# Cosmological observational constraints on the power law f(Q) type modified gravity theory

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Seminar Series in Gravitation, Cosmology and Astrophysics,
STAR-UBB Institute Babeș-Bolyai University
21st March, 2024

Universitatea Transilvania din Brașov
FACULTATEA DE MATEMATICĂ ȘI INFORMATICĂ

Universitatea Transilvania din Brașov

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TRADITIO ET EXCELLENTIA

# Outline of the talk

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- A.G. Riess et al., Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, Astron. J. **116**, 1009 (1998)

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  1. cosmological constant ($\Lambda$)
  2. dark energy
  3. dark matter
  4. **and many more related to GR**

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  1. a manifold $(\mathcal{M})$
  2. a metric structure $(g)$
  3. a connection $(\Gamma)$
- In differential geometry, the affine connection is defined as

$$\Gamma^{\lambda}_{\mu\nu} = \{^{\lambda}{}_{\mu\nu}\} + K^{\lambda}_{\mu\nu} + L^{\lambda}_{\mu\nu}$$

here, $\{^{\lambda}{}_{\mu\nu}\} \rightarrow$ **Levi-Civita connection**, $K^{\lambda}_{\mu\nu} \rightarrow$ **contortion**, $L^{\lambda}_{\mu\nu} \rightarrow$ **nonmetricity tensor**

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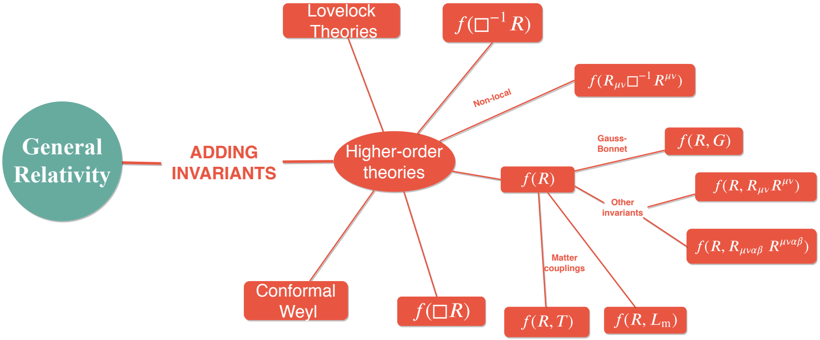
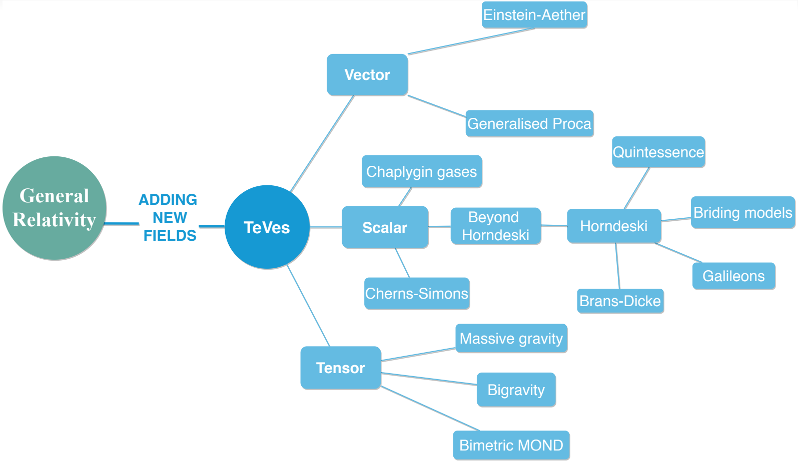
These three theories called **Geometric trinity of gravity**. Apart from these there are many more gravitational theories developed in the literature




**Source:** E.N. Saridakis, et al., Modified Gravity and Cosmology: An Update by the CANTATA Network, arXiv:2105.12582

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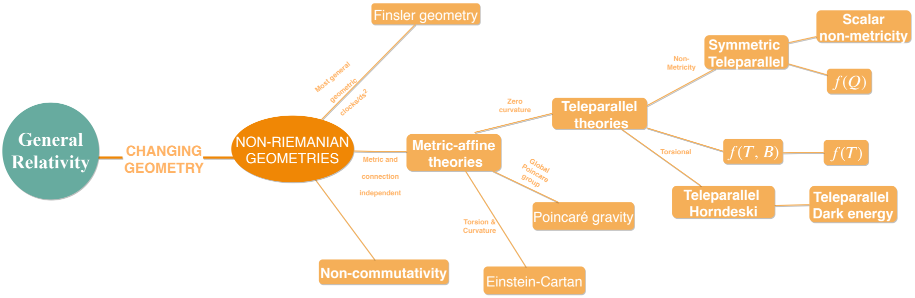


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## Einstein's theory of general relativity

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The Einstein field equations can derived from the following action

$$S = \frac{1}{2k^2} \int R\sqrt{-g} d^4x + \int \mathcal{L}_m(g, \chi, \nabla\chi)\sqrt{-g} d^4x, \quad (1)$$

where $R$ is the Ricci scalar, $g$ represents the determinant of the metric $g_{\mu\nu}$, and $\mathcal{L}_m$ is the matter Lagrangian density, $\sqrt{-g}d^4x$ is the volume element, k is gravitational coupling constant, $\chi$ is the matter field

# The $f(Q)$ Cosmological Model

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- The standard Friedmann-Lemaitre-Robertson-Walker line element, which describes our flat, homogeneous, and isotropic Universe, is given by,

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \tag{2}$$

Here $t$ is the cosmic time, and $x$, $y$, $z$ denote the Cartesian co-ordinates, $a(t)$ is the scale factor

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- We consider the matter content of the Universe as consisting of a perfect and isotropic fluid, with energy-momentum tensor given by

$$T_{\mu\nu} = (p + \rho)u_\mu u_\nu + p g_{\mu\nu}, \tag{3}$$

where $p$ and $\rho$ are the pressure and the energy density of the fluid, $u_\mu$ is the four-velocity vector normalized according to $u^\mu u_\mu = -1$

- Now, we introduce the action for the $f(Q)$ gravity theory, given by [1],

$$S = \int \left[ \frac{1}{2} f(Q) + \mathcal{L}_m \right] \sqrt{-g} d^4x, \tag{4}$$

where $f(Q)$ is a general function of the non-metricity scalar $Q$, ($Q = 6H^2$ for FLRW metric)

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$$Q + \Lambda = Q + F(Q)$$

$$\implies \Lambda = F(Q) = F(Q) = 6\gamma H_0^2 \left(\frac{Q}{Q_0}\right)^n,$$

[1] J. B. Jimenez et al. Coincident general relativity, Phys. Rev. D **98**, 044048, (2018)

The Friedmann equations becomes

$$3H^2 = \rho_r + \rho_m + \rho_{de}, \tag{5}$$

$$2\dot{H} + 3H^2 = -\frac{\rho_r}{3} - p_m - p_{de}, \tag{6}$$

where $\rho_r$, $\rho_m$, and $p_m$ are the energy densities of the radiation and matter components, $p_m$ is the matter pressure, while $\rho_{de}$ and $p_{de}$ are the DE's density and pressure contribution due to the geometry, given by

$$\rho_{de} = \frac{F}{2} - Q\,F_Q, \tag{7}$$

$$p_{de} = 2\dot{H}(2QF_{QQ} + F_Q) - \rho_{de}. \tag{8}$$

- When there are no interactions between the three fluids, the energy densities satisfy the following differential equations

$$\dot{\rho}_r + 4H\rho_r = 0, \quad (9)$$

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (10)$$

$$\dot{\rho}_{de} + 3H(1 + \omega_{de})\rho_{de} = 0. \quad (11)$$

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- The simplest form of the CPL model can be written as,

$$\omega_{de}(z) = \omega_0 + \omega_a \frac{z}{1+z}. \tag{12}$$

- Using equation (11) and (12)/ $p_{de} = \omega_{de}\rho_{de}$, we can find

$$H^2(z) = H_0^2(1+z)^{\frac{3(1+\omega_o+\omega_a)}{n}} e^{-\frac{3\omega_a z}{n(1+z)}} \tag{13}$$

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- Now, we can easily calculate $\rho_{de}$ as

$$\rho_{de}(z) = 3\gamma\,(1-2n)H_0^2(1+z)^{3(1+\omega_o+\omega_a)} e^{\frac{-3\omega_a z}{(1+z)}}. \tag{14}$$

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- From first Friedmann equation, we can find

$$\frac{H^2(z)}{H_0^2} = \Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \gamma\,(1-2n)(1+z)^{3(1+\omega_o+\omega_a)} e^{\frac{-3\omega_a z}{(1+z)}} \tag{15}$$

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**free parameters to be constraint** $\theta_s = (\gamma, n, \omega_0, \omega_a, H_0, \Omega_{m0})$

# Observational Constraints

- **Cosmic Chronometer (CC) Dataset:** Here, we have used 31 Hubble samples in the redshift range $0.07 < z < 2.42$ [2]. The chi-square function is defined to find the constraint values of the parameters $\gamma, n, \omega_0, \omega_a, H_0, \Omega_{m0}$

$$\chi^2_{CC} = \sum_{i=1}^{31} \frac{[H_i^{th}(\theta_s, z_i) - H_i^{obs}(z_i)]^2}{\sigma^2_{CC}(z_i)} \tag{16}$$

where $H_i^{obs}$ denotes the observed value, $H_i^{th}$ denotes the Hubble's theoretical value, $\sigma_{z_i}$ denotes the standard error in the observed value and $\theta_s = (\gamma, n, \omega_0, \omega_a, H_0, \Omega_{m0})$ is the cosmological background parameter space

[2] S. Mandal et al., $H_0$ tension in torsion-based modified gravity, Nuclear Physics B **993**, 116285 (2023)

- **Type Ia Supernovae:** Here have used Pantheon+ compilation of 1701 points in the redshift range $0.002122 < z < 2.26137$, which integrates Super-Nova samples[3]. The chi-square function is defined as,

$$\chi^2_{SNa} = \sum_{i,j=1}^{1701} \nabla\mu_i \left(C_{SN}^{-1}\right)_{ij} \nabla\mu_j, \tag{17}$$

Here $C_{SNa}$ is the covariance matrix and $\nabla\mu_i = \mu^{th}(z_i, \theta) - \mu_i^{obs}$ is the difference between the observed value of distance modulus extracted from the cosmic observations and its theoretical values calculated from the model with given parameter space $\theta$. $\mu_i^{th}$ and $\mu_i^{obs}$ are the theoretical and observed distance modulus respectively.

[3]D.M. Scolnic et al,, The Pantheon+ Analysis: The Full Data Set and Light-curve Release, ApJ **938** 113 (2022)

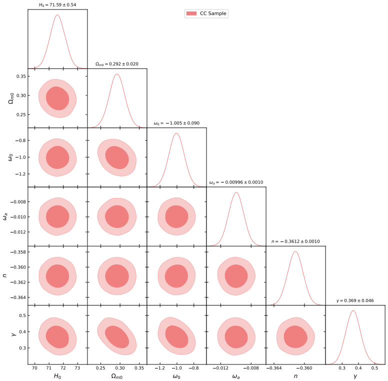



Figure: The dark orange shaded regions present the $1-\sigma$ confidence level (CL), and the light orange shaded regions present the $2-\sigma$ confidence level for the Hubble sample.

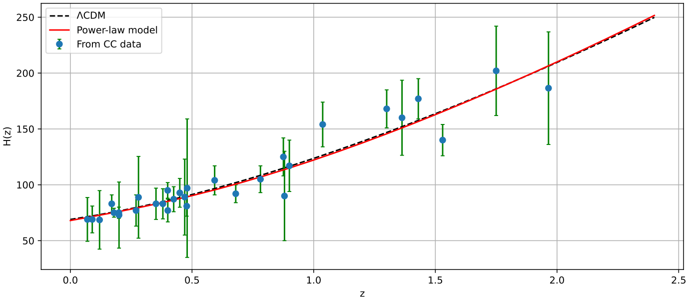



Figure: The red line represents the Hubble parameter profile of the power-law model $f(Q)$ model with the constraint values of $H_0, \Omega_{m0}, \omega_0, \omega_a, n, \gamma$. The blue dots with the green bars represent the CC sample, and the black dotted line represents the Hubble parameter profile of the $\Lambda$CDM model.

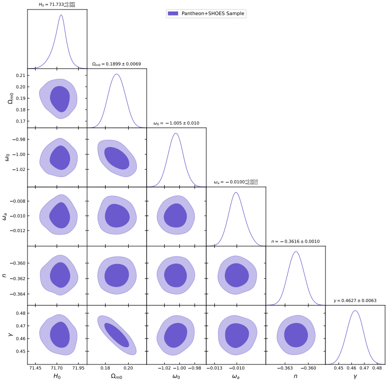



Figure: The dark blue shaded regions present the $1-\sigma$ confidence level (CL), and light blue shaded regions present the $2-\sigma$ confidence level for the Pantheon+ sample.

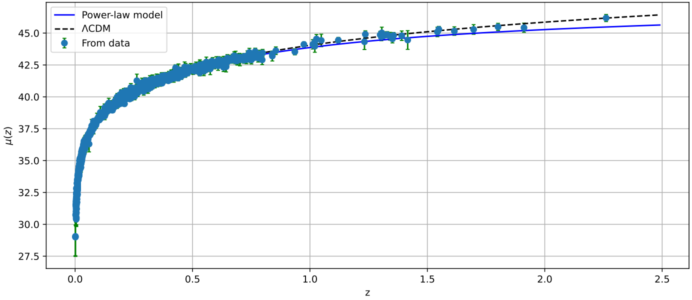



Figure: The blue line represents the distance modulus profile of the power-law $f(Q)$ model with the constraint values of $H_0, \Omega_{m0}, \omega_0, \omega_a, n, \gamma$. The blue dots with the green bars represent the Pantheon+SHOES sample, and the black dotted line represents the distance modulus profile of the $\Lambda$CDM model.

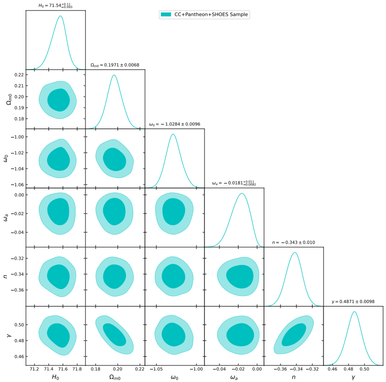
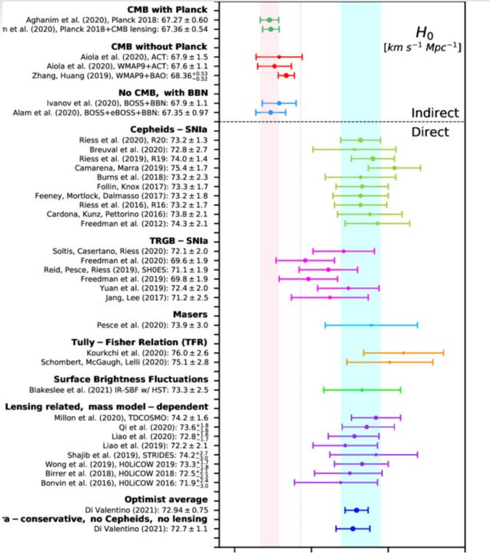



Figure: The dark-shaded regions present the $1-\sigma$ confidence level (CL), and the light-shaded regions present the $2-\sigma$ confidence level for the Hubble+Pantheon sample.

TABLE II. Marginalized constrained data of the parameters $H_0$, $\Omega_{m0}$, $\omega_0$, $\omega_a$, $\gamma$ and $n$ for different data samples with 68% and 95% confidence level.

| Model | $H_0$ | $\Omega_{m0}$ | $\omega_0$ | $\omega_a$ | $n$ | $\gamma$ |
|---|---|---|---|---|---|---|
| | | | **68% limits** | | | |
| | | | CC sample | | | |
| ΛCDM | $68.80 \pm 0.94$ | $0.318 \pm 0.034$ | - | - | - | - |
| Power-law | $71.59 \pm 0.54$ | $0.292 \pm 0.020$ | $-1.005 \pm 0.090$ | $-0.00996 \pm 0.0010$ | $-0.3612 \pm 0.0010$ | $0.369 \pm 0.046$ |
| | | | Pantheon+SHOES sample | | | |
| ΛCDM | $72.33 \pm 0.28$ | $0.383 \pm 0.022$ | - | - | - | - |
| Power-law | $71.733^{+0.085}_{-0.068}$ | $0.1899 \pm 0.0069$ | $-1.005 \pm 0.010$ | $-0.0100^{+0.0010}_{-0.0011}$ | $-0.3616 \pm 0.0010$ | $0.4627 \pm 0.0063$ |
| | | | CC+Pantheon+SHOES sample | | | |
| ΛCDM | $72.66 \pm 0.26$ | $0.342 \pm 0.019$ | - | - | - | - |
| Power-law | $71.54^{+0.11}_{-0.093}$ | $0.1971 \pm 0.0068$ | $-1.0284 \pm 0.0096$ | $-0.0181^{+0.011}_{-0.0082}$ | $-0.343 \pm 0.010$ | $0.4871 \pm 0.0098$ |
| | | | **95% limits** | | | |
| | | | CC sample | | | |
| ΛCDM | $68.8^{+1.9}_{-1.8}$ | $0.318^{+0.068}_{-0.063}$ | - | - | - | - |
| Power-law | $71.6^{+1.0}_{-1.0}$ | $0.292^{+0.040}_{-0.040}$ | $-1.00^{+0.18}_{-0.18}$ | $-0.00996^{+0.0020}_{-0.0020}$ | $-0.3612^{+0.0020}_{-0.0020}$ | $0.369^{+0.094}_{-0.089}$ |
| | | | Pantheon+SHOES sample | | | |
| ΛCDM | $72.33^{+0.55}_{-0.54}$ | $0.383^{+0.044}_{-0.044}$ | - | - | - | - |
| Power-law | $71.73^{+0.16}_{-0.19}$ | $0.190^{+0.013}_{-0.013}$ | $-1.005^{+0.020}_{-0.019}$ | $-0.0100^{+0.0022}_{-0.0021}$ | $-0.3616^{+0.0020}_{-0.0019}$ | $0.463^{+0.012}_{-0.012}$ |
| | | | CC+Pantheon+SHOES sample | | | |
| ΛCDM | $72.66^{+0.50}_{-0.53}$ | $0.342^{+0.038}_{-0.036}$ | - | - | - | - |
| Power-law | $71.54^{+0.19}_{-0.22}$ | $0.197^{+0.014}_{-0.014}$ | $-1.028^{+0.020}_{-0.018}$ | $-0.018^{+0.017}_{-0.018}$ | $-0.343^{+0.019}_{-0.020}$ | $0.487^{+0.020}_{-0.020}$ |

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[4]E. Valentino, et al., Cosmology Intertwined, arXiv:2203.06142

# Information Criteria and Model Selection Analysis

$$AIC = -2\ln\left(\mathcal{L}_{max}\right) + 2k + \frac{2k\left(k+1\right)}{N_{tot}-k-1}, \; BIC = -2\ln\left(\mathcal{L}_{max}\right) + k\log(N_{tot}), \qquad (18)$$

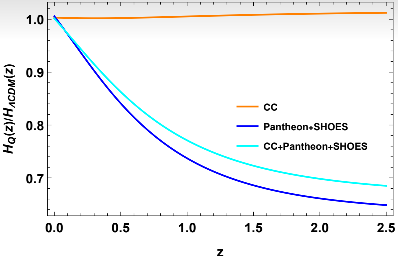
TABLE III. The corresponding $\chi^2_{min}$ of the models for each sample and the information criteria AIC, BIC for the examined cosmological models, along with the corresponding differences $\Delta IC_{model} = IC_{model} - IC_{min}$.

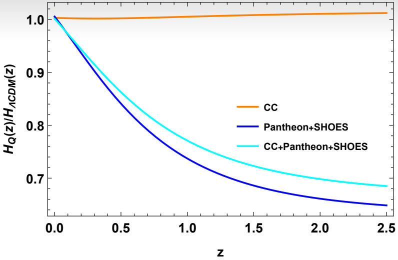
| Model | $\chi^2_{min}$ | red. $\chi^2$ | AIC | $\Delta$ AIC | BIC | $\Delta$ BIC |
|---|---|---|---|---|---|---|
| | | | CC | | | |
| ΛCDM | 16.07 | 0.64 | 20.07 | 0 | 22.93 | 0 |
| Power-law | 16.06 | 0.64 | 28.06 | 7.98 | 36.66 | 13.72 |
| | | | Pantheon+SHOES | | | |
| ΛCDM | 1696.84 | 1.0 | 1700.84 | 0 | 1719.15 | 0 |
| Power-law | 1683.20 | 0.99 | 1695.20 | 5.63 | 1727.83 | 8.6 |
| | | | CC+Pantheon+SHOES | | | |
| ΛCDM | 1712.9 | 1.0 | 1716.90 | 0 | 1735.28 | 0 |
| Power-law | 1699.33 | 0.99 | 1711.33 | 5.5 | 1744.07 | 8.79 |

# Constraint on cosmographic Parameters

TABLE IV. Present-day values of the cosmological parameters $q_0$, $j_0$, $s_0$ and $\Omega_{de0}$ as predicted by the power law $f(Q)$ model for different data samples with 68% confidence level.

| Model | $q_0$ | $j_0$ | $s_0$ | $\Omega_{de0}$ |
|---|---|---|---|---|
| | | CC sample | | |
| ΛCDM | $-0.523 \pm 0.0345$ | $1 \pm (< \mathcal{O}(10^{-16}))$ | $-0.431 \pm 0.1035$ | $0.682 \pm 0.034$ |
| Power-law | $-0.532^{+0.077}_{-0.070}$ | $1.001^{+0.298}_{-0.258}$ | $-0.439^{+0.469}_{-0.278}$ | $0.685^{+0.010}_{-0.013}$ |
| | | Pantheon+SHOES sample | | |
| ΛCDM | $-0.4255 \pm 0.033$ | $1 \pm (< \mathcal{O}(10^{-16}))$ | $-0.7235 \pm 0.099$ | $0.617 \pm 0.022$ |
| Power-law | $-0.717^{+0.017}_{-0.017}$ | $1.006^{+0.035}_{-0.035}$ | $0.108^{+0.075}_{-0.071}$ | $0.8076^{+0.0037}_{-0.0036}$ |
| | | CC+Pantheon+SHOES sample | | |
| ΛCDM | $-0.487 \pm 0.0285$ | $1 \pm (< \mathcal{O}(10^{-16}))$ | $-0.539 \pm 0.0855$ | $0.658 \pm 0.019$ |
| Power-law | $-0.744^{+0.015}_{-0.015}$ | $1.06^{+0.023}_{-0.038}$ | $0.198^{+0.011}_{-0.413}$ | $0.8064^{+0.0024}_{-0.0023}$ |


CC
Pantheon+SHOES
CC+Pantheon+SHOES
$H_Q(z)/H_{\Lambda CDM}(z)$
z
1.0
0.9
0.8
0.7
0.0
0.5
1.0
1.5
2.0
2.5


CC
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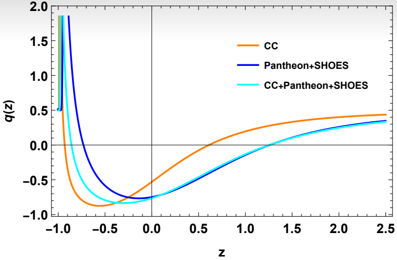
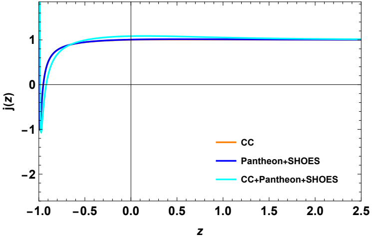
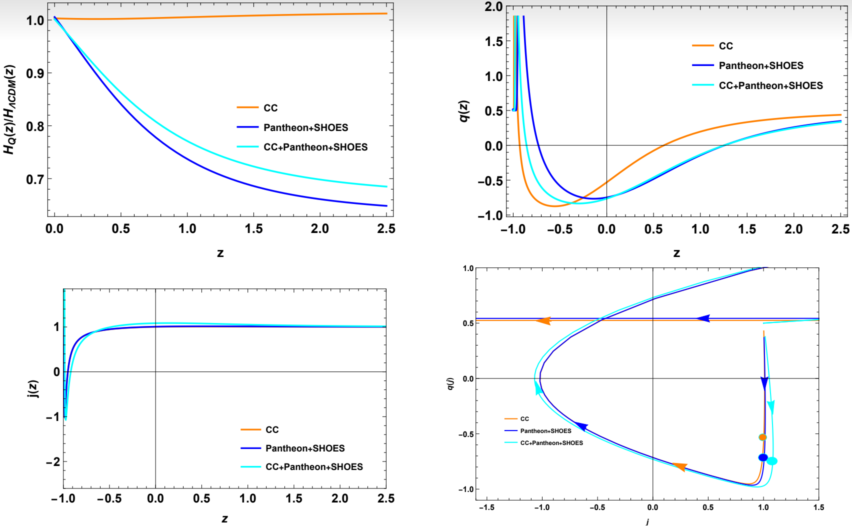
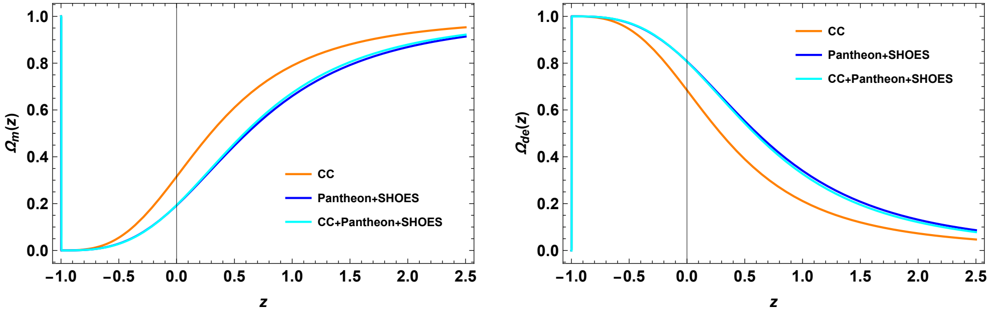

CC
Pantheon+SHOES
CC+Pantheon+SHOES
q(z)
z


CC
Pantheon+SHOES
CC+Pantheon+SHOES
j(z)
z




**Top-left:** $H_Q/H_{\Lambda CDM}$ vs $z$, **Top-right:** $q(z)$ vs $z$, **Bottom-left:** $j(z)$ vs $z$, **Bottom-right:** $q(z)$ vs $j(z)$

# Dimensionless density parameters




Profiles of the parameter of the energy densities as functions the redshift variable $z$ for the constraint values of $H_0, \Omega_{m0}, \omega_0, \omega_a, n, \gamma$ for the CC, Pantheon+SHOES, and CC+Pantheon+SHOES samples.

# *Om* Diagnostics

[5] Varun Sahni, Arman Shafieloo, and Alexei A. Starobinsky, PRD **78**, 103502 (2008).

# $Om$ Diagnostics

For the spatially flat Universe, it is defined as[5]

$$Om(x) = \frac{\mathcal{H}(x)^2 - 1}{(1+z)^3 - 1}, x = 1 + z, \mathcal{H}(x) = H(x)/H_0, \tag{19}$$

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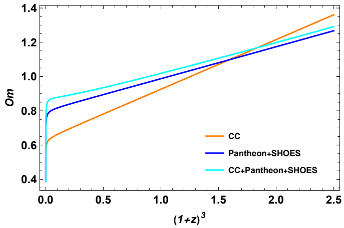


Figure: Profiles of the *Om* diagnostic parameter as a function of $1 + z$

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Cosmological observational constraints on the power law $f(Q)$ type modified gravity theory, EPJC **83** (12), 1141 (2023). arXiv:2310.00030
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**Thank you so much for your kind attention!**