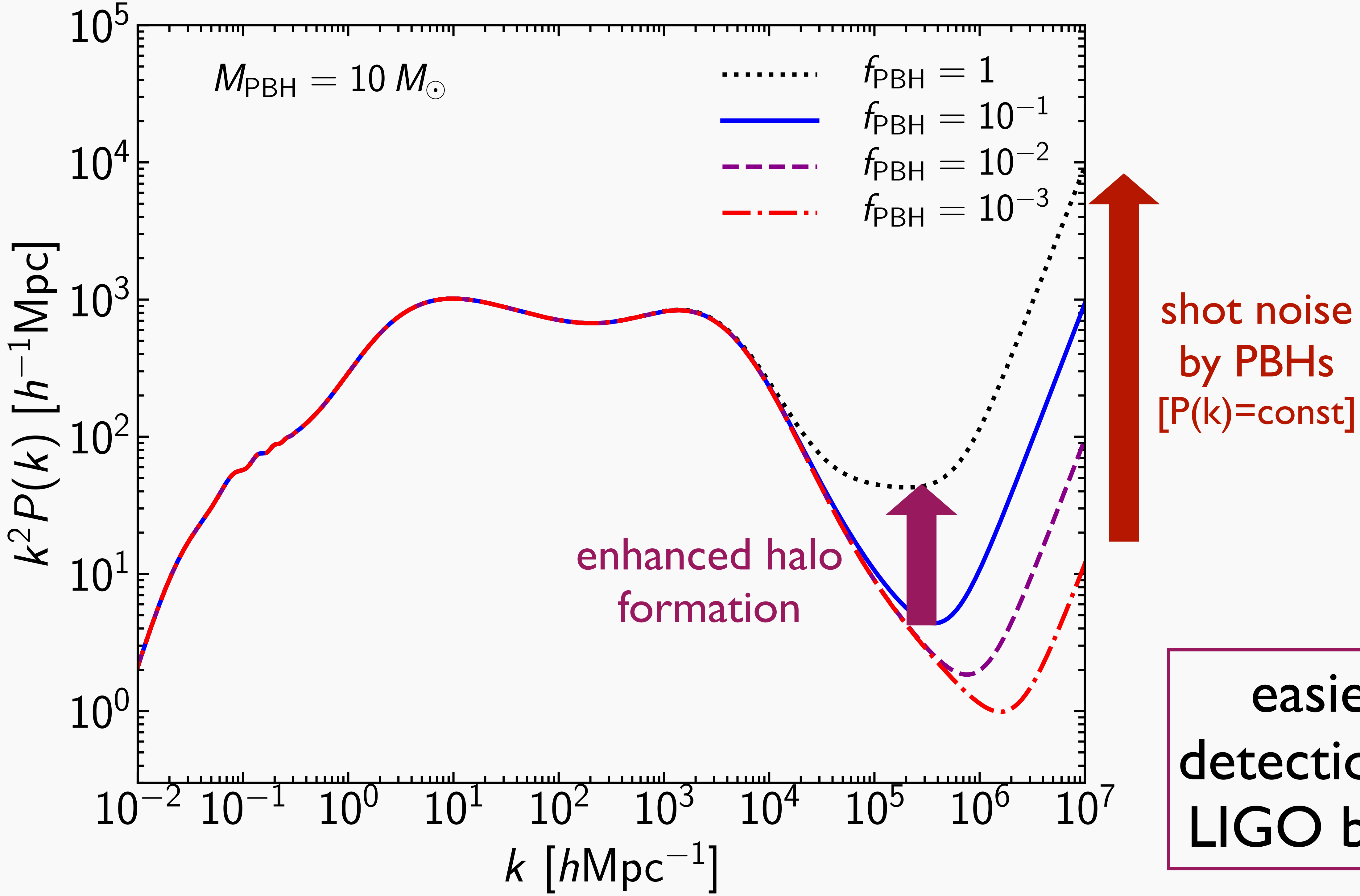
Expected signal

frequency evolution of amplitude fluctuation



- signal small, need many (~10^5) GWs for detection!

Primordial black hole (PBH) scenario

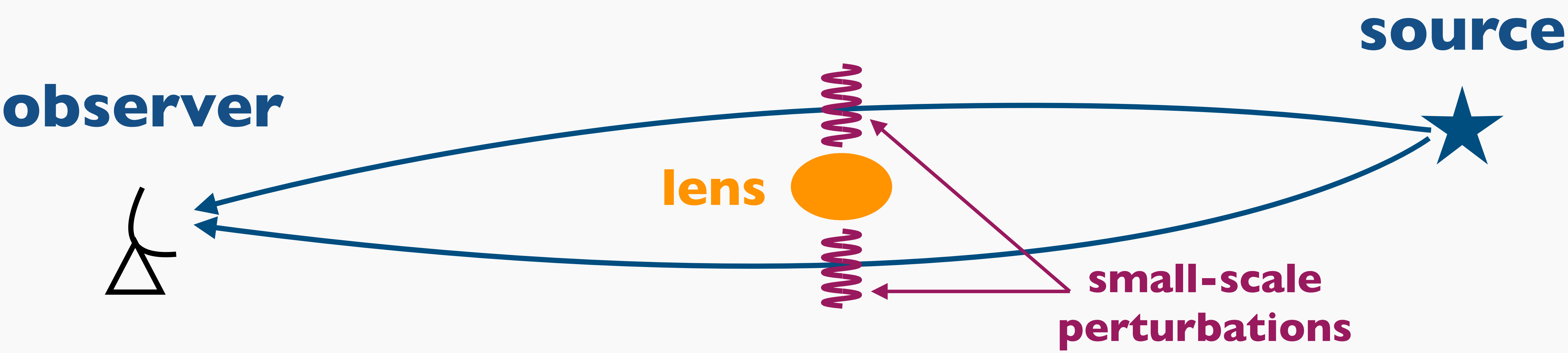


easier detection in LIGO band

Prospect?

- expected amplitude and phase fluctuations for random line-of-sight are small (**$\sim 10^{-3}$**)
- need to combine large number of events for detection, which is challenging
- **any cleverer way?**

Wave effects for lensed sources



- strong lensing by a massive lens, for which **geometric optics** is valid
- small-scale density perturbations that produce **perturbative wave optics effects** on highly magnified sources

MO & Takahashi PRD **106**(2022)043532

Fluctuations for lensed sources

- amplitude and phase fluctuations

$$\langle K_j^2(f)\rangle = \int \frac{k\,dk}{2\pi}\, P_\kappa^j(k)\, \underbrace{\frac{1 - J_0(A^{j,-})\cos(A^{j,+})}{2A^2}}_{\text{kernel}}$$

amplitude

$$\langle S_j^2(f)\rangle = \int \frac{k\,dk}{2\pi}\, P_\kappa^j(k)\, \underbrace{\frac{3 - 4J_0(A^{j,-}/2)\cos(A^{j,+}/2) + J_0(A^{j,-})\cos(A^{j,+})}{2A^2}}_{\text{kernel}}$$

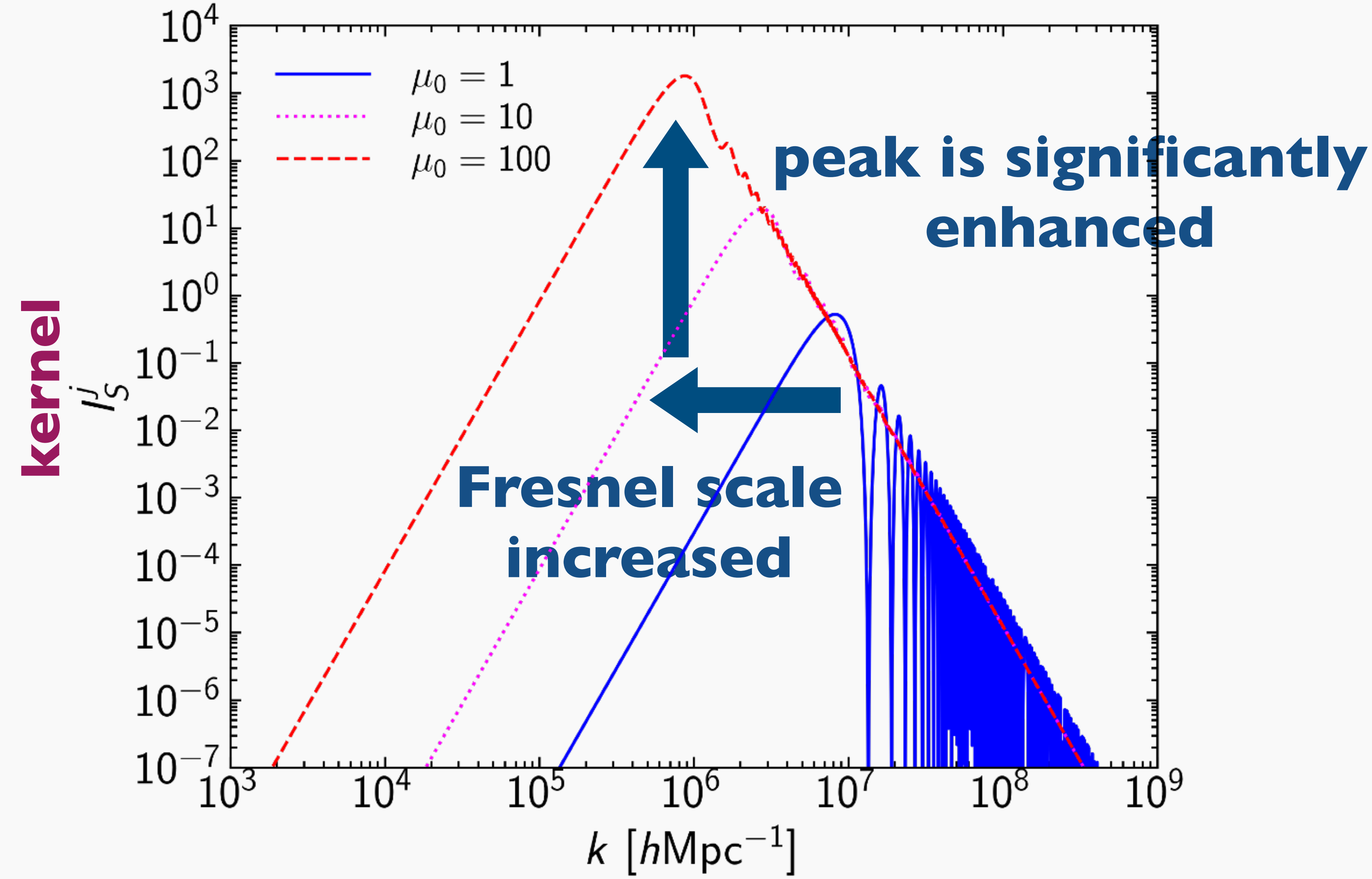
phase

$$A = r_{\rm F}^2 k^2/2 \qquad A^{j,\pm} = (\mu_{j,1} \pm \mu_{j,2}) r_{\rm F}^2 k^2/2$$

macro model magnification
$\mu_0 = \mu_{j,1}\mu_{j,2}$

MO & Takahashi PRD **106**(2022)043532

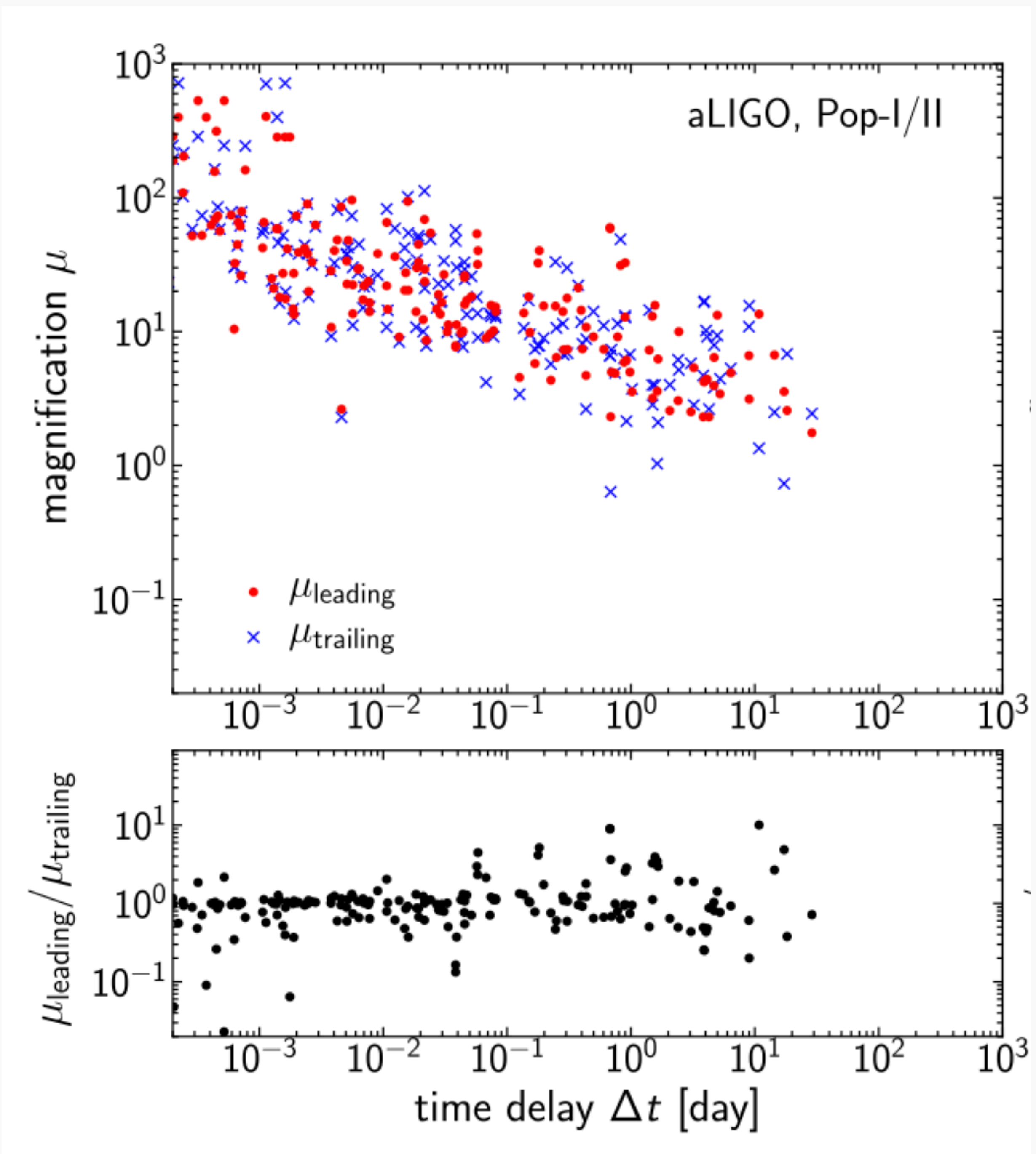
Enhanced signals

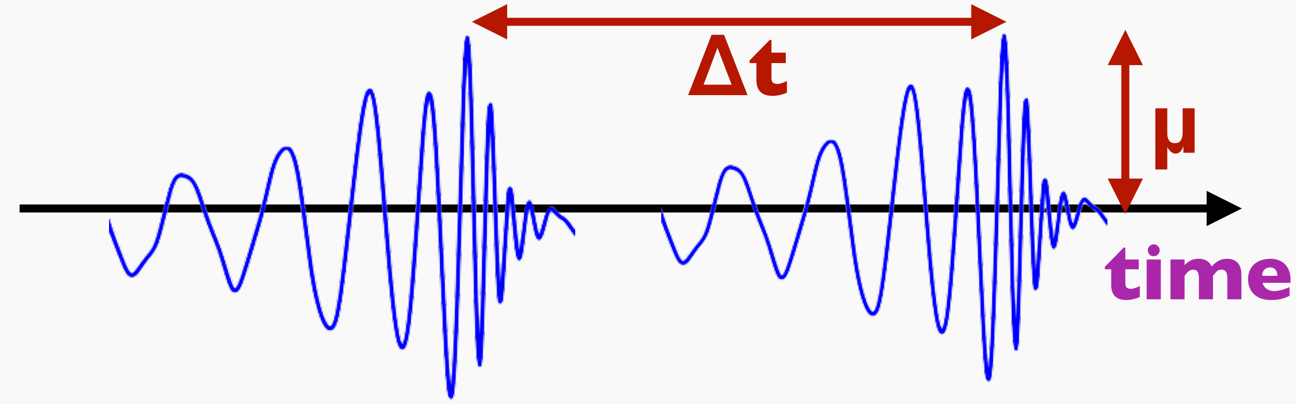


- μ_0 significantly modify kernel functions

➡ **detection possible even for a few events in LIGO band!**

Strongly lensed gravitational waves?





- even **advanced LIGO** can discover some events
- due to selection effect, those events have **large μ** and **small Δt**

Summary

- density fluctuations on the **Fresnel scale** (~pc) cause frequency-dependent amplitude and phase fluctuations
- expected signal is small (~10^{-3}) for random line-of-sight, but it can be **significantly enhanced** for strongly lensed, magnified events
- unique probe for dark matter subhalos, fuzzy dark matter (see also Kawai, MO+ ApJ **925**(2022)61), PBHs, …