Ricci inverse gravity wormholes

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Motivation

A wormhole (WH) is a hypothetical shortcut between two distant regions of space-time.

Imagine an intrinsically flat, two dimensional, space as a folded piece of paper embedded in a higher three dimensional space, where a tube connects two distant points, A and B, on the paper. The length through the tube can be much less than the distance from A to B along the paper, creating a shortcut.

Motivation

A full three dimensional WH would have entrances and exits that are three dimensional spheres rather than two dimensional rings like the mouths of the paper tube.

Such lower dimensional, human friendly, visualizations are termed embedding diagrams, and the iconic WH image is usually shown as the well known Schwarzchild embedding diagram, which is the WH analogue for a static, non-rotating, Schwarzchild black hole.

Motivation

In Einstein's General Relativity theory, WHs are expected to be filled with exotic matter, i.e. matter does not satisfy the energy conditions and may have negative density.

A WH, or an Einstein-Rosen bridge, is a topological feature that would fundamentally be a shortcut connecting two separate points in spacetime.

They contain two "mouths" and a "throat" connecting the two each at separate points in spacetime like a tunnel.

Motivation

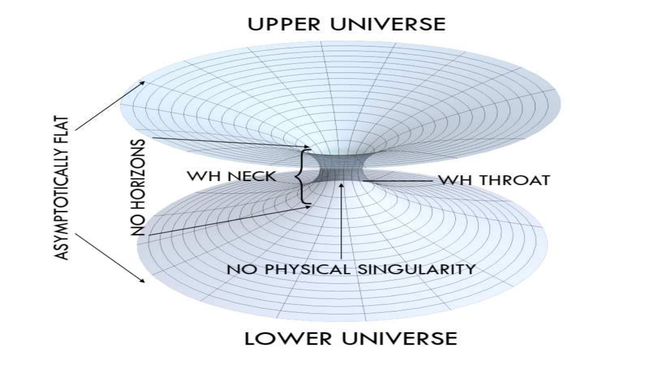
The term WH was introduced by the American theoretical physicist John Wheeler in 1957.

The name WH comes from an analogy used to explain the phenomenon.

If a worm is traveling over the skin of an apple, then the worm could take a shortcut to the opposite side of the apple's skin by burrowing through its center, rather than traveling the entire distance around, just as a WH traveler could take a shortcut to the opposite side of the universe through a hole in higher-dimensional space.

Motivation

Below Fig. shows the schematic view of WH Structure.



Motivation

WHs are not Black holes!

WHs are two-mouth tunnels, but black holes are a one way trip.

WHs have throat, but black holes have horizon.

Black holes have singularity expect regular black hole, while WHs, have no singularity.

Motivation

No observational evidence for WHs currently exists, but mathematical solutions describing WHs have long been known to be valid theoretical solutions to Einstein¡¯s field equations of General Relativity.

However, WHs made completely of normal matter with positive energy density would be inherently unstable, and would be likely to collapse in the presence of nearby matter or matter that tried to traverse the WH.

Stable, traversible WHs could exist if their entrances and exits were held open by exotic matter with a negative energy conditions.

Motivation

To study WH geometry, we need three basic assumption such as

- A static spherically symmetric metric
- A matter source
- A framework, which relates geometry and matter

WH Geometry

The static spherically symmetric WH metric within Schwarzschild coordinates is defined as

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{1}{(1 - X(r)/r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\Phi^2), \quad (1)$$

where $\Phi(r)$ is the redshift function and $X(r)$ is the shape function. Both these functions depend on radial coordinate r.

Moreover, r is the radial coordinate, which increases from a minimum radius value to ∞, i.e. $r_0 \leq r < \infty$, where r_0 is known as the throat radius.

WH Properties

WH Properties

- The behavior is non-monotonic as it decreases from ∞ to r_0 of the radial coordinate r, representing the location of throat of WH, i.e., $X(r_0) = r_0$ then its behavior increases back from r_0 to ∞.
- The most important constraint for WH existence is the flaring out property where the shape function satisfies the inequality: $(X - rX^{'})/X^2 > 0$, while at the WH throat, it satisfies $X(r_0) = r_0$.
- One further important property is $X^{'}(r_0) < 1$ which is a necessary condition to be satisfied for the WH solution.
- The radial distance should be properly finite throughout the space-time.

Exotic matter

Exotic matter, which should not be confused with dark matter or antimatter, contains negative energy density and a large negative pressure and mass. Such matter has only been seen in the behaviour of certain vacuum states as part of quantum field theory.

In case of Schwarzschild WH, it was found that it would collapse too quickly for anything to cross from one end to the other. It could be a traversable WH, would only be possible if exotic matter with negative energy density could be used to stabilize it.

For the development of WH structures an exotic fluid is required which violates the Null energy conditions (NEC) in GR.This violation of energy condition is regarded as one of the basic requirements for WH construction.

Energy Condtions

*The Raychaudhuri equations (Phys. Rev. D **98**, 1123 (1955)), which specify the action of correspondence of gravity for timelike, spacelike, or lightlike curves, provide one way to derive such conditions. In the case of an anisotropic fluid, the following energy conditions are given :*

- $\rho + p_k \geq 0$, $\rho + \sum_k p_k \geq 0$, $\forall k \Rrightarrow$ Strong Energy Condition (SEC).
- $\rho \geq 0$, $\rho \pm p_k \geq 0$, $\forall k \Rrightarrow$ Dominant energy condition (DEC) .
- $\rho \geq 0$, $\rho + p_k \geq 0$, $\forall k \Rrightarrow$ Weak Energy Condition (WEC) .
- $\rho + p_k \geq 0$, $\forall k \Rrightarrow$ Null Energy Condition (NEC).

where $k = r, t$.

Ricci Inverse Gravity

Recently, a very novel fourth-order gravity model, which is called the Ricci-inverse gravity, has been proposed by Amendola, Giani, and Laverda in (Phys. Lett. B, 811, 135923 (2020))

This model is constructed by introducing a very novel geometrical object called an anticurvature scalar denoted by the capital letter A*.*

More specific, the anticurvature scalar A *is nothing but the trace of anticurvature tensor denoted as* A^{ij} *, which is defined to be equal to the inverse Ricci tensor, i.e.,* $A^{ij} = R_{ij}^{-1}$ *.*

Ricci Inverse Gravity and Field Equations

The modified action for Ricci-inverse based gravity was provided by Amendola(Phys. Lett. B, 811, 135923 (2020)) andit is defined as:

$$S = \int \sqrt{-g} d^4 x (\alpha A + R). \tag{2}$$

In the above equation, the expression A is a trace of A^{ij}, which is calculated as

$$A^{ij} = R_{ij}^{-1}. \tag{3}$$

On differentiating Eq. (3), one can get the following relation

$$\delta A^{i\tau} = -A^{ij} \left(\delta R_{j\sigma}\right) A^{\sigma\tau}. \tag{4}$$

Ricci Inverse Gravity and Field Equations

Now, from the Eq. (2) we have the following expressions

$$\delta S = \int d^4x \left(A\delta\sqrt{-g} + \sqrt{-g}A^{iv}\delta g_{ij} + \sqrt{-g}g_{ij}\delta A^{ij}\right), \quad (5)$$

$$= \int d^4x\sqrt{-g}\left(\frac{1}{2}Ag^{iv}\delta g_{ij} + A^{iv}\delta g_{ij} + g_{ij}\delta A^{ij}\right), \quad (6)$$

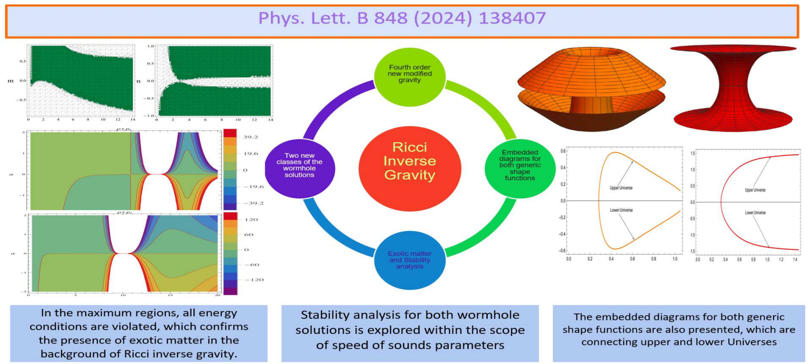
By adopting the procedure from (Phys. Lett. B, 811, 135923 (2020)), we have the following set of field equations

$$R^{ij} - \frac{1}{2}Rg^{ij} - \alpha A^{ij} - \frac{1}{2}\alpha A g^{ij} + \frac{\alpha}{2}\left(2g^{\varrho i}\nabla_\epsilon\nabla_\varrho A^\epsilon_\sigma A^{b\sigma} - \nabla^2 A^i_\sigma A^{j\sigma} - g^{ij}\nabla_\epsilon\nabla_\varrho A^\epsilon_\sigma A^{\varrho\sigma}\right) = \mathcal{T}^{ij} \quad (7)$$

where $A^\epsilon_\sigma A^{b\sigma} = A^{\epsilon\tau}g_{\tau\sigma}A^{\sigma b} = A^{\epsilon\tau}A^b_\tau = A^{\epsilon\sigma}A^b_\sigma = A^b_\sigma A^{\epsilon\sigma}$ with $8\pi G = 1$.

Graphical Abstract

Ricci inverse gravity WHs



Ricci Inverse Gravity Field Equations for WH Geometry

The energy density and pressure components in the background of Ricci inverse gravity are calculated as

$$\rho = \frac{(r-X(r))\left(-\frac{2\alpha(\rho_1-\rho_2)r^3}{(r-X(r))^2}+\frac{4\alpha\rho_{13}r^2}{\rho_{12}}-\frac{X'(r)}{r^2-rX(r)}\right)}{r}, \tag{8}$$

$$p_r = -4\alpha r\left(\frac{r\left(p_6r^4-p_9r^3X(r)+p_5r^2X(r)^2-p_4rX(r)^3+p_3X(r)^4\right)}{p_{10}^4}+\frac{p_2}{p_1^3}\right), \tag{9}$$

$$p_t = -\frac{r\left(-8\alpha\left(p_{12}+p_{13}\right)r^2+8\alpha p_{17}r^2+p_{11}\right)}{4(r-X(r))}. \tag{10}$$

where ρ_i, $\{i=1,...,13\}$ and p_i, $\{i=1,...,17\}$ are given in the Appendix of (Phys. Lett. B 848 (2024) 138407).

Redshift function and Shape function

For the current study, we consider the specific well-known redshift function $\Phi = -\frac{\zeta}{r}$, *(Phys. Rev. D 57 (1998) 829) where* χ *is constant. Further, we are considering two most generic series type shape functions, which are expressed as (Fortschr. Phys. 2200053 (2022))*

$$\begin{aligned} X(r) &= \frac{1}{2n^2}\Big(2\ln(r_0)(n(A_0+A_1)-A_0\ln(r))-2n\ln(r)(A_0+A_1)+A_0\ln(r^2) \\ &+ x2n^2(A_0+A_1+A_2)+A_0\ln(r_0^2)\Big), \end{aligned} \tag{11}$$

$$\begin{aligned} X(r) &= \frac{2}{2m^2}\Big(\Big(A_0\left(2n^2\left(r^m-r_0^m\right)^2-2mn\left(r^m-r_0^m\right)+m^2\right)+m\left(A_1\left(-2nr^m+2nr_0^m+m\right.\right. \\ &+ A_2 m))+m(\ln(r)-\ln(r_0))\left(A_0 m(\ln(r)-\ln(r_0))-2\left(A_0\left(2n\left(r_0^m-r^m\right)+m\right)+A_1 m\right)\right)\Big), \end{aligned} \tag{12}$$

where $0.1 < A_0 < A_1 < A_2 < 0.5$, $-0.9 < m < 0.9$, and $-0.9 < n < 0.9$.

Allowed-able region for the main part of WH geometry

Allowed-Able Region for Model-I and Model-II

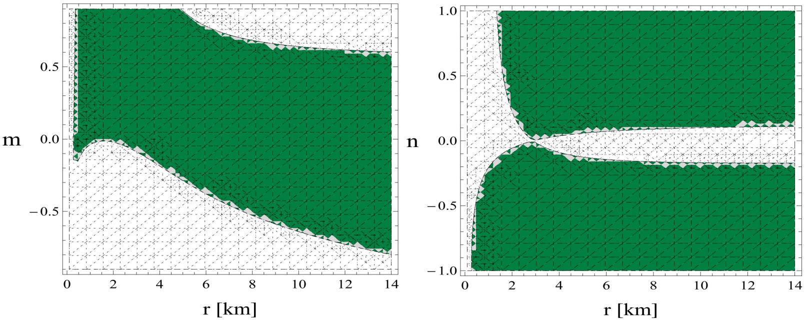


Fig. 1. shows the allowed-able region for both models with $\left(1-\frac{X(r)}{r}\right)>$ (shaded) and $\left(1-\frac{X(r)}{r}\right)<0$ (un-shaded).

Embedding surface diagram for WH solutions

The embedding surface diagram for WH solutions is now discussed using the $t = const.$ and $\theta = 2\pi$ in Eq.(1), one can get the following relation

$$ds^2 = \left(1 - \frac{X(r)}{r}\right)^{-1} dr^2 + r^2 d\phi^2, \tag{13}$$

The Eq.(13) can be revised into a 3-D Euclidean spacetime, which is further defined as

$$ds_\Sigma^2 = dg^2 + dr^2 + r^2 d\phi^2 = \left(1 + \left(\frac{dg}{dr}\right)^2\right) dr^2 + r^2 d\phi^2, \tag{14}$$

On matching Eqs. (13-14), one can get the following relation

$$\frac{dg}{dr} = \pm \left(\frac{r}{X(r)} - 1\right)^{-1/2}, \tag{15}$$

Embedding surface diagram for WH solutions

Embedding surface diagram for Model-I and Model-II

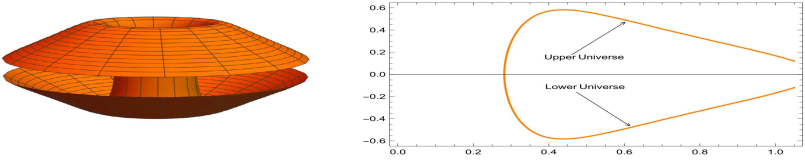


Fig. 2. shows the embedding surface with upper and lower Universes for WH model-I.

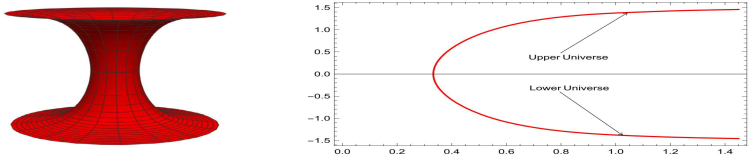


Fig. 3. shows the embedding surface with upper and lower Universes for WH model-II.

Energy density

Energy density

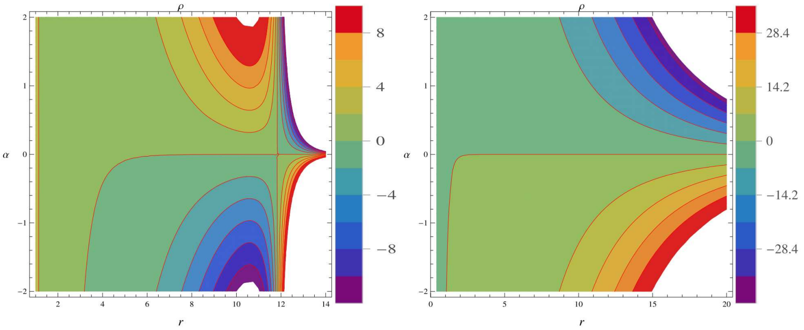


Fig. 4. shows the pictorial view of energy density for model-1 (left) and model-II (right).

Energy Conditions

Energy Conditions

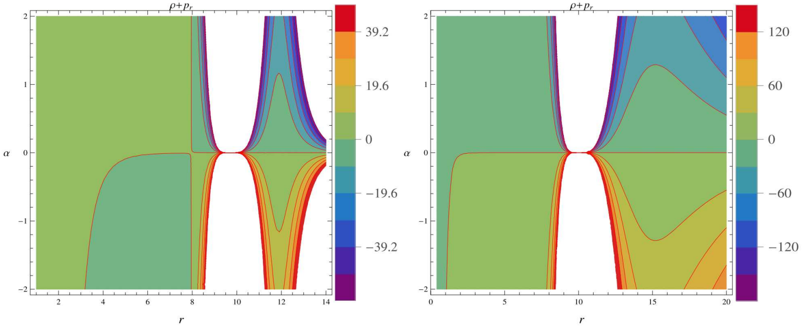


Fig. 5. shows the pictorial view of $\rho + p_r$ for model-1 (left) and model-II (right).

Energy Conditions

Energy Conditions

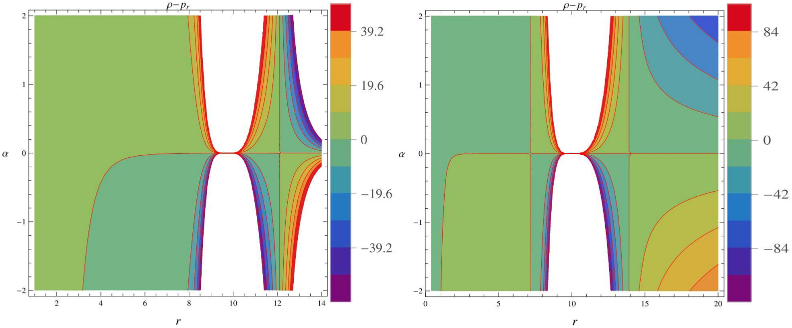


Fig. 6. shows the pictorial view of $\rho - p_r$ for model-1 (left) and model-II (right).

Energy Conditions

Energy Conditions

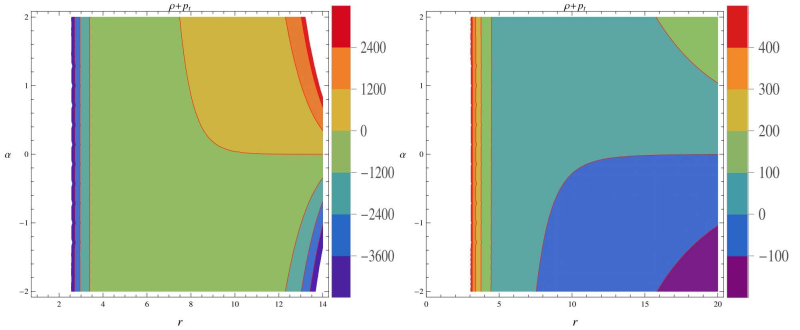


Fig. 7. shows the pictorial view of $\rho + p_t$ for model-1 (left) and model-II (right).

Energy Conditions

Energy Conditions

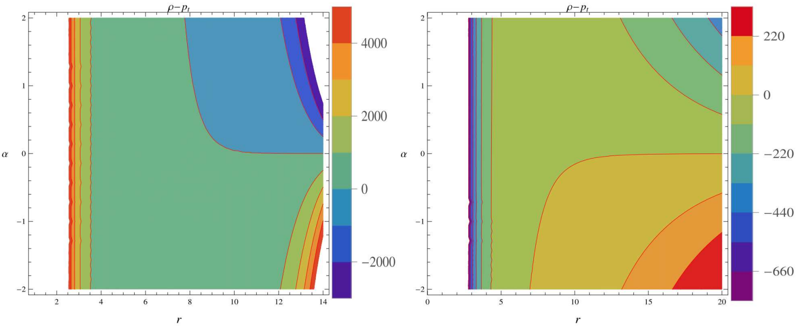


Fig. 8. shows the pictorial view of $\rho - p_t$ for model-1 (left) and model-II (right).

Energy Conditions

Energy Conditions

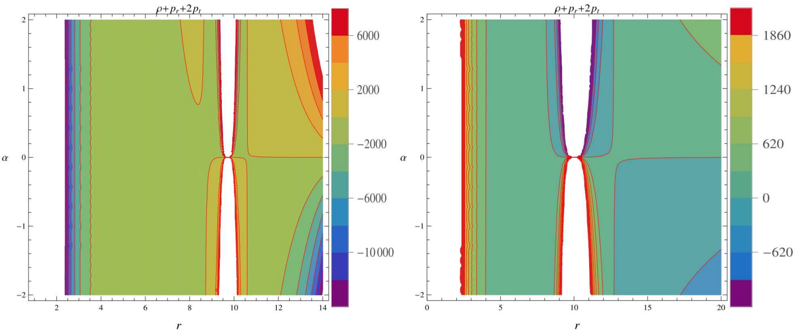


Fig. 9. shows the pictorial view of $\rho + p_r + 2p_t$ for model-1 (left) and model-II (right).

Stability Analysis

We examine the stability of the WH cosmic arrangement in this section. The propagation rate of sound waves via an anisotropic fluid distribution must be smaller than that of electromagnetic radiation in order for the system to be stable, i.e., $0 < v_r^2 < 1$ *and* $0 < v_t^2 < 1$*, where* v_r *and* v_t *are calculated as (Phys. Lett. B 835 (2022) 137572)*

$$v_r^2 = \frac{dp_r}{d\rho}, \quad v_t^2 = \frac{dp_t}{d\rho}. \tag{16}$$

For the various ranges of parameter α, the speed of sound parameters $\frac{dp_r}{d\rho}$ and $\frac{dp_t}{d\rho}$ have a value between 0 and 1 around the throat.

Stability analysis

Speed of sound parameters

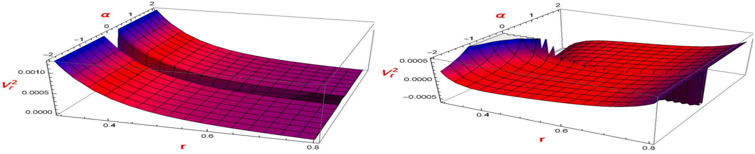


Fig. 10. shows the radial and tangential speed of sound parameters for model-I.

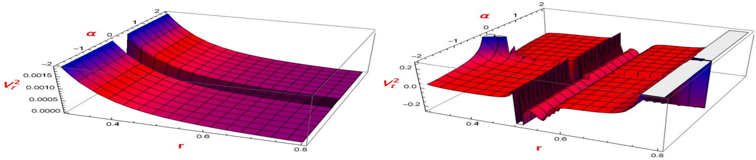


Fig. 11. shows the radial and tangential speed of sound parameters for model-II.

Discussion

Some Remarks

We have studied a recently proposed novel fourth order Ricci-inverse gravity, which introduces a very innovative geometrical component A known as the anti-curvature scalar. In this letter, we have studied WH solutions under Ricci-inverse gravitational framework. In order to achieve a physically interesting consequence, we have used specific shape functions to produce the WH structure. The gravitational lensing, time dilation, exotic matter, and the displacement of matter are the instances of observational evidence for WH solutions, or theoretical indications that may suggest the existence or presence of a WH. It is necessary to mention that exotic matter has the leading role for the existence of WHs solution in the current analysis. The creation of two new generic WH solutions has released a significant influence in the field of WH research.

The End

I want to say thanks to all and special thanks to Prof.T. Harko for his kind support, guidance, and time.

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