Dipole ΛCDM model:

Toward a realistic model building in dipole cosmology

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Based on my recent research program

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[GR-QC-Cosmo-Astro] online seminar series

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Introductory Remarks

- Science progresses through making assumptions and *testing them*.
- The assumption-verification cycle is more pronounced in cosmology, where we can only make observations which come with their own limitations.
- Verification of some of the assumptions in cosmology may remain out of reach for a long time.
- In some cases these assumptions are promoted to the level of principles.
- Cosmological Principle is one of the very basic assumptions in cosmology.

■ Cosmological Principle (CP)

- Historically Copernicus introduced the first cosmological principle:

  We are not special observers in space and in time.

- Copernican CP was challenged by Olbers' paradox (in 1820s) and refuted by Slipher's and Hubble's observations in 1920s. They established that the Universe is expanding and hence we are special in time.

- The same observations also laid the basis of modern cosmology which is about formulating evolution of the Universe and that Einstein's General Relativity (EGR) is the appropriate setup for cosmology.

- A. Friedmann in 1919 studied a prototype of what would become the widespread setup for cosmological models, even up to date.

- These models, which after the main contributors are known as Friedmann–Lemaítre-Robertson–Walker (FLRW) cosmologies, underlie the theoretical foundations of the (new) CP:

The Universe around us in constant time slices is
homogeneous and isotropic.

- Observationally, however, it is much harder to define and establish the CP.

- In cosmology we have background and perturbations.

- We have time scales (measured by redshift) and also length scales (distances).

- There are structures at different scales around us,
  - galactic scales, few 100 kpc;
  - our own local cluster few Mpc;
  - Farther cluster/superclusters, like Shapley supercluster, which extend up to few 100 Mpc;
  - Scales associated with globular clusters, recently pushed to as far as 3 Gpc by JWST;
  - Scales associated with Cosmic Microwave Background (CMB) beyond 4 Gpc.

- One may *observationally* define *statistical* or *coarse-grained* homogeneity or isotropy.

- In either case there is

  - a scale above which, and

  - a precision level below which,

  one is instructed to find homogeneity or isotropy.

- The above is about perturbations and do not tell us much about background level and where/when one is in fact moving with the *Hubble flow*.

- Observationally, the limits on the scale and precision has been constantly pushed, especially in the recent 10 years. [see arXiv: 2207.05765 for a recent review.]

- Observationally, probing inhomogeneity is much harder than anisotropy.

- (An)isotropy or (in)homogeneity are not observationally established facts.

- There are already various hints, from diverse observables, from CMB to supernovae, QSO and superstructures like Giant Arc and ..., for anisotropy and inhomogeneity.

- See our recent review on this: ”Is the observable Universe consistent with the cosmological principle?” [arXiv:2207.05765 [astro-ph.CO]].

- It is very timely to think about the CP and its verification.

FLRW cosmology

Real World

Figures: Courtesy of Keenan Crane

■ Outline of the Talk

- A theoretical statement of the Cosmological Principle.
- Dipole Cosmological Principle, a first step away from FLRW framework.
- Basics of Dipole Cosmology.
- Simple dipole models and their generic features.
- A dipole instability in FLRW framework.
- Dipole ΛCDM.
- Concluding remarks and outlook.

■ Cosmological Principle, a theoretical statement.

- A clean, unambiguous and yet practically useful statement of the CP is challenging.

- As a minimal (and maybe naive) theoretical definition, assuming one can unambiguously distinguish background from perturbations:

  CP ≡ background spatial homogeneity and isotropy.

- With this practical definition, CP necessitates FLRW background cosmology:

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 + kr^2} + r^2 d\Omega^2 \right]$$

  $a(t)$ is the scale factor, $d\Omega^2$ is the metric of the celestial sphere S^2.
  $k = 0, +1, -1$ is the spatial curvature (respectively corresponding to flat, closed and open cosmologies)

- Isotropy fixes existence of the rotationally symmetric S^2 factor.

- Homogeneity fixes the spatial dependence of the metric.

- In more abstract language,
  - isotropy implies existence of 3 Killing vectors forming $so(3)$ algebra.
  - homogeneity implies existence of 3 commuting Killing vectors, like algebra of translations.

- So, the CP guarantees existence of at least $6 = 3 + 3$ Killing vectors.

- In the same language, old Copernican CP implies having 10 Killing vectors, i.e. de Sitter, Minkowski or AdS space.

- In cosmological models, besides the metric we have the energy momentum of the stuff filling the universe $T_{\mu\nu}$.

- The CP restricts the form of $T_{\mu\nu}$:

$$\mathcal{L}_\xi T_{\mu\nu} = 0$$

where $\mathcal{L}_\xi$ denotes Lie derivative along either of the six Killing vectors.

- The above yields the following form for $T_{\mu\nu}$:

$$T^\mu{}_\nu = \text{diag}(-\rho(t), p(t), p(t), p(t)).$$

- So, $T_{\mu\nu}$ is specified by two functions of time, energy density $\rho(t)$ and pressure $p(t)$.

- The three functions $a(t), \rho(t), p(t)$ are governed by the Einstein equations,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}, \qquad 8\pi G \equiv 1.$$

- Einstein equations yield Friedmann equations:

$$3H(t)^2 = \rho(t), \qquad H(t) = \frac{\dot{a}}{a},$$

$$\dot{\rho} + 3H(t)(\rho + p) = 0$$

- Supplemented by an equation of state (EoS), $p = p(\rho)$ or in more usual notation

$$\omega := \frac{p}{\rho},$$

we have a complete set of Ordinary Differential Equations (ODEs) which may be solved.

■ Tilted Cosmologies [King & George Ellis ’1973]

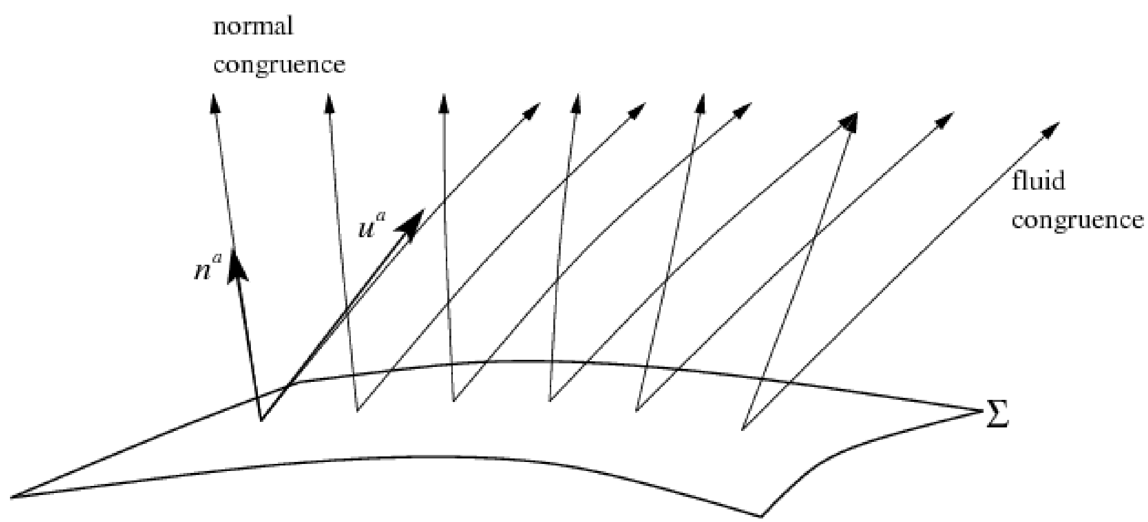
- In FLRW cosmology, flow of matter is zero at constant t (comoving time) surfaces, manifested in that $T^{\mu}{}_{\nu}$ has no time-space components in usual comoving coord. system.

- This is an observer dependent statement and in non-comoving frames we generically have flows.

- If the rest frame of different cosmic fluids and/or metric are not the same, we have cosmic tilts.

- Tilts are boost parameters parametrizing *relative rapidity* of different components in the cosmic fluid and the metric.

- Tilts need not be a constant and can evolve with cosmic time.



Tilt in Cosmic Fluids [Coley, Hervik, Lim'2006]

- Having cosmic tilts breaks isotropy.
- Tilted cosmologies maybe formulated within anisotropic Bianchi models.
- Cosmological tilt and anisotropic shear are features in tilted cosmologies.
- There is also noncomoving cosmology setting in which background metric is FLRW but cosmic fluids are in noncomoving frames.

Dipole Cosmological Principle.

- Various observational hints for the existence of a cosmic dipole, a preferred direction, in distribution of structures (QSO, SNe, radio galaxies) at redshifts $z \gtrsim 1$ and/or in the CMB, motivating considering anisotropic model. See e.g. [Subir Sarkar et al, Ashok Singal, Dominik Schwarz et al, Christos Tsagas et al, Leandros Perivolaropoulos, Eoin O Colgain, Ruth Durrer et al, ...]

- There are growing evidence that at lower distances ($\sim few 100$Mpc) there are nonzero bulk flows. See e.g. [Brent Tully et al]

- See our review [2207.05765 [astro-ph.CO]] for more.

- A cosmological framework which accommodates cosmological dipoles?

- We want to minimally move away from FLRW framework.

- Presence of a preferred (dipole) direction, breaks isotropy, while one can still keep homogeneity.

- "Maximally Copernican" model which minimally deviates from FLRW may be achieved through Dipole Cosmological Principle (DCP):

  homogeneity and axisymmetry plus "Locally Rotationally Symmetry" (LRS).

- Homogeneity restricts us to Bianchi cosmologies.

- Axisymmetry and LRS restrict us to Bianchi V model.

- DCP allows for 4 Killing vectors and Lie derivative of $T_{\mu\nu}$ along them should vanish. This yields $T^{\mu}{}_{\nu} = T^{\mu}{}_{\nu}(t)$.

- The LRS and axisymmetry implies that there is a boosted frame in which $T^{\mu}{}_{\nu}$ takes the same form as in the FLRW cosmologies.

- Cosmic fluids have flows in a specific direction on constant t slice of the Bianchi metrics.

- DCP assumes that this direction is fixed in time, but the rapidity or tilt parameter $\beta(t)$ varies in time.

■ Dipole Cosmologies [C.Krishnan, R. Mondol, MMShJ 2022]

- Metric ansatz for dipole cosmology (Bianchi V model):

$$ds^2 = -dt^2 + a^2(t)\left[e^{4b(t)}dz^2 + e^{-2b(t)-2A_0 z}(dx^2 + dy^2)\right], \tag{1}$$

  – $a(t)$ is the over all scale factor,

  – $b(t)$ parameterizes the anisotropy,

  – $A_0 \neq 0$ is a constant of dimension of inverse length and may be set to 1 by a choice of units, we keep it for later convenience.

- Overall Hubble parameter $H(t)$ and the shear σ:

$$H(t) := \frac{\dot{a}}{a}, \qquad \sigma(t) := 3\dot{b}$$

where dot denotes derivative w.r.t. cosmic time t.

- Tilted energy-momentum ansatz for dipole cosmology:

$$T^{\mu}{}_{\nu} = T_{\text{iso}}{}^{\mu}{}_{\nu} + T_{\text{tilt}}{}^{\mu}{}_{\nu}, \qquad T_{\text{iso}}{}^{\mu}{}_{\nu} = \text{diag}(-\rho, p, p, p)$$

$$T_{\text{tilt}}{}^{\mu}{}_{\nu} = (\rho + p)\sinh\beta \begin{pmatrix} -\sinh\beta & ae^{2b}\cosh\beta & 0 & 0 \\ -\cosh\beta/(ae^{2b}) & \sinh\beta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2}$$

  - $T_{\text{iso}}{}^{\mu}{}_{\nu}$ is energy momentum tensor of a usual isotropic perfect fluid.
  - Tilt parameter β denotes the bulk flow (rapidity of cosmic fluid) along the z direction.
  - β drops out when $p = -\rho$.
  - Parity along the tilt direction z direction is also broken once we choose a sign (say positive) for A_0.
  - With the choice of sign of A_0, sign of β has physical significance.

Dynamics in Dipole Cosmology

- Dipole cosmology has 5 dynamical variable (functions of time): $H(t), \sigma(t)$ appearing in the metric and $\rho(t), p(t), \beta(t)$ coming from matter sector.

- Einstein equations yields 4 equations (extensions of Friedmann equations).

- Addition of EoS of the cosmic fluid makes these an autonomous set of ODEs, as in the FLRW case.

- Likewise, one may replace 2 of Einstein equations with continuity equations.

Dynamical Equations

$$\dot{H} + 3H^2 - 2\frac{{A_0}^2}{a^2}e^{-4b} = \frac{1}{2}(\rho - p) + \frac{1}{3}(\rho + p)\sinh^2\beta + \Lambda \tag{3a}$$

$$\dot{\sigma} + 3H\sigma = (\rho + p)\sinh^2\beta \tag{3b}$$

$$H^2 - \frac{1}{9}\sigma^2 - \frac{{A_0}^2}{a^2}e^{-4b} = \frac{\rho}{3} + \frac{1}{3}(\rho + p)\sinh^2\beta + \frac{\Lambda}{3} \tag{3c}$$

$$\frac{2A_0}{a}e^{-2b}\sigma = (\rho + p)\sinh\beta\cosh\beta \tag{3d}$$

$$\dot{\rho} + 3H(\rho + p) = -(\rho + p)\tanh\beta(\dot{\beta} - \frac{2A_0}{a}e^{-2b}) \tag{4a}$$

$$\dot{p} + H(\rho + p) = -(\rho + p)\left(\frac{2}{3}\sigma + \dot{\beta}\coth\beta\right). \tag{4b}$$

Only 4 out of the above 6 are independent.

- Shear σ,

$$\sigma = \frac{1}{4A_0} a e^{2b} (\rho + p) \sinh 2\beta, \tag{5}$$

is sourced by the tilt.

- Signs of β, A_0 and σ are correlated.

- For $\beta = 0$ we get either $A_0 = 0$ or $\sigma = 0$.

- We do not consider $A_0 = 0$ case, which yields a Bianchi I model that does not allow for a tilted (dipole) cosmology.

- So, $\sigma = 0 \Longrightarrow \beta = 0$, an open FLRW universe with an untilted matter.

- When $\rho + p = 0$ the tilt β drops out of the equations; β remains unspecified by the equations. For this case, $\sigma = 0$. The solution is a (A)dS$_4$ space in an H^3 slicing.

▶ Model Building within Dipole Cosmology Framework

- Cosmic fluid in cosmology has typically various non-interacting components:

$$p = \sum_i \omega_i \, \rho_i, \qquad \rho = \sum_i \rho_i$$

- One can show that EoM are not consistent if different components have the same tilt.

- Dipole Cosmological Principle force all cosmic fluid to move in the same direction, but each component can have its own tilt, $\beta_i(t)$.

- A generic dipole cosmology model is hence described by

parameters ω_i and dynamical d.o.f ρ_i, $\beta_i(t)$; $H(t)$, $\sigma(t)$.

- For a positive Λ, which we assume, we define energy density ratios:

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}, \qquad \Omega_k := \frac{{A_0}^2}{H^2 a^2 e^{4b}}, \qquad \Omega_\sigma := \frac{\sigma^2}{9H^2}, \qquad \Omega_i := \frac{\rho_i + (\rho_i + p_i)\sinh^2\beta_i}{3H^2}.$$

- Weak energy condition (WEC) $\rho_i \geq 0$ which we assume together with generalized Friedman eq. yield,

$$\Omega_\Lambda + \Omega_k + \Omega_\sigma + \sum_i \Omega_i = 1, \qquad 0 \leq \Omega_X \leq 1, \quad \forall X. \tag{6}$$

- It will be handy to introduce,

$$\kappa := \sqrt{\Omega_k} = \frac{A_0}{Hae^{2b}}, \qquad \sigma := 3H\mathfrak{S}K, \qquad 0 \leq \kappa, K \leq 1, \quad \mathfrak{S} = \pm 1. \tag{7}$$

- $\mathfrak{S}$ is defining sign of σ. $\mathfrak{S}$ is in principle a function of time and can change sign in the course of evolution. However, $\mathfrak{S}$ can only evolve from negative to positive (and not the reverse).

■ Example 1: Single fluid with constant EoS, $p = \omega\rho, \quad -1 < \omega \leq 1$

- $\omega = 1, 1/3, 0, -1/3$ respectively correspond to stiff matter, radiation, pressureless matter, spatial curvature.

- For a generic ω,

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = \frac{\Lambda}{3} - \frac{\rho}{6}(1+3\omega) - \frac{2}{9}\sigma^2 - \frac{\rho}{3}(1+\omega)\sinh^2\beta \tag{8a}$$

$$\dot{\beta}(\coth\beta - \omega\tanh\beta) = (3\omega - 1)H - \frac{2}{3}\sigma - \frac{2\omega A_0}{a(t)}e^{-2b}\tanh\beta \tag{8b}$$

$$\rho^{\frac{\omega}{1+\omega}}\, a e^{2b}\sinh\beta = C = const. \tag{8c}$$

$$\sigma = \frac{1+\omega}{4A_0}\rho\, a e^{2b}\sinh 2\beta, \tag{8d}$$

▶ Simple Analytical Remarks

- Consider non-negative cosmological constant $\Lambda \geq 0$

- For $\beta > 0$

1. As in the FLRW case, for $\omega \leq -1/3$ we get an accelerated expansion for any $\Lambda \geq 0$.

2. As universe expands $a(t)$ grows and $\rho(t)$ drops.

3. The shear $\sigma(t)$ goes to zero (exponentially fast for accelerated expansion) and the universe isotropizes rapidly.

4. Sign of β does not change in the course of evolution, so one can safely choose a sign for β.

$$\dot{\beta}(\coth\beta - \omega\tanh\beta) = (3\omega - 1)H - \frac{2}{3}\sigma - \frac{2\omega A_0}{a(t)}e^{-2b}\tanh\beta \tag{9}$$

5. Since $-1 < \omega \leq 1$, and $\coth\beta > 1$ and $0 \leq \tanh\beta < 1$, the coefficient of $\dot{\beta}$ term is always positive. While the last term in the above can take positive or negative values, it becomes insignificant at late times due to the expansion.

6. Therefore, *for $\omega > 1/3$ the sign of $\dot{\beta}$ is positive and the tilt grows at late times.*

7. Cosmological constant Λ does not explicitly appear in the above and therefore β growth only depends on ω and not the value of Λ;

8. β growth can happen in accelerating/decelerating cosmologies. In fact, β growth is faster in cases with Λ, due to faster growth of H.

9. ***β growth is an instability in FLRW cosmologies w.r.t bulk flow, tilt.***

- For $\beta < 0$:

1. Signs of σ, β, A_0 are correlated with each other. If we fix $A_0 = 1$ (choosing a specific direction along z axis), positive (negative) β means positive (negative) σ.

2. Recall β equation,
$$\dot{\beta}(\coth\beta - \omega\tanh\beta) = (3\omega - 1)H - \frac{2}{3}\sigma - \frac{2\omega A_0}{a(t)}e^{-2b}\tanh\beta$$

3. σ, A_0 terms are both contribute with positive sign. So, for $\omega > 1/3$ all terms in the LHS are positive, hence β growth.

4. For $\omega = 1/3$, again we get β growth (with a lower rate than the $\omega > 1/3$ case).

5. That is, **Tilt in radiation sector (CMB) can grow**.

6. For $\omega < 1/3$, the H term is negative while the last two are positive. We can have **tilt growth** **in** **some cosmological epoch for any** ω.

▸ General results for single fluid, $\sigma < 0, \beta < 0$ case

- In future infinity, β goes to zero for $\omega < 1/3$ and grows (indefinitely) for $\omega \geq 1/3$.

- $\sigma \simeq -3H$ at the BB and goes to zero in future.

- At the Big Bang, β goes to zero for $\omega > -1/3$ and it blows up for $\omega < -1/3$.

- Near the BB metric is

$$
\begin{aligned}
a^3 = 3H_0 t, \quad H = \frac{1}{3t}, \quad \rho \sinh 2\beta = -\frac{6A_0H_0}{1+\omega}(3H_0 t)^{-2/3} \\
ds^2 = -dt^2 + t^{-\frac{2}{3}} dz^2 + t^{\frac{4}{3}} e^{-2A_0 z}(dx^2 + dy^2)
\end{aligned}
\tag{10}
$$

- The Universe is filament-type at the BB.

- We have a curvature singularity.

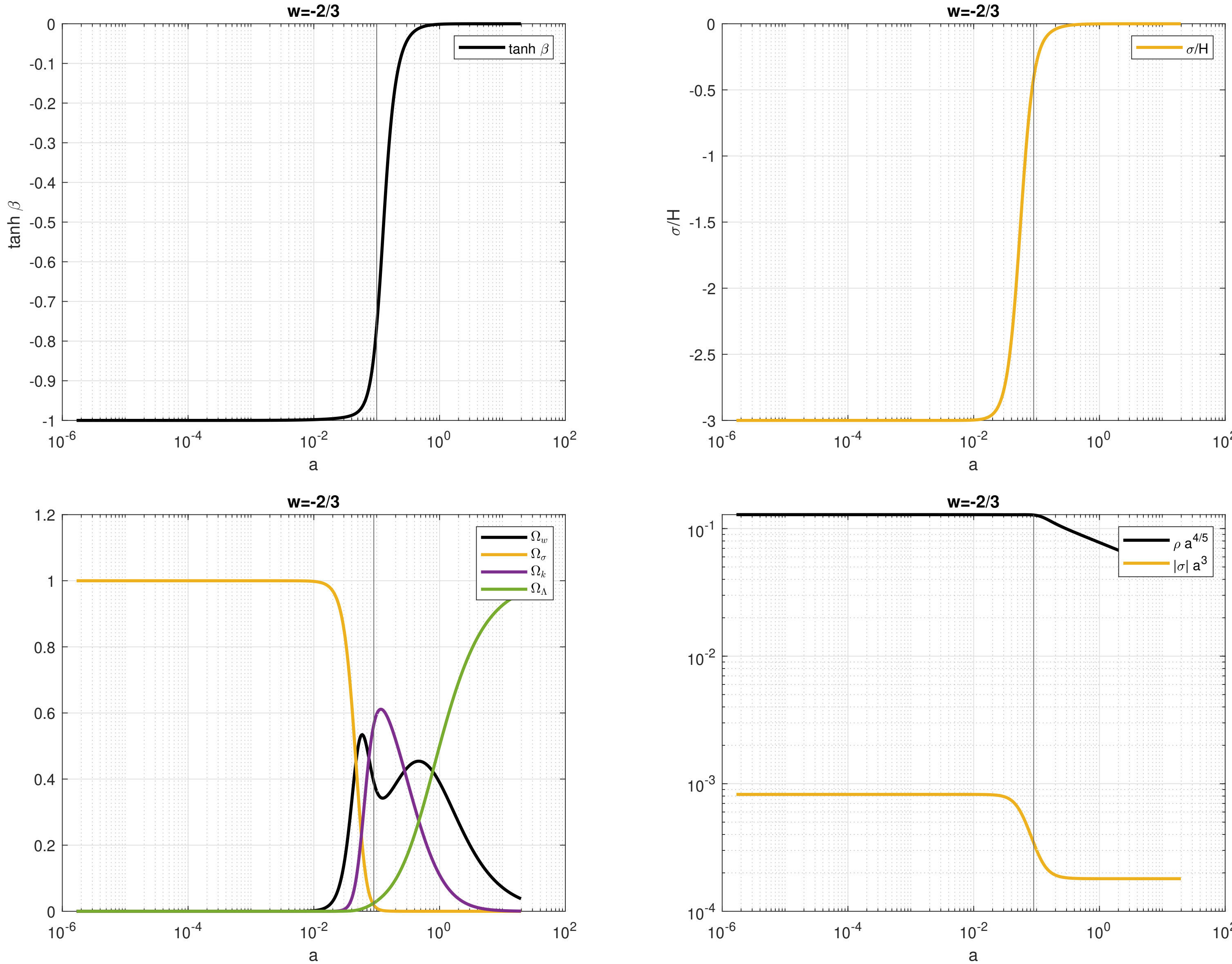
▸ General results for single fluid, $\sigma > 0, \beta > 0$ case

- In future infinity, β goes to zero for $\omega < 1/3$ and grows (indefinitely) for $\omega \geq 1/3$.
- β blows up at the Big Bang for any ω.
- We have a whimper singularity [King & Ellis, (1974)].
- Metric near the BB

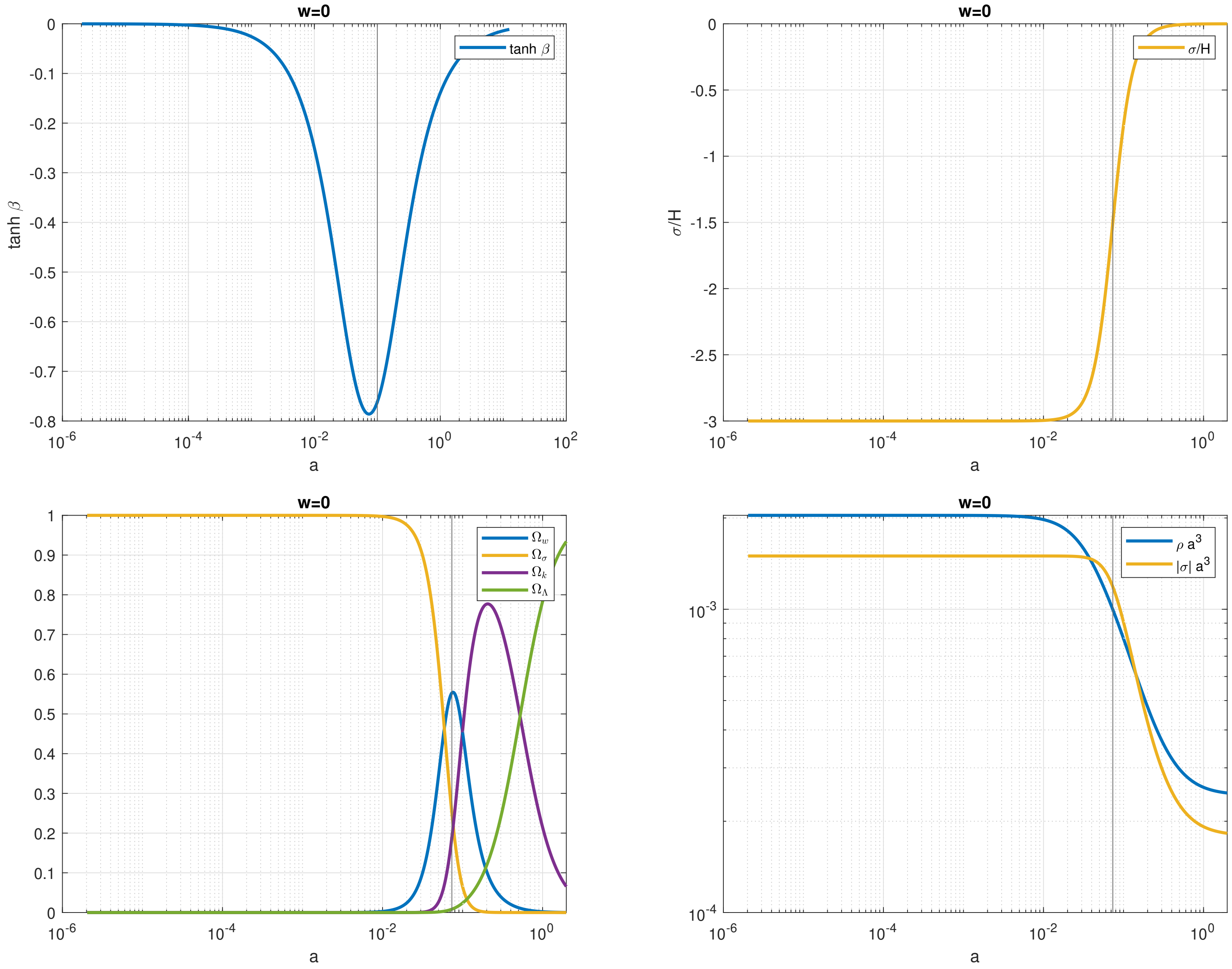
$$\begin{aligned} H &\simeq H_0 \frac{\sinh \beta}{\sinh \beta_0}, \\ ds^2 &= -dt^2 + t^2 dz^2 + t^{\frac{2-2K}{1+2K}} e^{-2A_0 z}(dx^2 + dy^2). \end{aligned} \tag{11}$$

- $0 < \sigma/3H < 1$, the z direction shrinks while the x, y directions are large in the early $t \to 0$ Universe. We have a *pancake type Universe*.

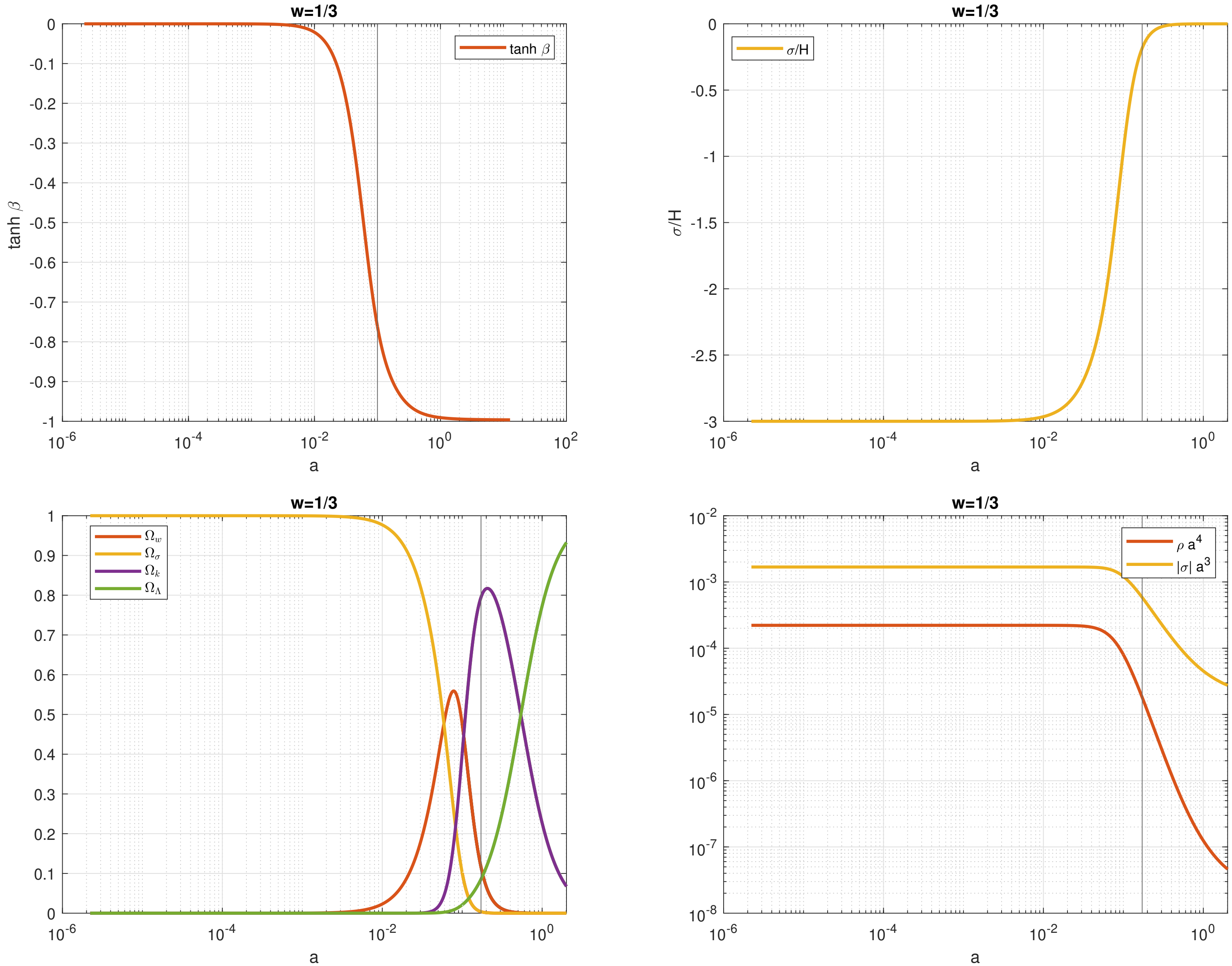
SEC-violating case $\omega = -2/3$ & $\sigma < 0$. We have curvature singularity



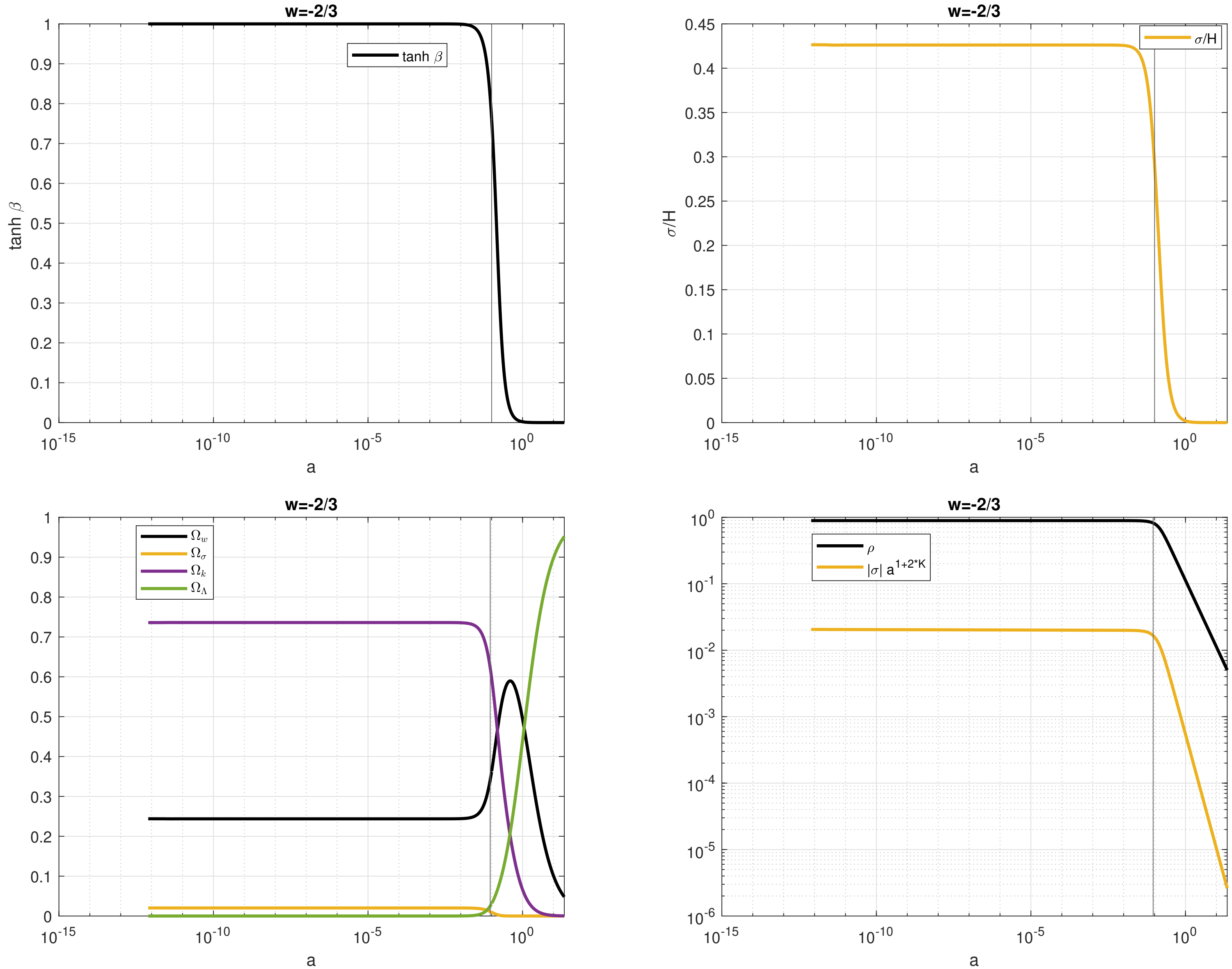
$\omega = 0$ case & $\sigma < 0$. We have curvature singularity.



$\omega = 1/3$ case & $\sigma < 0$. We have curvature singularity.



Whimper singularity $\omega = -2/3$ with $\sigma > 0$.



■ Example 2: Dipole ΛCDM Cosmology

[E. Ebrahimian, C. Krishnan, R. Mondol, MMShJ, 2305.16177 [astro-ph.CO]].
[A. Allahyari, E. Ebrahimian, R. Mondol, MMShJ, arXiv:2307.15791[astro-ph.CO].].

- ΛCDM has $\omega = 0, 1/3$ (on top of Λ)

- Radiation and pressureless mater each has its own tilt:
$$\rho_r,\ \beta_r,\ \rho_m,\ \beta_m;\ H,\ \sigma.$$

- We have an autonomous system of ODEs which may be explored analytically and numerically

- Our analysis shows β_m damps down in late universe, whereas β_r can (mildly) grow if $\beta_r < 0$. In Dipole ΛCDM a generic feature is sizeable relative tilt/dipole between matter and radiation sectors.

- Shear σ also damps out roughly as $\sigma \sim a^{-3}$ in the late Universe.

Depending on the signs of β_m, σ, β_r we have 6 different cases:

(1) $\beta_m > 0, \beta_r > 0$ for which $\sigma > 0$;

(2) $\beta_m < 0, \beta_r < 0$ for which $\sigma < 0$;

(3) $\beta_m < 0, \beta_r > 0$ and $\sigma < 0$;

(4) $\beta_m < 0, \beta_r > 0$ and σ starts negative at the BB and becomes positive;

(5) $\beta_m > 0, \beta_r < 0$ and $\sigma < 0$;

(6) $\beta_m > 0, \beta_r < 0$ and σ starts negative at the BB and becomes positive.

- We evolve equation from near Big Bang to today and also to (near or far) future).

- We choose Planck ΛCDM parameter values:

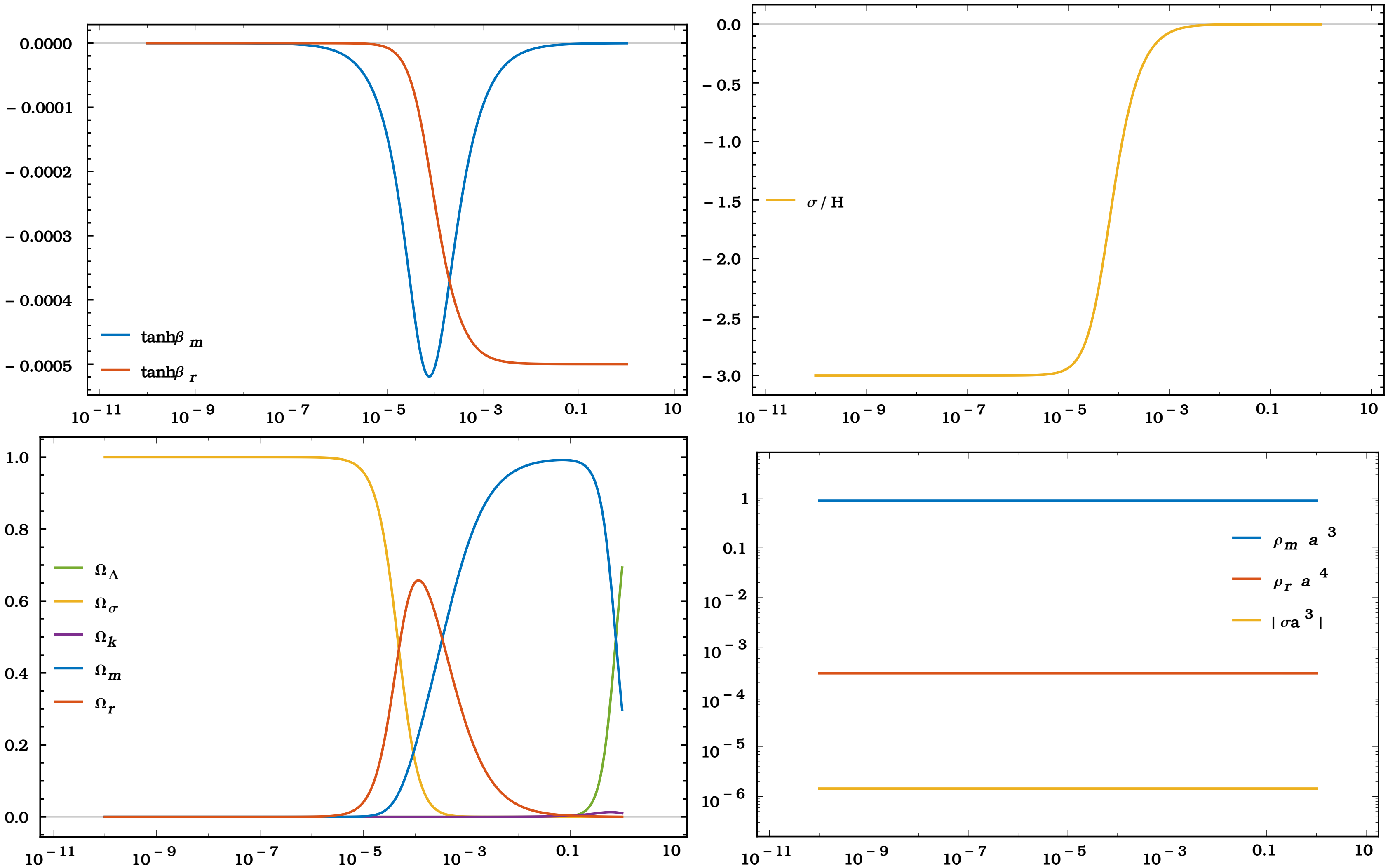
$$t = 0,\ a(0) = 1, b(0) = 1, \rho_r(0) = 3 \times 10^{-4}, \rho_m(0) = 0.9, \Lambda = 2.1, A_0 = 0.1$$

- This data is consistent with Planck values

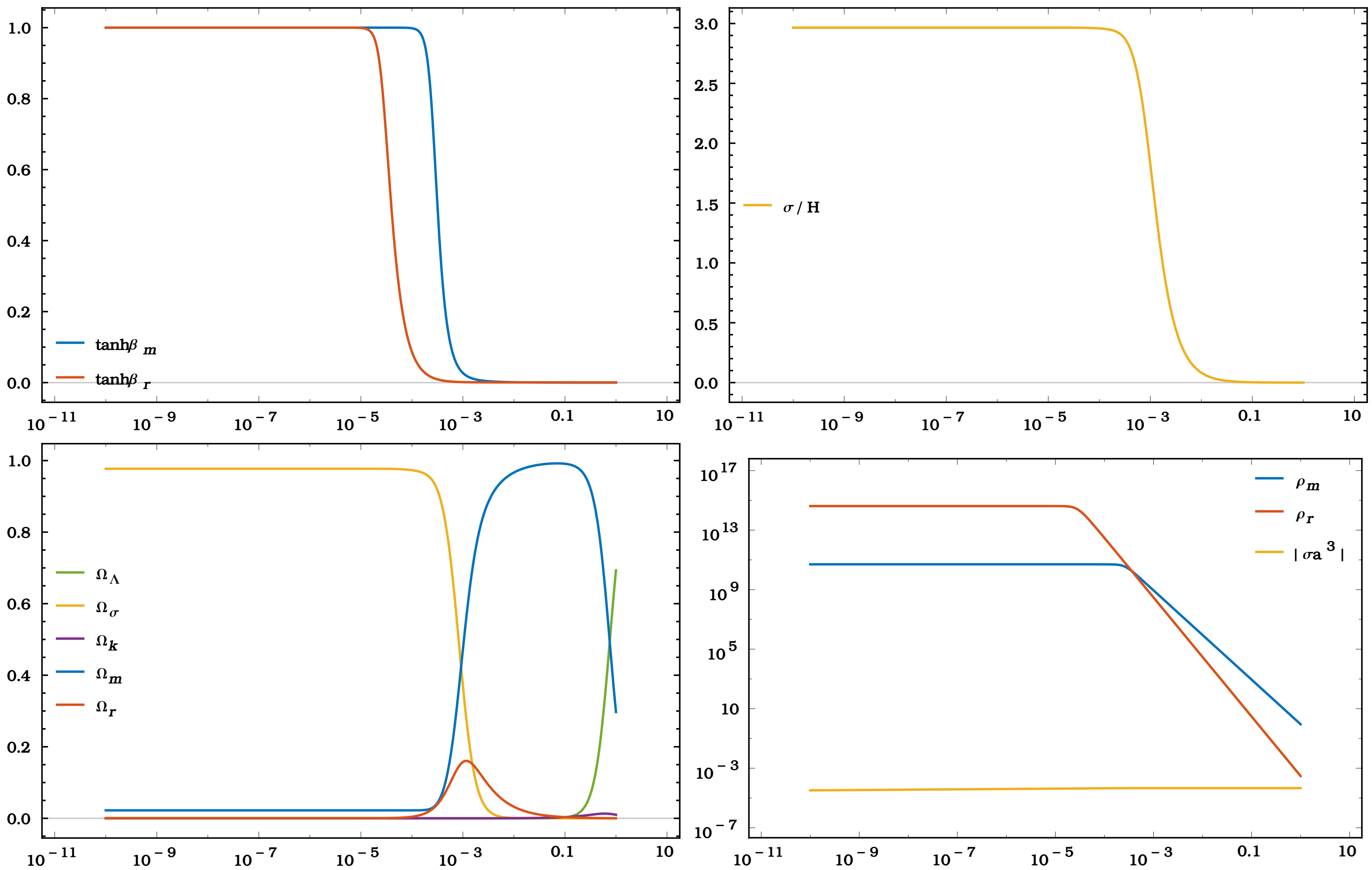
$$\Omega_r \approx 10^{-4}, \Omega_\Lambda \approx 0.7, \Omega_k \approx 0.01, \Omega_m \approx 0.3, \& \quad \Omega_\sigma \sim 10^{-13}.$$

- For β_r, β_m boundary values we choose some typical numbers.

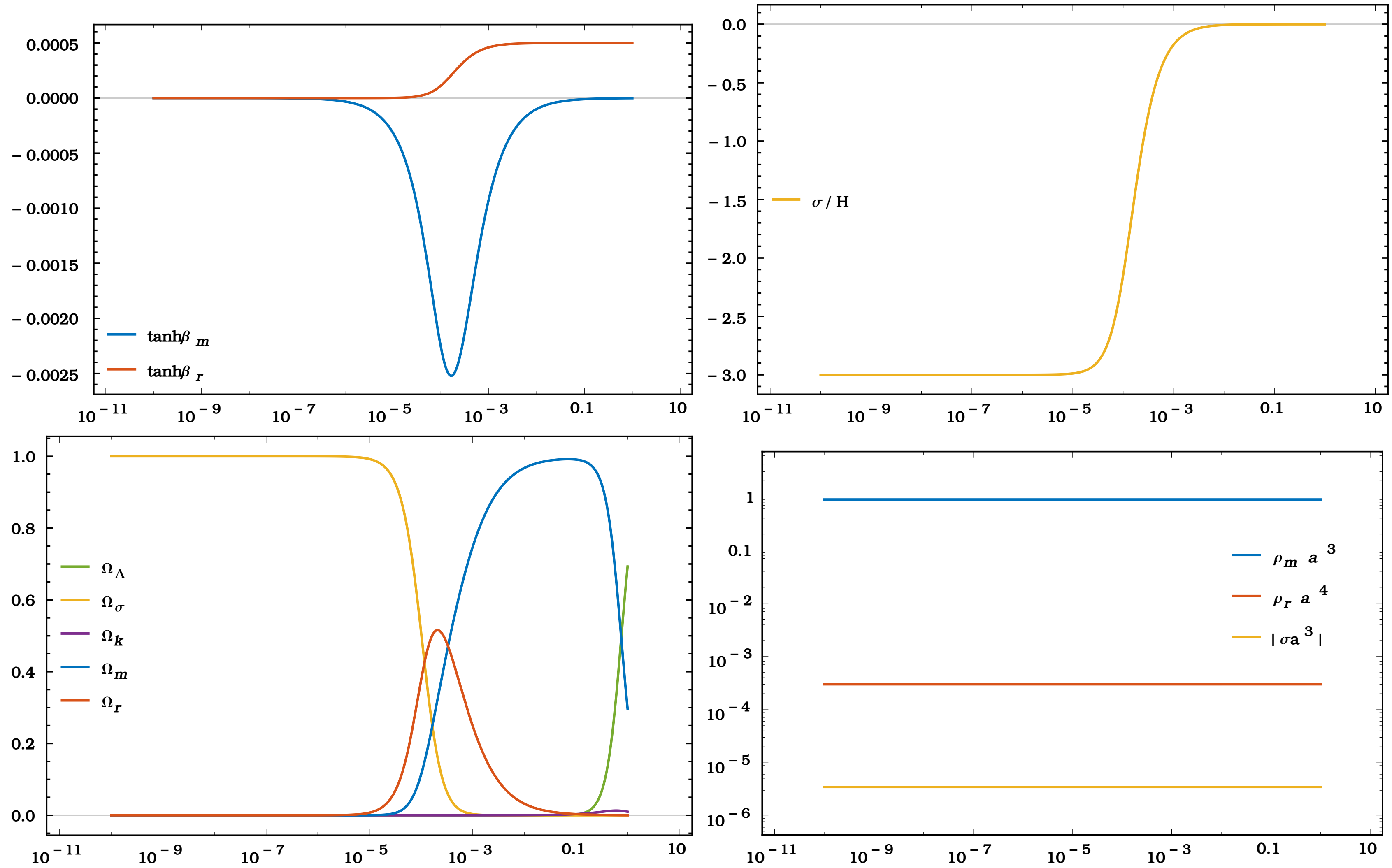
Plots for $\beta_r, \beta_m < 0$ case for boundary conditions $\beta_r(1) = -5 \times 10^{-4}$, $\beta_m(1) = -10^{-7}$.
BB is shear-dominated and the curvature singularity is Weyl-dominated.



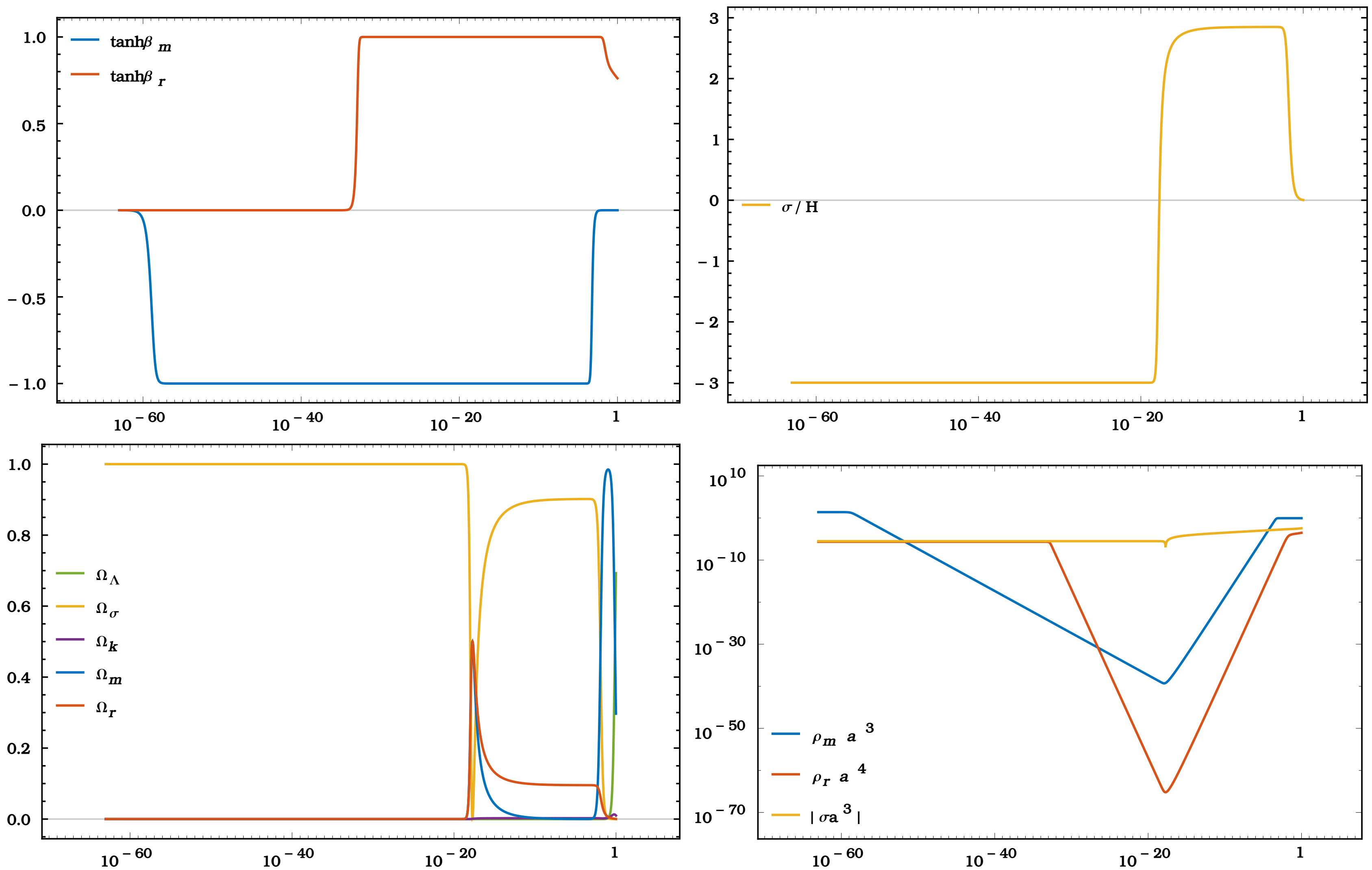
Plots for $\beta_r, \beta_m > 0$ case for boundary conditions $\beta_r(1) = 5 \times 10^{-4}$, $\beta_m(1) = 10^{-5}$. ρ_r, ρ_m go to a constant near the BB & BB is a whimper singularity.



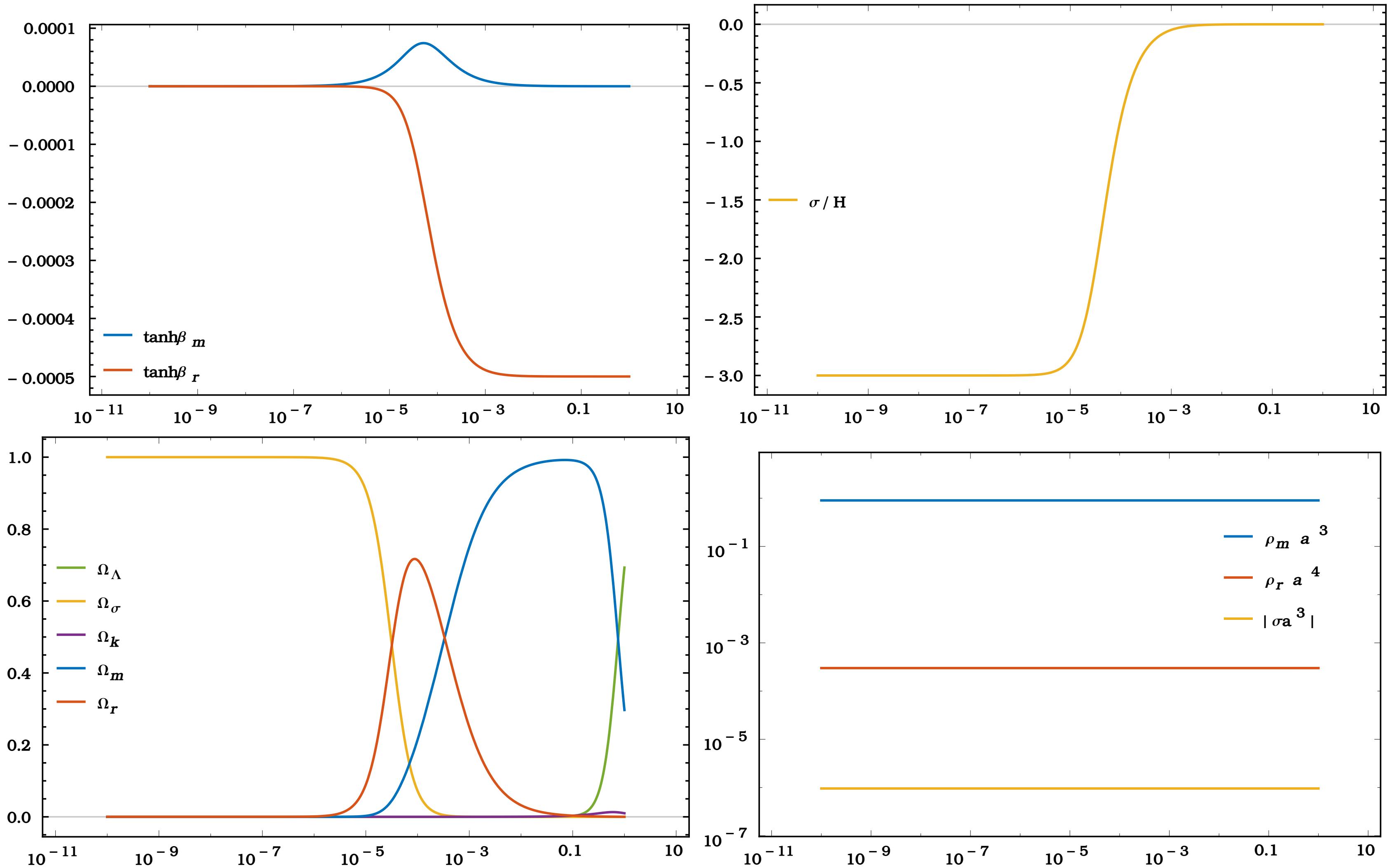
Plots for $\beta_r > 0, \beta_m < 0$, $\sigma < 0$ & boundary conditions $\beta_r(1) = 5 \times 10^{-4}$, $\beta_m(1) = -10^{-6}$.
BB is shear-dominated and the curvature singularity is Weyl-dominated.



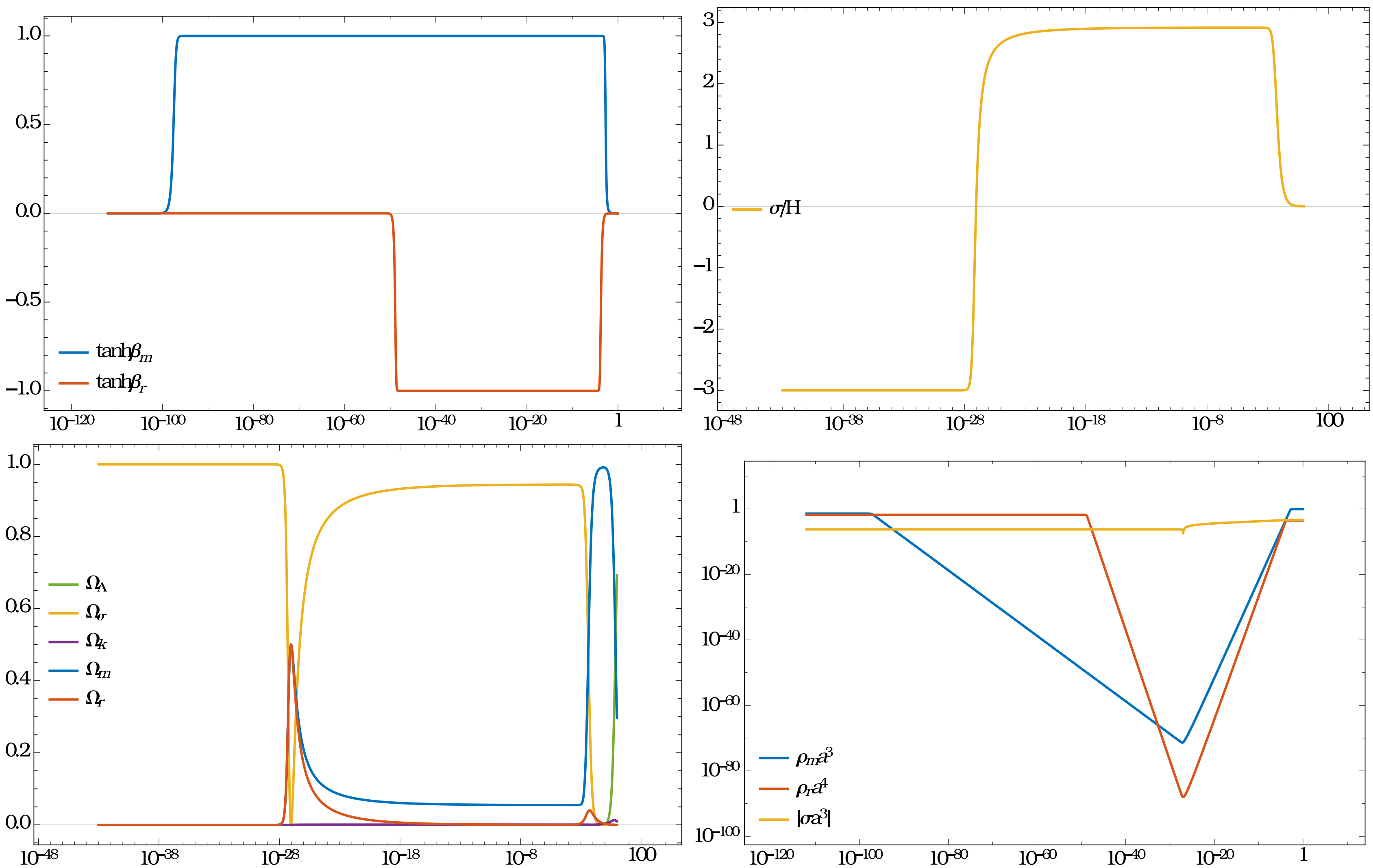
Plots for $\beta_r > 0, \beta_m < 0$, with $\beta_r(1) = 1$, $\beta_m(1) = -5 \times 10^{-7}$. σ changes sign. BB is shear-dominated and the curvature singularity is Weyl-dominated.



Plots for $\beta_r < 0, \beta_m > 0$, with $\beta_r(1) = -5 \times 10^{-4}$, $\beta_m(1) = 10^{-8}$. $\sigma < 0$ in the whole evolution. BB is shear-dominated and the curvature singularity is Weyl-dominated.



Plots for $\beta_r < 0, \beta_m > 0$, with $\beta_r(1) = -5 \times 10^{-4}$, $\beta_m(1) = 10^{-4}$. σ changes sign.
BB is shear-dominated and the curvature singularity is Weyl-dominated.



Discussion, Concluding Remarks and Outlook

⊛ There are various yet-to-be substantiated observational hints for breakdown of homogeneity and isotropy at cosmic scales and redshifts.

⊛ The most significant among them are the evidence for *dipole anisotropy*.

⊛ These may have deep implications for the currently hotly debated cosmic tensions, in particular H_0 tension.

⊛ Cosmic bulk flows may be formulated through the notion of tilt, within the so-called Titled Cosmology setup of George Ellis et al.

⊛ Elaborating on the tilted cosmology ideas, we formulated Dipole Cosmology framework, as the setup replacing FLRW framework with a new maximally Copernican cosmological principle, assuming homogeneity, axisymmetry and Local Rotational Symmetry.

⊛ Worked out and analyzed dynamical equations governing the dipole cosmology, analogue of Friedmann equations for the dipole cosmology.

⊛ In particular, we focused on the evolution of shear σ and the tilt β.

⊛ In a general expanding universe we found that shear σ damps out (essentially as $1/a^3$). This is of course expected recalling Wald's cosmic no-hair theorem.

⊛ In an accelerated expanding universe (e.g. in presence of $\Lambda > 0$) shear damps out exponentially fast.

⊛ The tilt β, grows if the EoS of the cosmic fluid $\omega > 1/3$. The growth is faster in an accelerated expanding universe.

⊛ The radiation tilt β_r, when $\omega = 1/3$ can also grow if $\beta_r < 0$.

⊛ Tilt growth for any ω can happen in some cosmic epoch for some range of initial values of parameters.

⊛ One can hence think about an extension of Wald's theorem in a dipole setting:

While the metric isotropizes (exponentially) fast, in a couple of e-folds,
the radiation sector can grow dipole hair.

⊛ *FLRW cosmology is unstable against homogeneous dipole (tilt) perturbations.*
[Krishnan,Mondol, MMShJ, arXiv:2211.08093[astro-ph.CO]]

⊛ So, theoretically one should expect our universe NOT to be an FLRW cosmology.

⊛ Dipole Cosmologies are hence expected to be very relevant to cosmological model buildings.

⊛ In particular for the Dipole ΛCDM model, a generic feature is sizeable relative dipole between matter and radiation, what seems to be suggested by current observations.

⊛ The Big Bang singularity is different for the dipole cosmologies, [Allahyari, Ebrahimian, Mondol, MMShJ 2307.15791 [astro-ph.CO]]

⊛ For single fluid case, depending on the sign of β we have different singularity structures:

- $\beta > 0$, : we have a whimper singularity (a la King & Ellis).
  In this case β blows up at the BB and we have a Milne type Universe. Tilt goes to zero at late times.

- $\beta < 0$: we have filament universe at the BB, with a curvature singularity.
  Tilt is small at the BB for $\omega > -1/3$, while it may grow at later times $\omega \geq 1/3$.
  Tilt blows up at the BB for $\omega < -1/3$, while it goes to zero later times.

⊛ For Dipole ΛCDM model one can distinguish four distinct cases [Allahyari, Ebrahimian, Mondol, MMShJ 2307.15791 [astro-ph.CO]]

- $\beta_r > 0, \beta_m > 0$: $\sigma > 0$ and drops to zero very fast; whimper singularity, $\Delta\beta \neq 0$ at late times.

- $\beta_r < 0, \beta_m < 0$: $\sigma < 0$ and drops to zero very fast. filament universe at the BB, with a curvature singularity and $\Delta\beta \neq 0$ at late times.

- $\beta_r > 0, \beta_m < 0$: $\sigma < 0$ at the BB, shear-dominated curvature singularity; σ can change sign and $\Delta\beta \neq 0$ at late times.

- $\beta_r < 0, \beta_m > 0$: $\sigma < 0$ at the BB, σ may change sign for which $\Delta\beta \neq 0$ at late times; OR $\sigma < 0$ in the whole evolution for which case $\Delta\beta = 0$ at late times.

There are various

theoretical, *model building*, *observational aspects and features* of

dipole cosmology framework in

both *early and late universe* and

background and cosmic perturbation levels

to be explored and uncovered.

Thank You For Your Attention