Seminar Series in Gravitation, Cosmology and Astrophysics

A Quantum Geometrical Approach: Quantizing GR || Gravitizing QM

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11th April 2024, 11:00

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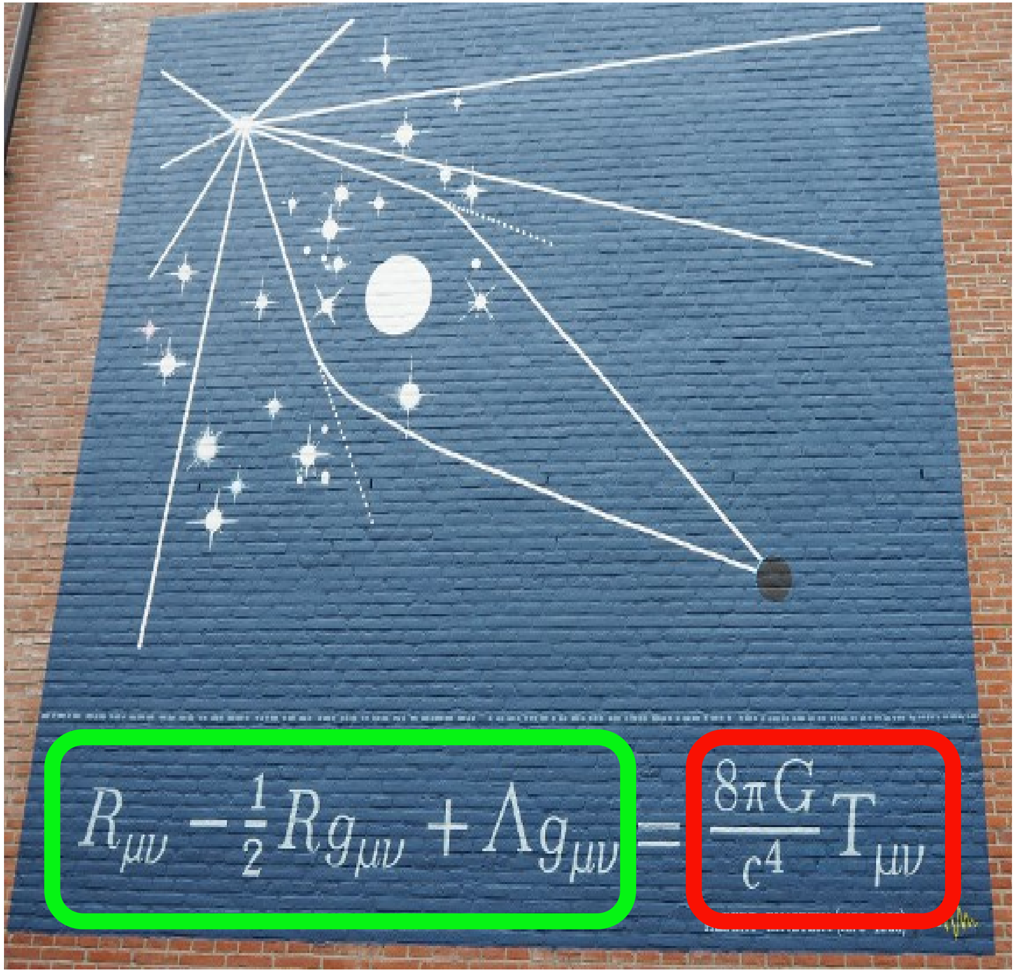
Conceptional Problem

Einstein Field Equations (EFE) combine
classical geometry
with quantizable energy-momentum tensor!

EFE introduce an astonishingly-correct approximation at large scales!

Gravity (similar to EM for instance) is represented as a form of geometry or covariant derivative. While in Gravity, gravitational/inertial mass ratio is unity, i.e., universal geometry but no neutrally gravitational mass, in EM charge/mass ratio differs with the particles, i.e., different geometries

Leiden, Netherlands



Motivations

GR and QM are fundamentally distinct theories explaining how nature works but with genuinely incompatible perception of reality.

General Relativity (Smoothed)

- Large scale and gravity force (?)
- Objects are point particles
- Events happen continuously with deterministic outcomes
- No instant connections between apart events
- Noncoherent and certain measurement; commutation
- Nonsensical predictions at low scale

Quantum Mechanics (roughed)

- Low scale and three forces of nature
- Objects are wave functions
- Events happen in jumps with probabilistic outcomes
- Instant entanglement between apart events
- Coherent and uncertain measurement; noncommutation
- Nonsensical predictions at large scale

Motivations

Unification principles of GR and QM requires

Quantization of GR and simultaneously Gravitization of QM

General Relativity

(roughed at some scale)

- **Spacetime discretization,**
- **Measurement uncertainty,**
- **Noncommutative relations,**
- **Generalized Riemannian manifold**

Quantum Mechanics

(smoothed at some scale)

- **Gravitational field impacts,**
- **Generalized noncommutation relation,**
- **Relativity principle,**
- **Isotropy and Lorentz covariance**

Quantizing GR allows for corrections to GR at low scale

Gravitating QM allows for corrections to QM at large scale

We suggest to start from scratch, quantizing metric tensor in a quantum-geometrical approach

Quantizing Rank-2 Tensor Field

1) **Loop quantum gravity, string theory** and **QFT** suggest quantization of the gravitational field, including that of the metric tensor. **QFT** treats metric tensor as a quantum field, with the associated creation and annihilation operators, and applies principles of **QM** to its behavior.

2) **Quantum geometry** combines a set of mathematical concepts generalizing the geometric properties of spacetime at the quantum scale, where **QM** effects become significant.

3) Because of its geometric nature, **Quantum geometry** likely succeeds in reconciling principles of GR and QM!

Einstein Field Equations in GR

$$\tilde{G}_{\beta\nu} = \tilde{R}_{\beta\nu} - \frac{1}{2} g_{\beta\nu} R$$

$$\tilde{R}_{\beta\mu} = R^{\mu}_{\beta\mu\nu} = g^{\mu}_{\gamma} R^{\gamma}_{\beta\mu\nu},$$

$$\tilde{R} = \tilde{g}^{\beta\nu} \tilde{R}_{\beta\nu},$$

$$\tilde{R}^{\gamma}_{\beta\mu\nu} = \tilde{\Gamma}^{\gamma}_{\beta\nu,\mu} - \tilde{\Gamma}^{\gamma}_{\beta\mu,\nu} + \tilde{\Gamma}^{\gamma}_{\sigma\mu} \tilde{\Gamma}^{\sigma}_{\beta\nu} - \tilde{\Gamma}^{\gamma}_{\sigma\nu} \tilde{\Gamma}^{\sigma}_{\beta\mu}$$

$$\tilde{\Gamma}^{\gamma}_{\beta\mu} = \frac{1}{2} \tilde{g}^{\alpha\gamma} \left(\tilde{g}_{\alpha\beta,\mu} + \tilde{g}_{\alpha\mu,\beta} - \tilde{g}_{\beta\mu,\alpha} \right)$$

Nothing would quantize EFE

$$\tilde{g}_{\alpha\beta}(x_0^{\alpha}) = C \, g_{\alpha\beta}(x_0^{\alpha})$$

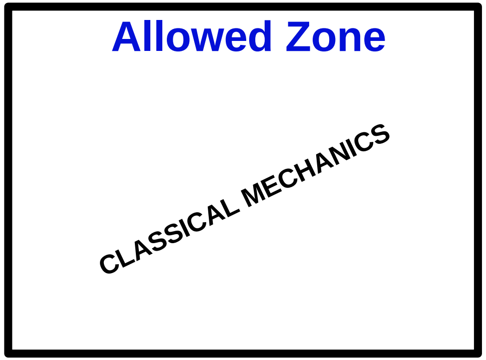
more profounder than the fundamental metric

Our Approch to gravitize QM

Qravitizing QM:
Relativistic Generalized Uncertainty Principle

Position Uncertainty, Δx

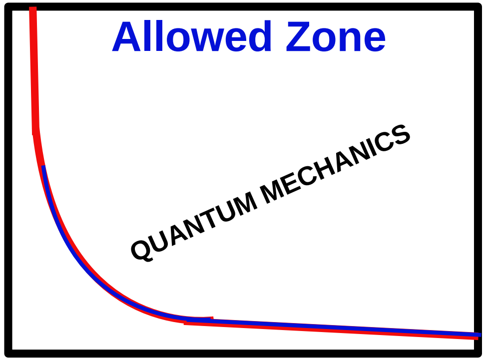
Δx and Δp are not correlated



Momentum Uncertainty, Δp

Without Gravitational Fields

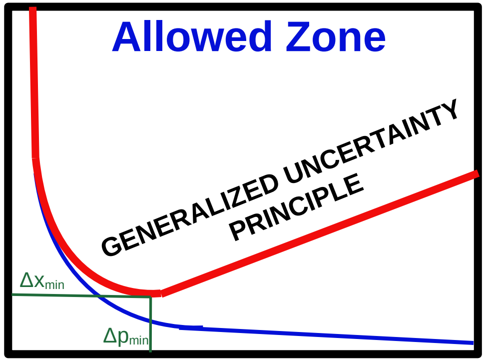
$\Delta x \Delta p \geqq \hbar/2$



Momentum Uncertainty, Δp

With Gravitational Fields

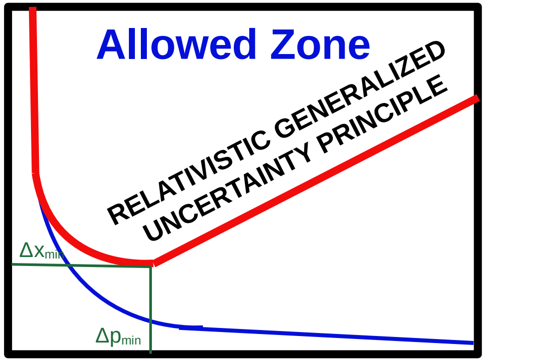
$\Delta x \Delta p \geqq \hbar[1+\beta(\Delta p)^2+\beta\langle p\rangle^2]/2$



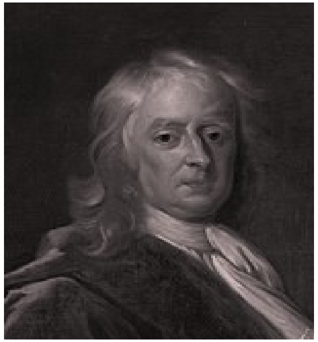
Momentum Uncertainty, Δp

With Relativistic Gravitational Fields

$\Delta x^\mu \Delta p^\nu \geqq \hbar[g^{\mu\nu}+\beta_1\langle p^2\rangle+\beta_2\langle p^\mu\rangle^2\langle p^\nu\rangle^2 -(\beta_2/2)\Delta p^\mu)^2(\Delta p^\nu)^2]/2$



Momentum Uncertainty, Δp



Sir Isaac Newton 1687

Werner Karl Heisenberg 1927

Hartland Sweet Snyder 1949

Qravitizing QM:

Relativistic Generalized Uncertainty Principle

For a test particle in curved spacetime, the physical position and momentum coordinates are given in terms of their auxiliary 4-vectors

$$\hat{x}^\mu\left(\hat{x}_0^\mu, \hat{p}_0^\mu\right) \qquad \hat{p}^\mu\left(\hat{x}_0^\mu, \hat{p}_0^\mu\right) \qquad [\hat{x}_0^\mu, \hat{p}_0^\nu] = i\hbar g^{\mu\nu} \text{ and } \hat{p}_0^\mu = -i\hbar\partial/\partial\hat{x}_{0\mu}$$

With Snyder algebra, $\hat{x}^\mu = (\hat{x}_0^0, \hat{x}_0^i), \qquad \hat{p}^\mu = (\hat{p}_0^0, \hat{p}_0^i\,(1 + \beta\hat{p}_0^\rho\hat{p}_0^\rho))$

Then, $$[\hat{x}^\mu, \hat{p}^\nu] = i\hbar\left[\left(1 + \beta_1\hat{p}^\rho\hat{p}_\rho\right) g^{\mu\nu} + 2\beta_2\hat{p}^\mu\hat{p}^\nu\right]$$

$$\boxed{\hat{x}^\mu = \hat{x}_0^\mu,} \qquad \boxed{\hat{p}^\mu = \left(1 + \beta\hat{p}_0^\rho\hat{p}_{0\rho}\right)\hat{p}_0^\mu = \phi\,\hat{p}_0^\mu}$$

$$\Delta x^\mu\,\Delta p^\nu \geq \frac{1}{2}\left|\langle[x^\mu, p^\nu]\rangle\right| = \frac{i\hbar}{2}\left[\langle g\rangle^{\mu\nu} + \beta_1\langle p^2\rangle + \beta_2\langle p^\mu\rangle^2\langle p^\nu\rangle^2 - \frac{\beta_2}{2}\left[(\Delta p^\mu)^2 + (\Delta p^\nu)^2\right]\right]$$

Alternative Approaches to “Qravitizing the Quantum”

“Gravitizing the quantum”, 2202.06890 [hep-th], IJMPD31(2022)2242024 suggests

- Extending the dynamical aspects of general covariance (dynamical physical quantities) to QM structures

- Dynamizing the quantum geometry so that the quantum gravity
  - becomes consistent with the principles of unitarity and
  - gains fundamental aspects of gravity, such as topology change.

Turning the Hilbert-space scalar product into dynamical similar to the GR’s dynamical metric

Alternative Approaches to “Qravitizing the Quantum”

Collapse model modifies the standard QM by a physical mechanism responsible for the collapse of the wavefunction "measurement problem".

- Examples: Spontaneous/Continuous Spontaneous Localisation model.
- Also, a non-relativistic spontaneous collapse model based on the idea of collapse points situated at fixed spacetime coordinates was proposed.
- Penrose suggests gravity as the physical process modifying QM, GERG8(1996)581, FoundPhys44(2014)557
- In summary, by including nonlinear and stochastic terms for the collapse of the wavefunction, the Schrodinger equation is modified.

Extending our Approach of "Qravitizing the Quantum"

- With the integration of quantum gravitational degrees of freedom an effective description of quantum particle's kinematics becomes feasible.

- A modification in relativistic kinematics could be geometrically described in Finsler and Hamilton geometry, in which the vacuum states of quantum gravity could be characterized, at low energies.

- Also, the Finslerian length element must contribute to the quantization of spacetime.

- The proposed quantization of the fundamental tensor predicts a maximal proper force as a new physical constant, which gravitationally drives the quantum particle's motion and causes its maximal proper acceleration along the additional curvatures.

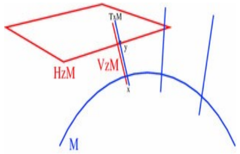
Attempts to generalize/quantize GR

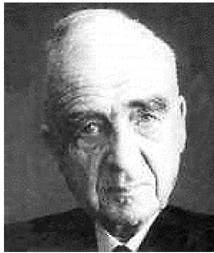
Attempts to generalize GR

- We start with Weyl's attempt to unifying Gravitation and Electromagnetism, H. Weyl, Sitzungsberichte der Preussinske Akademie der Wissenschaften, 465, (1918)

- Increasing the number of dimensions as in Kaluza-Klein approach T. Kaluza, Zum Unitötsproblem der Physik, Sitz. Preuss. Akad. Wiss. Phys. Math. K1 (1921) 966. O. Klein, Quantentheorie und fünfdimensionale Relativitaetstheorie, Zeits. Phys. 37 (1926) 895.

- Using Finsler spaces, complex manifolds, scalar-tensor coupling, extended particles in form of strings or bubbles, etc. Paul Finsler, Über Kurven und Flächen in allgemeinen Räumen, Dissertation, Göttingen (1918)





Attempts to generalize GR

- Amelino-Camelia proposed a model based on the existence of a Length scale G. Amelino-Camelia, Int. J. Mod. Phys. D 11, 35 (2002); Phys. Lett. B 510, 255 (2001)
- Magueijo and Smolin preferred to use an Energy scale J.Magueijo and L.Smolin, Phys. Rev. Lett. 88, 190403 (2002)
- This leads to a resulting maximal momentum found in many theories S.G. Low, J. Math. Phys., 38, 2197 (1997)
- Ahluwalia and Kirchbach argued that gravitational and quantum realms require two invariant scales => gravitationally modified de Broglie wavelength D.V.Ahluwalia, Phys. Lett, A 275, 31 (2000)

Attempts to generalize GR

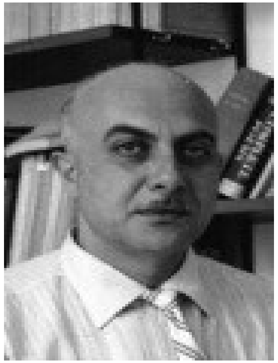
- Ketsaris starts from a 7d-manifold and obtained a Maximal Acceleration and Maximal Angular Velocity A. A. Ketsaris, Accelerated motion and special relativity transformations
- Quantum Special Relativity (QSR), Poincaré covariant formulation of QM, which is applicable to massive particles propagating at velocities up to c, Martin, B.R.; Shaw, G.. Particle Physics, (2008)
- Amelino-Camelia suggests Quantum General Relativity (QGR) which requires an appropriate extension of the k-Minkowski spacetime to some sort of k-phase space intended as space x_i, t, p_i, E rather than just x_i, p_i Giovanni Amelino-Camelia, AIP Conf.Proc. 589, 137-150 (2001)

CENTER OF EXCELLENCE

Attempts to generalize GR

- The "Quantum Geometry" proposed in E.R.Caianiello, Nuovo Cimento, 59B, 350 (1980); E.R.Caianiello, G.Marmo, G.Scarpetta, Nuovo Cimento, 86A, 337 (1985) predicts a maximal acceleration E.R.Caianiello, Lett. Nuovo Cimento, 32, 65 (1981)
- With an upper limit on acceleration Scarpetta suggested deformation of QSR G.Scarpetta, Lett. Nuovo Cimento, 41, 51 (1984)
- QGR was proposed by Brandt and Schuller H.E. Brandt, Found. Phys. Lett., 2, 39, (1989), F.P. Schuller, Annals Phys., 299, 174, (2002)
- Due to Maximal Acceleration, quantum corrections to the classical spacetime could be analyzed E.R. Caianiello, A.Feoli, M.Gasperini and G.Scarpetta, International Journal of Theoretical Physics, 29, 131 (1990)

Attempts to generalize GR



- Caianiello's idea is that the simplest theoretical framework, the one including the maximal proper acceleration, must fulfill physical invariance.

$$g_{\mu\nu} = \begin{bmatrix} -\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{bmatrix}.$$

$$\mathrm{d}s^2 = -c^2\left(1-\frac{2GM}{c^2 r}\right)\mathrm{d}t^2 + \left(1-\frac{2GM}{c^2 r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin(\theta)^2\mathrm{d}\phi^2$$

- **Generic line element defined in an eight-dimensional phase-space.**

$$d\tilde{s}^2 = g_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu + \frac{c^4}{A_{max}^2} g_{\mu\nu}d\dot{x}^\mu d\dot{x}^\nu \qquad d\tilde{s}^2 = ds^2\left(1 - \frac{c^4|g_{\mu\nu}\ddot{x}^\mu\ddot{x}^\nu|}{A_m^2}\right)$$

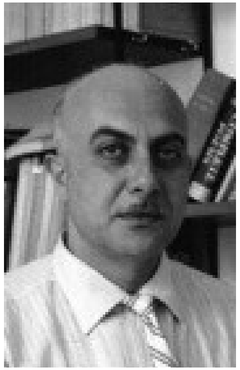
Attempts to generalize GR

- Quantization as geometry in phase space or the quantum corrections to the spacetime metric from the geometric phase space quantization.
- Additional dimensions or additional curvatures, i.e., geometry, mimic the possible quantization of the fundamental tensor.
- In spacetime the quantization are interpreted as curvature emerged by the relativistic eight-dimensional spacetime tangent bundle

$$TM = M_4 \otimes TM_4$$

- The geometric theory for spacetime (quantum geometry) predicts that the world lines are associated with physical particles having an upper bound for the proper (physical) acceleration.
- Caianiello derived the maximal proper acceleration from the principles of quantum mechanics and relativity

$$A_{max} = \frac{2mc^3}{\hbar}$$



Attempts to generalize GR

- **Brandt proposed QGR**

$$a^2 = -c^4 g_{\mu\nu} \frac{Dv^\mu}{ds} \frac{Dv^\nu}{ds}$$

$$\frac{Dv^\mu}{ds} \equiv \frac{dv^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta} v^\alpha v^\beta \qquad v^\mu \equiv dx^\mu / ds$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$\Gamma^\mu{}_{\alpha\beta}$ is the symmetric affine connection

Attempts to generalize GR

Born suggested duality-symmetry configuration of distance and momentum of a free particle $x^\mu \rightarrow \gamma\, p^\mu$ and $p^\mu \rightarrow -\lambda x^\mu$

Born Reciprocity Principle

where distance and momentum are four-vectors

$$\hat{x}^\mu := (ct, \hat{x}^1, \hat{x}^2, \hat{x}^3),$$
$$\hat{p}^\mu := (E/c, \hat{p}^1, \hat{p}^2, \hat{p}^3).$$

Based on the assumption of invariant wave functions, the canonical functions

$$\dot{x}^i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_j = \frac{\partial H}{\partial x^j},$$

- And components of the angular momentum, $m_{ij} = m_i p_j - m_j p_i$ are also invariant under the proposed transformation.
- All these conclusions are assume to be equally valid in classical and quantum mechanics.

Attempts to generalize GR

Therefore, following noncommutative relations were suggested

$$[x^i, p_j] = x^i p_j - p_j x^i = i\hbar\delta^i_j$$

Validity of QM implies fundamental symmetry between space and momentum: BRP tells that classical & quantum laws of Nature are symmetric.

Born also suggested another invariant transformation combining both distance the momentum operators, quantum metric operator $\hat{x}_i\hat{x}^i + \hat{p}_i\hat{p}^i$

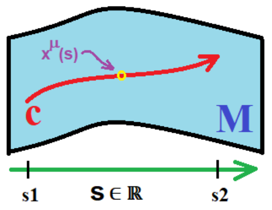
Coupling GR metric structure with spacetime coordinates combining or exchanging distance Δx^μ, and momentum uncertainties Δp^ν draw the conclusion that BRP introduces dynamics to GR and reveals its phase-space structure

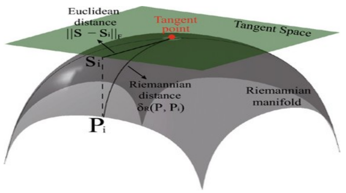
Our Attempt to generalize/quantize GR

Quantum Geometry:
Finsler/Hamilton Geometry

Riemann geometry (M,g) so that at point x:

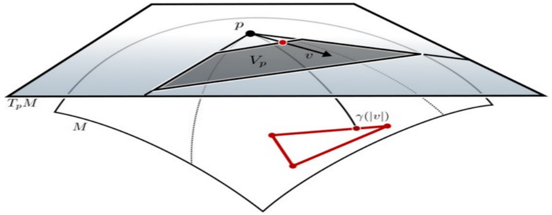
- metric tensor $g = g^{\mu\nu}(x) dx^\mu \otimes dx^\nu$,
- length of curve c, $\int_c \sqrt{g^{\mu\nu}(x) dx^\mu dx^\nu}$ or $\int_{s_1}^{s_2} \sqrt{g^{\mu\nu}(s) d\dot{x}^\mu d\dot{x}^\nu} ds$, where $\dot{x} = \frac{dx}{ds}$





Finsler geometry (M,F) so that at point x on M, Finsler structure $F(x,\dot{x})$ is related to the generalized metric tensor

$$F = \sqrt{g^{\mu\nu}(x) d\dot{x}^\mu d\dot{x}^\nu} \text{ and } g := \frac{1}{2}\frac{\partial^2 F^2}{\partial \dot{x}^\mu \partial \dot{x}^\nu}.$$



With the auxiliary four-momentum, $F(x,\dot{x}) = F(x_0, p_0)$, where the resulting F is fortunately 1-homogeneous of p_0.

This allows the direct implication of RGUP, $F(x_0,p_0)=F(x_0,\phi p_0)$, where $\phi = 1 + \beta p_0^\rho p_{o\rho}$

The resulting $F(x_0,\phi p_0)$ is 1-homogeneous of p_0, as ϕ is indeed 0-homogeneous in p_0.

Finsler-Hamilton Manifolds

- F is +ive for $\dot{x} \neq 0$ on tangent bundle ***TM*** & homogeneous of degree ***1*** in $\dot{x}$,
- therefore, on ***TM***, at local coordinates $(x, \dot{x})$,

$$F(x, a\dot{x}) = aF(x, \dot{x}), \quad \forall \lambda \in R^{+}$$

- and the ratio of lengths of any two collinear vectors doesn't include metric functions.

- In Finsler geometry, the special case $F(x, \dot{x}) = \sqrt{g^{\mu\nu}(x) d\dot{x}^{\mu} d\dot{x}^{\nu}}$ distinguishes Finsler from Riemann geometry; a relaxation of quadratic restriction
  Notices Amer. Math. Soc. 43(1996) 95.

- The length of a curve ***c*** is given as $\int_{c} F(x, \dot{x})$ or $\int_{s1}^{s2} F\left(x^{\mu}(s), \dot{x}^{\mu}(s)\right) ds$.

Finsler-Hamilton Manifolds

- Based on 1-homogeneous $F(x, a\dot{x}) = aF(x, \dot{x}), \forall a \in R^+$ we assume a free particle with mass m
- Then, $F(x, m\dot{x}) = m\, F(x, \dot{x}) = F(x, m\dot{x}), \quad \forall m \in R^+$
- With the auxiliary four-momentum, $F(x, \dot{x}) = F(x_0, p_0)$, where the resulting F is fortunately 1-homogeneous of p_0.
- This allows the direct implication of RGUP, $F(x_0, p_0) = F(x_0, \phi p_0)$, where $\phi = 1 + \beta p_0^{\rho} p_{o\rho}$
- The resulting $F(x_0, \phi p_0)$ is 1-homogeneous of p_0, as ϕ is indeed 0-homogeneous in p_0.
- The Klein metric can be generalized as

$$F(\hat{x}_0^{\mu}, \hat{\tilde{p}}_0^{\nu}) = \left[\frac{|\hat{\tilde{p}}_0^{\nu}|^2 - |\hat{x}_0^{\mu}|^2 |\hat{\tilde{p}}_0^{\nu}|^2 + \langle \hat{x}_0^{\mu} \cdot \hat{\tilde{p}}_0^{\nu}\rangle^2}{1 - |\hat{x}_0^{\mu}|^2}\right]^{1/2}$$

R. S. Hamilton, Three manifolds with positive Ricci curvature, J. Differential Geom. 17, (1982), 255-306.

Finsler-Hamilton Manifolds

- **Finsler manifold:** tangent bundle (TM,π,M), M is nd C^{∞}-manifold.
- **Hamilton manifold:** cotangent bundle $T^{*}M$ of finite-dimensional manifold M equipped with a regular Hamiltonian. Hamilton manifold: cotangent bundle $(T^{*}M, \pi^{*}, M)$.

- Dualization between Finsler and Hamilton geometries, i.e., a nonlinear connection $(TM, \pi, M) \rightarrow (T^{*}M, \pi^{*}, M)$, because the geometric structure of $T^{*}M$, $TT^{*}M$ is different from TM, TTM,

- Hamilton $H(x,p)$ can only be obtained from Finsler $F(x,v)$ by Legendre transformation.
- $F(x,v) \rightarrow F(x,p)$ is obtained solely from homogeneity of Finslerian F.

Finsler-Hamilton Manifolds

Klein metric is among simplest Finsler metric

$$F^2(x_0^\mu, \phi p_0^\nu) = \phi^2 \frac{|p_0^\nu|^2 - |x_0^\mu|^2 |p_0^\nu|^2 + \langle x_0^\mu, p_0^\nu \rangle^2}{(1 - |x_0^\mu|^2)^2}$$

With the Hessian and anisotropic transformation of the components of the corresponding **Finsler** metric tensor

$$\tilde{g}_{ab}(x) = \frac{1}{2} \frac{\partial^2}{\partial p_0^\mu \partial p_0^\nu} F^2(x_0^\mu, \phi p_0^\nu)$$

$$\begin{aligned} \tilde{g}_{ab}(x) \; = \; & \left(\phi^2 + \frac{2\kappa\phi F^2}{(p_0^0)^2}\right) g_{ab}(x) + \frac{4\kappa F^2}{(p_0^0)^2}\left(2\phi + \frac{\kappa F^2}{(p_0^0)^2}\right) \ell_\mu \ell_\nu \\ & - \frac{4\kappa F^3}{(p_0^0)^3}\left(2\phi + \frac{\kappa F^2}{(p_0^0)^2}\right)(\ell_\mu \delta_{0\nu} + \ell_\nu \delta_{0\mu}) + \frac{2\kappa F^4}{(p_0^0)^4}\left(3\phi + \frac{2\kappa F^2}{(p_0^0)^2}\right) \delta_{0\mu}\delta_{0\nu} \end{aligned}$$

Quantized Fundamental Tensor

By equating the line element in Finsler space with the one in Riemann space, the generalized fundamental tensor in Riemann space becomes

$$\tilde{g}_{\mu\nu} = g_{ab}(x) \frac{dx_0^a}{d\zeta^\mu} \cdot \frac{dx_0^b}{d\zeta^\nu}.$$

$$\tilde{g}_{\mu\nu} = \left(\frac{1}{2} \frac{\partial^2}{\partial p_0^\mu \partial x_0^\nu} \phi^2 F^2(x_0^\mu, p_0^\nu)\right) \left(\frac{dx_0^\mu}{d\zeta^\mu} \frac{dx_0^\nu}{d\zeta^\nu} + \frac{\bar{m}^2}{\mathscr{F}^2} (1 + 2\beta p_0^\rho p_{0\rho}) \frac{dp_0^\mu}{d\zeta^\mu} \frac{dp_0^\nu}{d\zeta^\nu}\right)$$

where $\mathscr{F} = \bar{m}\mathscr{A}$ represents a proper maximal force with $\mathscr{A}$ is the maximum proper acceleration, $\mathscr{A} = c^7/G\hbar$.

Quantized Fundamental Tensor

$$\tilde{g}_{\mu\nu} = \left(\phi^2 + 2\frac{\kappa\phi}{(p_0^0)^2}F^2\right)\left[1 + \frac{\bar{m}^2}{\mathscr{F}^2}\left(1 + 2\beta p_0^\rho p_{0\rho}\right)\dot{p}_0^\mu \dot{p}_0^\nu\right] g_{\mu\nu}$$

$$+ \left[\frac{dx_0^\mu}{d\zeta^\mu}\frac{dx_0^\nu}{d\zeta^\nu} + \frac{\bar{m}^2}{\mathscr{F}^2}\left(1 + 2\beta p_0^\rho p_{0\rho}\right)\frac{dp_0^\mu}{d\zeta^\mu}\frac{dp_0^\nu}{d\zeta^\nu}\right] d_{\mu\nu},$$

$$d_{\mu\nu}(x) = \frac{6\kappa\phi}{(p_0^0)^2}\left\{F^2 \ell_\mu \ell^\sigma g_{\sigma\mu} - F^3 \ell^\sigma \left[\delta_{0\nu} g_{\sigma\mu} + \delta_{0\mu} g_{\sigma\nu}\right] + \frac{2+\phi}{8\phi} F^4 \delta_{0\mu}\delta_0^\sigma g_{\sigma\nu}\right\}$$

The symmetric metric d_{ab} should NOT be vanishing as ϕ is positive finite

Quantized Fundamental Tensor

$$d_{\mu\nu}(x) = \frac{6\kappa\phi}{(p_0^0)^2}\left\{F^2\ell_\mu\ell^\sigma g_{\sigma\mu} - F^3\ell^\sigma\left[\delta_{0\nu}g_{\sigma\mu} + \delta_{0\mu}g_{\sigma\nu}\right] + \frac{2+\phi}{8\phi}F^4\delta_{0\mu}\delta_0^\sigma g_{\sigma\nu}\right\}$$

$$\phi = 1 + \beta p_0^\rho p_{0\rho} = 1 + \frac{\kappa}{(p_0^0)^2}F^2,$$

$$\phi_\mu = \frac{2\kappa F}{(p_0^0)^3}(p_0^0\ell_\mu - F\delta_{0\mu}),$$

$$\phi_{\mu\nu} = \frac{2\kappa}{(p_0^0)^2}g_{\mu\nu}(x) - \frac{4\kappa F}{(p_0^0)^3}(\ell_\nu\delta_{0\mu} + \ell_\mu\delta_{0\nu}) + \frac{6\kappa F^2}{(p_0^0)^4}\delta_{0\nu}\delta_{0\mu},$$

$$F^2 = \phi^2\,\frac{|p_0^\nu|^2 - |x_0^\mu|^2|p_0^\nu|^2 + \langle x_0^\mu \cdot p_0^\nu\rangle^2}{1 - |x_0^\mu|^2},$$

$$\ell_\gamma = \frac{p_0^\gamma}{F} + \frac{\langle x_0, p_0\rangle}{(1 - |x_0|^2)\,F}x_0^\gamma.$$

Quantized Fundamental Tensor

$$\tilde{g}_{\mu\nu} = \left(\phi^2 + 2\frac{\kappa\phi}{(p_0^0)^2}F^2\right)\left[1 + \frac{\bar{m}^2}{\mathscr{F}^2}\left(1 + 2\beta p_0^\rho p_{0\rho}\right)\dot{p}_0^\mu \dot{p}_0^\nu\right] g_{\mu\nu}$$

$$\phi = 1 + \beta \hat{p}_0^\rho \hat{p}_{0\rho}$$

RGUP Approach

$$\beta = \beta_0 G/(c^3\hbar) = \beta_0(\ell_{\mathrm{p}}/\hbar)^2$$

$$\mathscr{A} = (c^7/\hbar G)^{1/2}$$

$$\mathscr{F} = (m\mathscr{A})^{-1}$$

Klein Metric

Phase-Space Finsler Geometry

Quantized Metric Tensor on Riemann Manifold

Classical Metric Tensor on Riemann Manifold

Quantized Fundamental Tensor

$$\tilde{g}_{\mu\nu} = \left(\phi^2 + 2\frac{\kappa\phi}{(p_0^0)^2}F^2\right)\left[1 + \frac{\bar{m}^2}{\mathscr{F}^2}\left(1 + 2\beta p_0^\rho p_{0\rho}\right)\dot{p}_0^\mu \dot{p}_0^\nu\right] g_{\mu\nu}$$

$$F(\hat{x}_0^\mu, \hat{\tilde{p}}_0^\nu) = \left[\frac{|\hat{\tilde{p}}_0^\nu|^2 - |\hat{x}_0^\mu|^2 |\hat{\tilde{p}}_0^\nu|^2 + \langle \hat{x}_0^\mu \cdot \hat{\tilde{p}}_0^\nu \rangle^2}{1 - |\hat{x}_0^\mu|^2}\right]^{1/2}$$

$$\phi = 1 + \beta \hat{p}_0^\rho \hat{p}_{0\rho} \qquad \hat{p}_0^\mu \hat{p}_0^\nu \qquad \hat{\dot{p}}_0^\mu \hat{\dot{p}}_0^\nu$$

Quantum Operators

$$\beta = \beta_0 G/(c^3 \hbar) = \beta_0 (\ell_{\mathrm{p}}/\hbar)^2$$

$$\mathscr{F} = (m\mathscr{A})^{-1} \qquad \mathscr{A} = (c^7/\hbar G)^{1/2}$$

Quantum Constants

Roles of $\phi = 1 + \beta \hat{p}_0^{\rho} \hat{p}_{0\rho}$

0-homogeneous Phi plays multiple roles. The phase-space metric and whose relation to Finsler structure

$$\phi = 1 + \beta F^2(x_0^{\rho}, p_0^{\rho})$$

By multiplying both sides with F and expressing ϕF as F̄

$$\bar{F} = (1 + \beta F^2) F$$

By assuming that $\bar{\beta} = \beta F^3$

$$\bar{F} = F + \bar{\beta}$$

The first term F could be related to the Riemann metric

$$F^2 = g_{\mu\nu} y^{\mu} y^{\nu}, \text{ where } y = \dot{x} \in T_p M$$

The second term must be one-form on manifold M at point p

$\bar{\beta}$ should be expressed as $\bar{\beta} = b_i(x) y^i$ with $||\bar{\beta}||_p := \sup_{y \in T_p M} (\bar{\beta}(y)/F(y))$

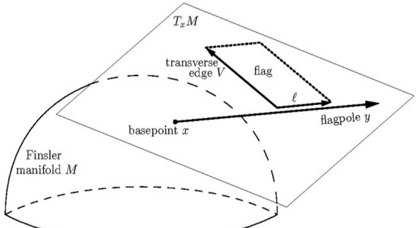
ϕ is capable of retaining the special curvature properties of the Randers metric. geometrical "*unification*" of the gravitation and electromagnetism interactions Phys. Rev. 59 (1941) 195-199

F function of x^μ, dx^ν

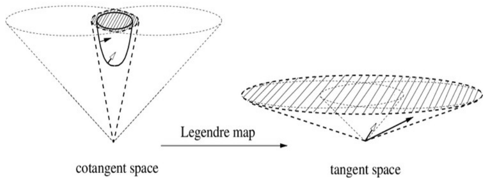
Non-linear metric space

Quadratic restriction on $F^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$

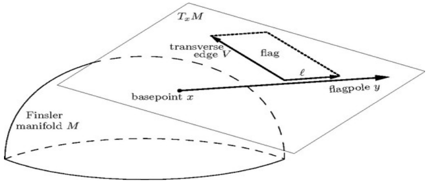
→ arc length, curvature & Euler-Lagrange



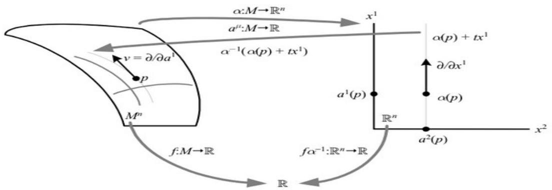
Dynamics of a quantum particle
=> F(x, dx) => F(x, v)



Legendre transform F(x, v) =>
H(x, p), Hamilton Structure



Homogeneous Finsler Structure:
F(x, mv) => F(x, p)



Descritized Finsler Structure

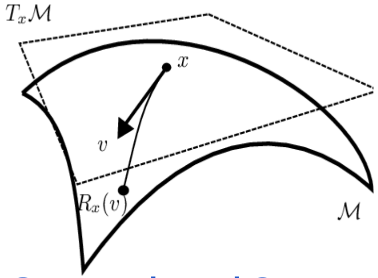
quadratic curves →
$F(x, \phi p) = \phi[g_{AB}(x)\, dp^A dp^B]^{1/2}$,

quartic curves →
$F(x, \phi p) = \phi[g_{ABCD}(x)\, dp^A dp^B dp^C dp^D]^{1/4}$

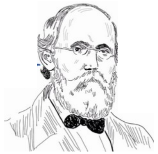
M function of x^μ

Linear metric space

Smoothly varying positive definite quadratic form



Without $F^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$
quadratic restriction

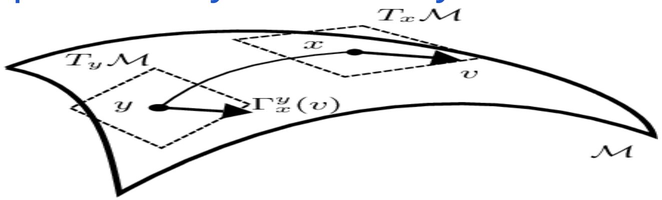


$g_{\mu\nu} = e_\mu \cdot e_\nu$

Symmetric and Constant (Linear) Connection wrt g

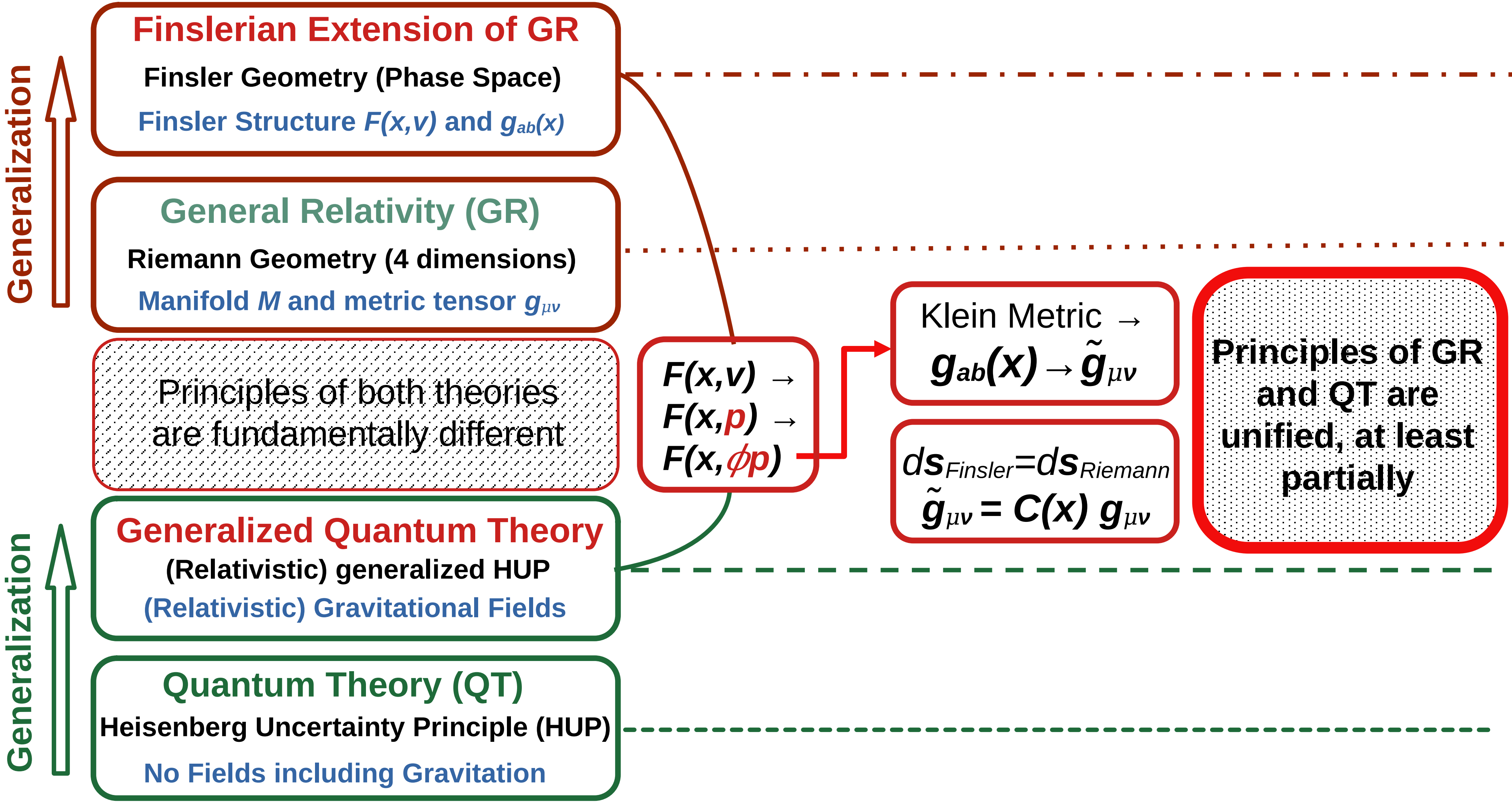
$\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$ $\quad\quad \nabla_\alpha g_{\mu\nu} = \nabla_\beta g^{\mu\nu} = 0$

Similar to Weyl metric, $g_{\mu\nu}$ is
approximately conformally transformed



$\tilde{g}_{\mu\nu} = C(x)\, g_{\mu\nu}$

α, β, μ, ν = 0,1,…,3
A,B,C,D = 0,1,2…,7


Generalization
Finslerian Extension of GR
Finsler Geometry (Phase Space)
Finsler Structure F(x,v) and g_ab(x)
General Relativity (GR)
Riemann Geometry (4 dimensions)
Manifold M and metric tensor g_μν
Principles of both theories are fundamentally different
F(x,v) →
F(x,p) →
F(x,ϕp)
Klein Metric →
g_ab(x) → g̃_μν
ds_Finsler=ds_Riemann
g̃_μν = C(x) g_μν
Principles of GR and QT are unified, at least partially
Generalization
Generalized Quantum Theory
(Relativistic) generalized HUP
(Relativistic) Gravitational Fields
Quantum Theory (QT)
Heisenberg Uncertainty Principle (HUP)
No Fields including Gravitation

Constructing GR: Metric Tensor

$$\tilde{g}_{\mu\nu,\alpha} = \frac{g_{\mu\nu}}{g_{\mu\nu}} g_{\mu\nu,\alpha}$$

$$+ \; 2\beta\phi\hat{p}_0^\sigma\hat{p}_0^\rho \left(1 + \dot{\hat{p}}_0^\mu\dot{\hat{p}}_0^\nu \mathscr{T}\right) \hat{g}_{\rho\sigma,\alpha}$$

$$+ \; \frac{1}{\dot{\hat{x}}^\alpha}\left\{\left[8 + 12\beta\hat{p}_0^\delta\hat{p}_{0\delta} + 4\beta^2\left(\hat{p}_0^\delta\hat{p}_{0\delta}\right)^2\right]\left(1 + \dot{\hat{p}}_0^\mu\dot{\hat{p}}_0^\nu\mathscr{T}\right) + 12\beta\left(\hat{p}_0^\mu\dot{\hat{p}}_0^\nu + \dot{\hat{p}}_0^\nu\hat{p}_0^\mu\right) + \phi^2\mathscr{T}\left(\dot{\hat{p}}_0^\mu\ddot{\hat{p}}_0^\nu + \ddot{\hat{p}}_0^\nu\dot{\hat{p}}_0^\mu\right)\right\}$$

For simplicity, we assume that

$$\bar{\phi} \;=\; \phi^2\big|_{\mathtt{tiny}\beta} = 1 + 2\beta\hat{p}_0^\delta\hat{p}_{0\delta},$$

$$\bar{\mathscr{T}} \;=\; \mathscr{T}\big|_{\mathtt{tiny}\beta} = \bar{\phi} + 3\beta F^2 + 12\beta\hat{p}_0^\mu\hat{p}_0^\nu,$$

and therefore at small β, we also find that $\beta\phi = \beta$ and

Constructing GR: Metric Tensor

at small β, we also find that $\beta\phi = \beta$ and

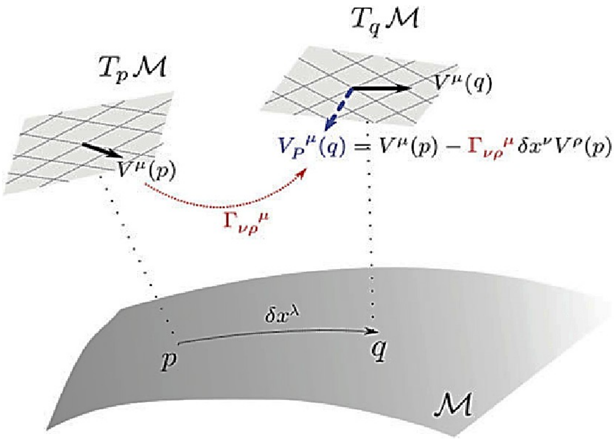
$$
\begin{aligned}
\tilde{g}_{\mu\nu,\alpha} &= \frac{g_{\mu\nu}}{g_{\mu\nu}} g_{\mu\nu,\alpha} \\
&+ 2\beta \hat{p}_0^{\sigma}\hat{p}_0^{\rho}\left(1+\dot{\hat{p}}_0^{\mu}\dot{\hat{p}}_0^{\nu}\mathscr{T}\right) g_{\rho\sigma,\alpha} \\
&+ \frac{1}{\dot{\hat{x}}^{\alpha}}\left[4\left(2+3\beta\hat{p}_0^{\delta}\hat{p}_{0\delta}\right)\left(1+\dot{\hat{p}}_0^{\mu}\dot{\hat{p}}_0^{\nu}\mathscr{T}\right)+12\beta\left(\hat{p}_0^{\mu}\dot{\hat{p}}_0^{\nu}+\dot{\hat{p}}_0^{\nu}\hat{p}_0^{\mu}\right)+\left(\bar{\phi}+\bar{\mathscr{T}}-1\right)\left(\dot{\hat{p}}_0^{\mu}\ddot{\hat{p}}_0^{\nu}+\ddot{\hat{p}}_0^{\nu}\dot{\hat{p}}_0^{\mu}\right)\right]
\end{aligned}
$$

For contravariant metric tensor

$$
\tilde{g}^{\mu\nu} = \left[\left(1+\phi^2\mathscr{F}^2\dot{p}_0^{\mu}\dot{p}_0^{\nu}\right)\mathscr{T}\right]^{-1} g^{\mu\nu}
$$

Constructing GR: Affine Connection

- a geometric object connecting nearby tangent (curved) spaces, i.e., permitting differentiability of the tangent vector fields or assuring them restrict dependence on manifold in a fixed vector space,
- a function assigning to each tangent vector and each vector field a covariant derivative or a new tangent vector.



In differential geometry, the generic form of affine connection reads

$$\Gamma^\mu_{\lambda\nu} = \left\{{}^{\mu}_{\lambda\nu}\right\} + K^\mu_{\lambda\nu} + \frac{1}{2}\left(Q^\mu_{\lambda\nu.} + Q^\mu_{\nu\lambda.} - Q^\mu_{.\nu\lambda}\right)$$

Christoffel symbol $\left\{{}^{\mu}_{\lambda\nu}\right\}$

Levi–Civita connection $K^\mu_{\lambda\nu} = \frac{1}{2}\left(T^\mu_{.\lambda\,\nu} - T^\mu_{\lambda.\nu} - T^\mu_{\nu.\lambda}\right)$

Covariant derivative of metric $Q_{\mu\nu\lambda} = -D_\mu(\Gamma) g_{\nu\lambda}$

Torsion $T^\mu_{\lambda\nu} = \Gamma^\mu_{\lambda\nu} - \Gamma^\mu_{\nu\lambda} = 2\Gamma^\mu_{[\lambda\nu]}$

Constructing GR: Affine Connection

GR assumes torsion-free and metric compatibility.
The latter implies linear independence of partial derivative tangent vectors and a flat space that can be found locally in a suitable frame (like Mikowski space),

$$\Gamma^{\mu}_{\lambda\nu} = \left\{{}^{\mu}_{\lambda\nu}\right\} + K^{\mu}_{\lambda\nu} + \frac{1}{2}(Q^{\mu}_{\lambda\nu.} + Q^{\mu}_{\nu\lambda.} - Q^{\mu}_{.\nu\lambda})$$

Christoffel symbol

$\left\{{}^{\mu}_{\lambda\nu}\right\}$ ✓

Levi-Civita connection

$K^{\mu}_{\lambda\nu} = \frac{1}{2}(T^{\mu}_{.\lambda\nu} - T^{\mu}_{\lambda.\nu} - T^{\mu}_{\nu.\lambda})$ ✗

Covariant derivative of metric

$Q_{\mu\nu\lambda} = -D_{\mu}(\Gamma)g_{\nu\lambda}$ ✗

Torsion

$T^{\mu}_{\lambda\nu} = \Gamma^{\mu}_{\lambda\nu} - \Gamma^{\mu}_{\nu\lambda} = 2\Gamma^{\mu}_{[\lambda\nu]}$ ✗

$$\Gamma^{\gamma}_{\beta\mu} = \frac{1}{2}g^{\alpha\gamma}(g_{\alpha\beta,\mu} + g_{\alpha\mu,\beta} - g_{\beta\mu,\alpha})$$

Constructing GR: Affine Connection

$$\Gamma^{\gamma}_{\beta\mu} = \frac{1}{2} g^{\alpha\gamma} \left(g_{\alpha\beta,\mu} + g_{\alpha\mu,\beta} - g_{\beta\mu,\alpha} \right)$$

$$\tilde{\Gamma}^{\alpha}_{\delta\beta} = \Gamma^{\alpha}_{\delta\beta} + \frac{F^{2}_{,\gamma}}{2C} \left(\delta^{\alpha}_{\beta} + \delta^{\alpha}_{\delta} - g^{\alpha\gamma} g_{\delta\beta} \right),$$

$$C = \left(\phi^2 + 2 \frac{\kappa}{(p^0_0)^2} F^2 \right) \left[1 + \frac{\bar{m}^2}{\mathcal{F}^2} \left(1 + 2\beta p^{\rho}_0 p_{0\rho} \right) \dot{p}^{\mu}_0 \dot{p}^{\nu}_0 \right]$$

$$F^{2}_{,\gamma} = 2 \frac{\left\langle x^{\alpha}_0 \cdot p^{\beta}_0 \right\rangle}{\left[(x^{\alpha}_0)^2 - 1 \right]^2} \left\{ \left[1 - (x^{\alpha}_0)^2 \right] p^{\beta}_0 - x^{\alpha}_0 \left\langle x^{\alpha}_0 \cdot p^{\beta}_0 \right\rangle \right\}$$

Constructing GR: Geodesic Congruence

The proper time can be expressed as $-c^2 d\tau^2 = ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$

$$\tilde{\tau}_{ab} = \int_0^1 \sqrt{-\tilde{g}_{\alpha\beta}(x)\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}} = \int_0^1 L\left(\frac{dx^\alpha}{d\sigma}, x^\alpha\right) d\sigma$$

The Euler-Lagrange equations can be derived by the variational methods

$$-\frac{d}{d\sigma}\frac{\partial L}{\partial(dx^\gamma/d\sigma)} + \frac{\partial L}{\partial x^\gamma} = 0,$$

Constructing GR: Geodesic Congruence

$$\frac{\partial L}{\partial \hat{x}^{\gamma}} = \frac{-L}{2}\left\{C\frac{\partial g_{\alpha\beta}}{\partial \hat{x}^{\gamma}}\frac{d\hat{x}^{\alpha}}{d\tau}\frac{d\hat{x}^{\beta}}{d\tau} + g_{\alpha\beta}\frac{2\kappa}{(\hat{p}^{0}_{0})^{2}}\left[1+\frac{\tilde{m}^{2}}{\mathcal{F}^{2}}\left(1+2\beta\hat{p}^{\rho}_{0}\hat{p}^{\rho}_{0}\right)\right]F^{2}_{,\gamma}\frac{d\hat{x}^{\alpha}}{d\tau}\frac{d\hat{x}^{\beta}}{d\tau}\right\}$$

$$\frac{d}{d\sigma}\frac{\partial L}{\partial(d\hat{x}^{\gamma}/d\sigma)} = -L\left[\tilde{g}_{\alpha\gamma}\frac{d^{2}\hat{x}^{\alpha}}{d\tau^{2}} + \frac{1}{2}\left(\frac{\partial\tilde{g}_{\alpha\gamma}}{\partial\hat{x}^{\beta}} + \frac{\partial\tilde{g}_{\gamma\beta}}{\partial\hat{x}^{\alpha}}\right)\frac{d\hat{x}^{\alpha}}{d\tau}\frac{d\hat{x}^{\beta}}{d\tau}\right].$$

Then, the generalized geodesic equations read

$$\frac{d^{2}x^{\alpha}}{d\tau^{2}} + \widetilde{\Gamma}^{\alpha}_{\delta\beta}\frac{dx^{\delta}}{d\tau}\frac{dx^{\beta}}{d\tau} = -\frac{g_{\delta\beta}}{2\tilde{g}_{\alpha\gamma}}F^{2}_{,\gamma}C\frac{dx^{\delta}}{d\tau}\frac{dx^{\beta}}{d\tau}, \qquad \frac{d^{2}x^{\alpha}}{d\tau^{2}} + \Gamma^{\alpha}_{\delta\beta}\frac{dx^{\delta}}{d\tau}\frac{dx^{\beta}}{d\tau} = 0,$$

Constructing GR: Geodesic Equations

- The difference between the geodesic equations for g$\alpha\beta$ and the ones for g̃$\alpha\beta$ is not negligible.

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\delta\beta}\frac{dx^\delta}{d\tau}\frac{dx^\beta}{d\tau} = -\frac{F^2_{,\gamma}}{2C}\left[\frac{g_{\delta\beta}}{\tilde{g}_{\alpha\gamma}}C^2\frac{dx^\delta}{d\tau}\frac{dx^\beta}{d\tau} + \delta^\alpha_\beta + \delta^\alpha_\delta - g^{\alpha\gamma}g_{\delta\beta}\right]$$

- The impact of such additional contributions to the emerged geometric structures and curvatures extends the GR's applicability to cover both low and large scales.

GR's Sensical Predictions

$$\tilde{g}_{\mu\nu} = \left(\frac{1}{2}\frac{\partial^2}{\partial p_0^\mu \partial x_0^\nu}\phi^2 F^2(x_0^\mu, p_0^\nu)\right)\left(\frac{dx_0^\mu}{d\zeta^\mu}\frac{dx_0^\nu}{d\zeta^\nu} + \frac{\bar{m}^2}{\mathscr{F}^2}(1 + 2\beta p_0^\rho p_{0\rho})\frac{dp_0^\mu}{d\zeta^\mu}\frac{dp_0^\nu}{d\zeta^\nu}\right),$$

Additional Curvatures

Additional Connections

Additional Structures

Insights on Quantum Gravity

NEWLAND

Einstein-Gilbert Straus Metric

- When combining FLRW and Schwarzschild metrics so that the Schwarzschild metric becomes embedded in an expanding Universe, one refers to the Swiss–cheese model of the Universe which was suggested by Einstein and Straus in the 1940s. Rev. Mod. Phys., 17(1945)120–124

- The Swiss-cheese model was an early attempt to approach the Universe as lumps of cosmic substance which is inhomogeneously interspersed with holes or voids.

- This was correctly formulated by Gilbert Monthly Notices of the Royal Astronomical Society, 116(1956)678–683

Einstein-Gilbert Straus Metric

- The EGS metric rads

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{a(t)}{1 - \frac{2M}{r}} dr^2 - a(t) r^2 d\Omega^2,$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the standard metric on the surface of two-sphere.

- Imposing M (r, t) requires incorporating an overall expansion, for instance, finite cosmological constant Λ

- For the sake of simplicity, let us assume that the so-called Einstein–Straus vacuole forms exact spheres

Einstein-Gilbert Straus Metric

$$M(r,t) = \frac{4}{3}\pi f(t) r^3.$$

- The arbitrary function f(t) expresses the evolution of the cosmic substance with the cosmic time t. To scale r with t, let us suggest that

$$f(t) = \frac{1}{\mu^2}\tanh\left(\frac{t}{\mu}\right)$$

$$ds^2 = \left(1 - \frac{2M(r,t)}{r} - \frac{1}{3}\Lambda r^2\right)dt^2 - a(t)\left(1 - \frac{2M(r,t)}{r} - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 - a(t)r^2 d\Omega^2$$

Riamann Curvature Tensor

Accordingly, the Riemann curvature tensor reads

$$\tilde{R}^{\gamma}_{\beta\mu\nu} = R^{\gamma}_{\beta\mu\nu}$$

Conventional Riemann Curvature Tensor

$$+ \tilde{\Gamma}^{\gamma}_{\sigma\mu} + \tilde{\Gamma}^{\gamma}_{\sigma\nu,\mu} + \tilde{\Gamma}^{\gamma}_{\beta\mu,\nu} - \left(\Gamma^{\gamma}_{\sigma\nu,\mu} + \Gamma^{\gamma}_{\beta\mu,\nu} \right)$$

Connections & QM imprints

$$+ \frac{F^2_{,\gamma}}{2C(x)} \left(g^{\alpha\gamma}_{,\mu} g_{\sigma\nu} + g^{\alpha\gamma} g_{\sigma\nu,\mu} + g^{\alpha\gamma}_{,\mu} g_{\beta\mu} + g^{\alpha\gamma} g_{\beta\mu,\nu} \right)$$

Metrics, Geometric Structures & QM imprints

Ricci Curvature Tensor

Ricci curvature tensor can be deduced by contraction of Riemann curvature tensor or derivations of the affine connections

$$\tilde{R}_{\beta\nu} = R_{\beta\nu}$$

Conventional Ricci Curvature Tensor

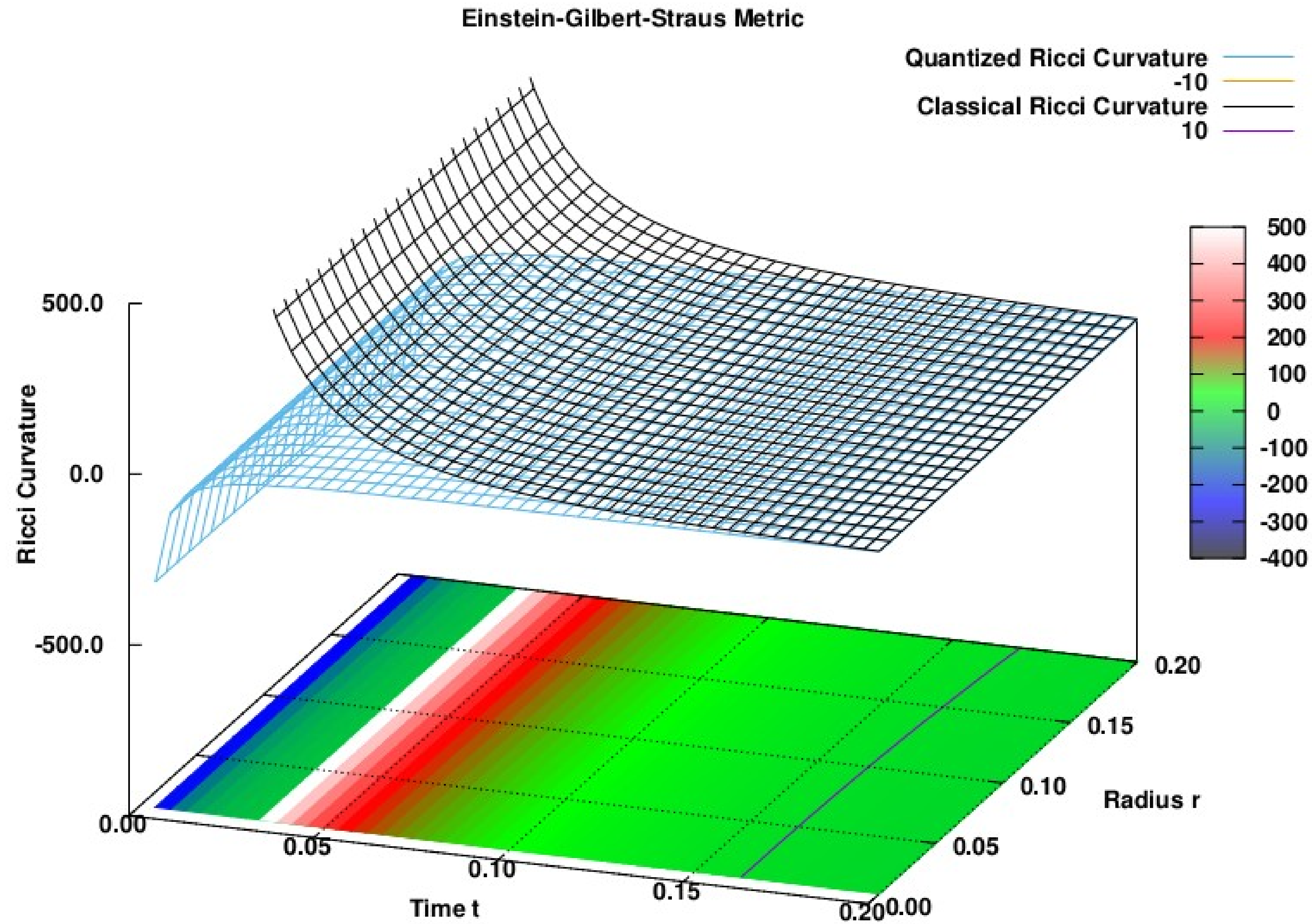
$$+ \tilde{\Gamma}^{\gamma}_{\sigma\gamma} + \tilde{\Gamma}^{\gamma}_{\sigma\nu,\gamma} + \tilde{\Gamma}^{\gamma}_{\beta\gamma,\nu} - \left(\Gamma^{\gamma}_{\sigma\nu,\gamma} + \Gamma^{\gamma}_{\beta\gamma,\nu}\right)$$

Connections & QM imprints

$$+ \frac{F^2_{,\gamma}}{2C(x)} \left(g^{\alpha\gamma}_{,\gamma} g_{\sigma\nu} + g^{\alpha\gamma} g_{\sigma\nu,\gamma} + g^{\alpha\gamma}_{,\nu} g_{\beta\gamma} + g^{\alpha\gamma} g_{\beta\gamma,\nu}\right)$$

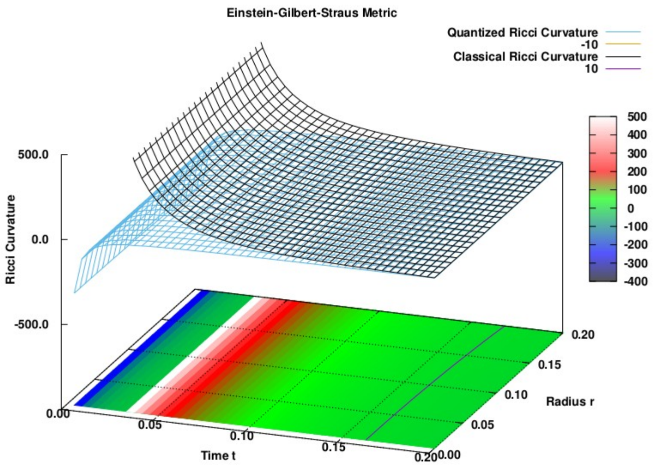
Metrics, Geometric Structures & QM imprints

$$
\begin{aligned}
\tilde{R}_{\beta\nu} &= \frac{3r\dot{a}^2(t)}{4a(t)A(r,t)} + \frac{9ra(t)}{A^3(r,t)}\left[-12\frac{\partial M^2(r,t)}{\partial t} + A(r,t)\frac{\partial^2 M(r,t)}{\partial t^2}\right] \\
&- \frac{r^2\left[1+\sin^2(\theta)\right]}{12a(t)}\left\{3\dot{a}^2(t) - 6a(t)\ddot{a}(t) + \frac{18a(t)\dot{a}(t)\frac{\partial M(r,t)}{\partial t}}{A(r,t)}\right. \\
&\quad \left. + \frac{4a(t)}{3r^4}\left[A(r,t)\left(r^3\Lambda - 3M(r,t) + 3r\frac{\partial M(r,t)}{\partial r}\right)\right]\right\} \\
&+ \frac{1}{6r^3A^2(r,t)}\left\{432M^3(r,t) + 216rM^2(r,t)\left[-2 + r^2\Lambda - 2\frac{\partial M(r,t)}{\partial r} + r\frac{\partial^2 M(r,t)}{\partial r^2}\right]\right. \\
&\quad +18r^2M(r,t)\left[-3r^2\ddot{a}(t) + 2\left(-3+r^2\Lambda\right)\left(-1 + r^2\Lambda - 4\frac{\partial M(r,t)}{\partial r} + 2r\frac{\partial^2 M(r,t)}{\partial r^2}\right)\right] \\
&\quad +r^3\left[-9r^2\left(-3+r^2\Lambda\right)\ddot{a}(t) + 2(-3+r^2\Lambda)^2\left(-6\frac{\partial M(r,t)}{\partial r} + r\left[r\Lambda + 3\frac{\partial^2 M(r,t)}{\partial r^2}\right]\right)\right. \\
&\qquad \left.\left. +81r\dot{a}(t)\frac{\partial M(r,t)}{\partial t}\right]\right\}, \qquad (
\end{aligned}
$$

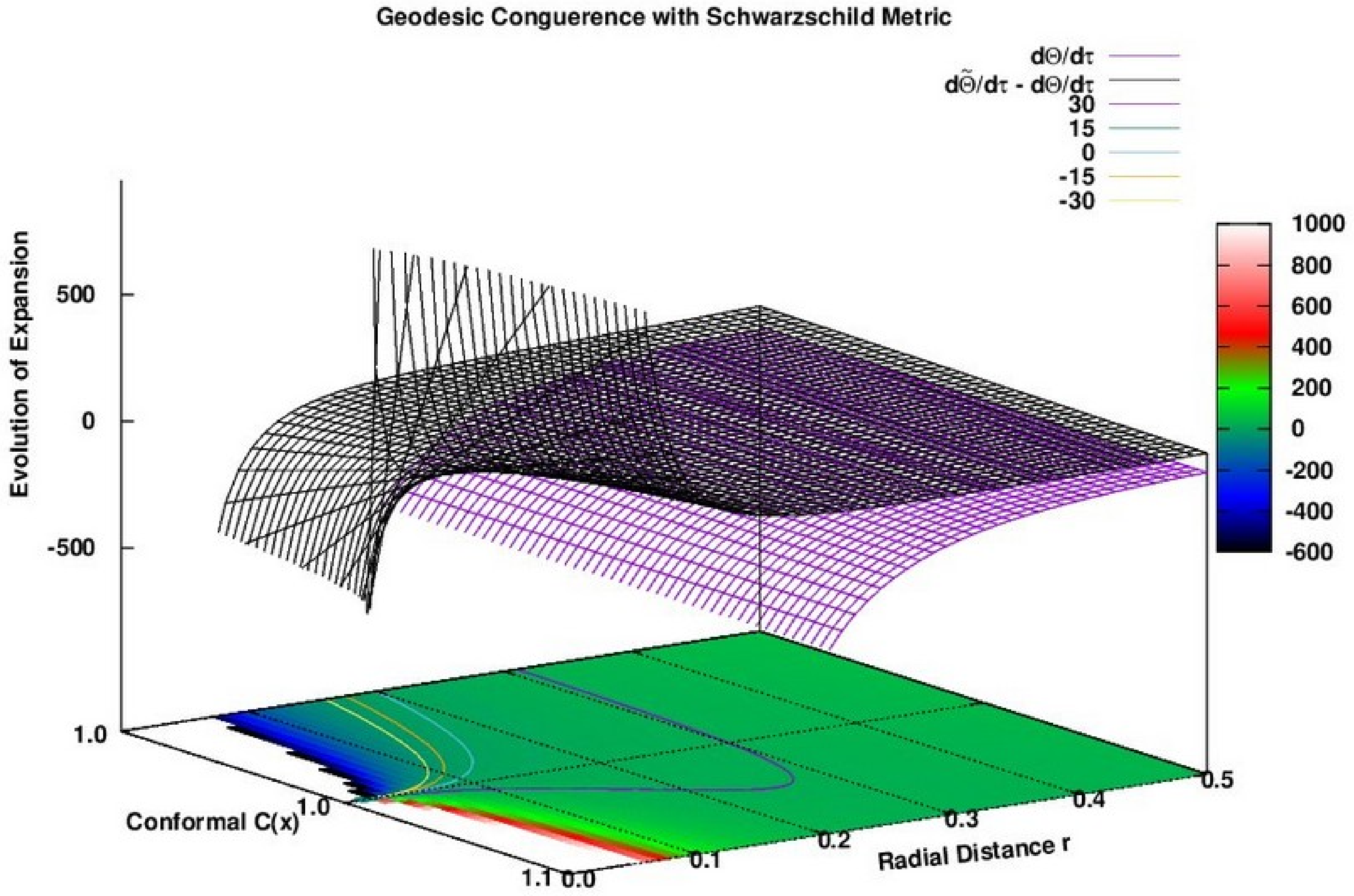

Einstein-Gilbert-Straus Metric
Quantized Ricci Curvature
-10
Classical Ricci Curvature
10
500
400
300
200
100
0
-100
-200
-300
-400
500.0
0.0
-500.0
Ricci Curvature
0.00
0.05
0.10
0.15
0.20
Time t
0.00
0.05
0.10
0.15
0.20
Radius r

Additional Curvature/Gravitation

- The classical Ricci curvatures are positive and long-lived.
- Decreasing t the positivity of $R_{\beta\nu}$ increases.
- There is almost no impact of varying r on $R_{\beta\nu}$.
- The results of $\tilde{R}_{\beta\nu}$ are negative but short-lived.



Athough the absence of topological consequences, there are no topological obstructions for the negativity of the Ricci curvatures



Geodesic Conguerence with Schwarzschild Metric
dΘ/dτ
dΘ̃/dτ - dΘ/dτ
30
15
0
-15
-30
1000
800
600
400
200
0
-200
-400
-600
Evolution of Expansion
500
0
-500
1.0
Conformal C(x)
1.0
1.1
0.0
0.1
0.2
0.3
0.4
0.5
Radial Distance r

Additional Curvature/Gravitation

- One would rightfully argue that the emerged curvatures might be artifacts in some coordinate systems.

- An assessment would be feasible by the basic polynomial curvature invariants in GR, namely, the Ricci and Kretschmann invariant scalars.

$$R = g^{\beta\nu} R_{\beta\nu}, \qquad \tilde{R} = \tilde{g}^{\beta\nu} \tilde{R}_{\beta\nu}$$

$$K = R^{\gamma\beta\mu\nu} R_{\gamma\beta\mu\nu}, \qquad \tilde{K} = \tilde{R}^{\gamma\beta\mu\nu} \tilde{R}_{\gamma\beta\mu\nu}$$

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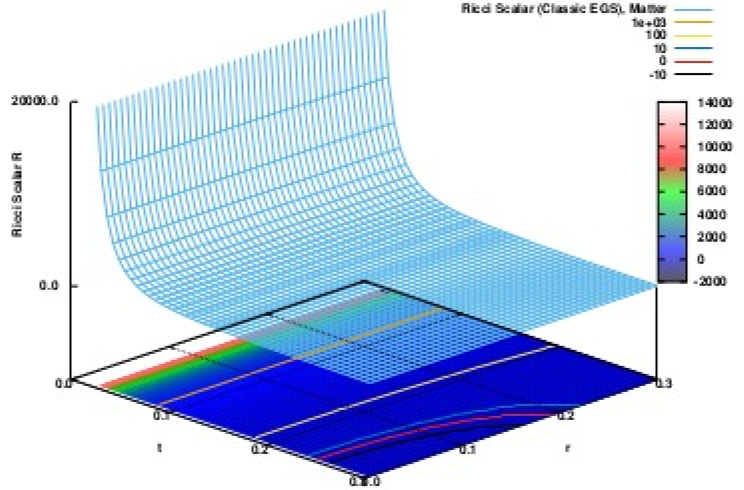

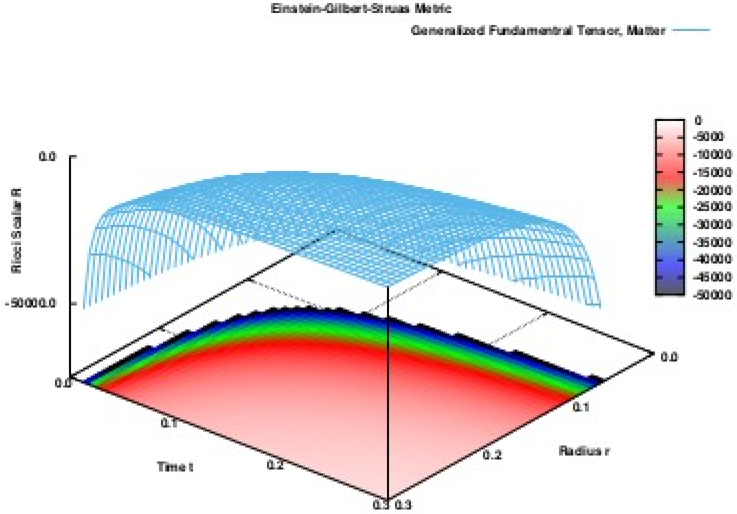

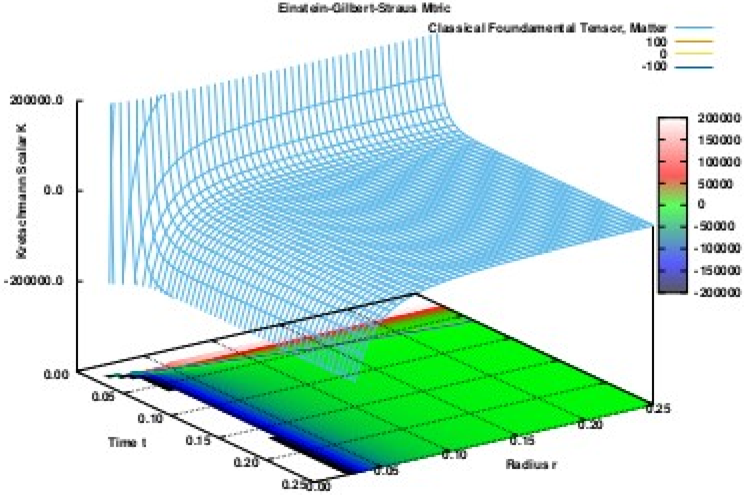

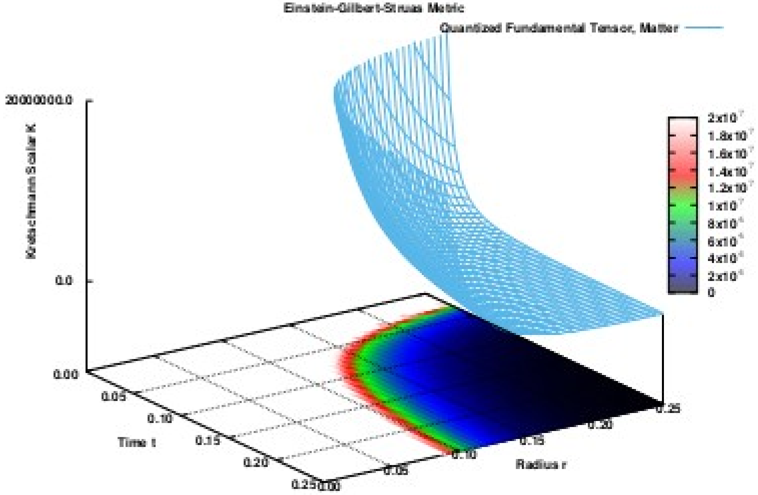

Conclusions

- The proposed quantization seems to unveil quantum-conditioned curvatures whose intrinsicality, essentiality, and reality are assessed by finite Ricci and Kreschmann invariant scalars.

- The additional sources of gravity
  - i) arise in the relativistic quantum regime,
  - ii) are overcast at nonrelativistic classical regime, and
  - iii) are apparently overlooked in Einstein’s GR.

- We conclude that the proposed quantum geometrical approach would be rather supposed to represent an appropriate mathematical framework for the emergence of quantum gravity.

Thank You!