# Bootstrapped Newtonian compact objects

*Star-UBB Institute*

*Seminar Series in Gravitation, Cosmology and Astrophysics*

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# Plan of the talk

- The Lagrangian and the equation of motion
- Outer vacuum and boundary conditions
- Stars and black holes with uniform density
- Polytropic stars
- Masses of bootstrapped Newtonian objects
- Binary mergers, mass gap and area law
- Stability of bootstrapped Netwonian dense stars
- Conclusions

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# Bootstrapped Newtonian gravity

- Motivation:
  - Gravity is tested in the weak-field regime, many orders of magnitude below where it becomes dominant, regime in which results are in very good agreement with general relativity;
  - Perturbative approaches fail in strong gravitational fields (reason being that all terms in the series contribute roughly the same an the series cannot be truncated);
  - Singularity theorems of general relativity require black holes to collapse all the way into a region of vanishing volume and infinite density;
  - There are some corpuscular proposals for black hole interiors which would solve the problem of the singularities.
- Bootstrapped Newtonian gravity
  - Bottom-up approach;
  - It allows us a fresh new look into (extremely) dense self-gravitating stars;
  - It allows for highly compact objects with regular densities due to the absence of a Buchdahl limit.

# Bootstrapped Newtonian Lagrangian [Phys.Rev.D 98 (2018) 10, Eur.Phys.J.C 79 (2019) 11]

- The bootstrapped Newtonian Lagrangian

$$
\begin{aligned}
L[V] &= L_{\mathrm{N}}[V] - 4\,\pi \int_0^\infty r^2\, dr [q_V\, \mathcal{J}_V\, V + q_p\, \mathcal{J}_p\, V + q_\rho\, \mathcal{J}_\rho\, (\rho + q_p\, \mathcal{J}_p)] \\
&= -4\,\pi \int_0^\infty r^2\, dr \left[ \frac{(V')^2}{8\,\pi\, G_N} (1 - 4\, q_V\, V) + (\rho + 3\, q_p\, p) V (1 - 2\, q_\rho\, V) \right]
\end{aligned}
$$

Newtonian part:

$$
L_{\mathrm{N}}[V] = -4\,\pi \int_0^\infty r^2\, dr \left[ \frac{(V')^2}{8\,\pi\, G_N} + \rho\, V \right]
$$

$$
r^{-2} \left( r^2\, V' \right)' \equiv \triangle V = 4\,\pi\, G_N\, \rho
$$

gravitational self-coupling :

$$
\mathcal{J}_V \simeq \frac{dU_{\mathrm{N}}}{d\mathcal{V}} = -\frac{[V'(r)]^2}{2\,\pi\, G_N}
$$

(non-negligible) pressure contribution:

$$
\mathcal{J}_p \simeq -\frac{dU_p}{d\mathcal{V}} = 3\, p
$$

higher order term: $\mathcal{J}_\rho = -2\, V^2$

- Euler-Lagrange equation:

$$
\triangle V = 4\,\pi\, G_N\, (\rho + 3\, q_p\, p) \frac{1 - 4\, q_\rho\, V}{1 - 4\, q_V\, V} + \frac{2\, q_V\, (V')^2}{1 - 4\, q_V\, V}
$$

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# Outer Vacuum Solutions and Boundary Conditions [Phys.Rev.D 98 (2018) 10, Eur.Phys.J.C 79 (2019) 11]

- Outside the source: $\rho = 0, \quad p = 0$

  and, after solving the EOM, the potential in vacuum becomes:

$$V_{\text{out}} = \frac{1}{4\,q_V}\left[1 - \left(1 + \frac{6\,q_V\,G_N\,M}{r}\right)^{2/3}\right]$$

$$V_{\text{out}} \underset{r\to\infty}{\simeq} -\frac{G_N\,M}{r} + q_V\,\frac{G_N^2\,M^2}{r^2} - q_V^2\,\frac{8\,G_N^3\,M^3}{3\,r^3}$$

- Boundary conditions:

$$V_{\text{in}}(R) = V_{\text{out}}(R) \equiv V_R = \frac{1}{4\,q_V}\left[1 - (1 + 6\,q_V\,\mathcal{X})^{2/3}\right]$$

$$V'_{\text{in}}(R) = V'_{\text{out}}(R) \equiv V'_R = \frac{\mathcal{X}}{R\,(1 + 6\,q_V\,\mathcal{X})^{1/3}}$$

$$V'_{\text{in}}(0) = 0$$

$$\mathcal{X} \equiv \frac{G_N\,M}{R}$$

represents the compactness.

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# Bootstrapped Newtonian stars and black holes [Phys.Rev.D 98 (2018) 10, Eur.Phys.J.C 79 (2019) 11]

- Stars and black holes of uniform density:

$$\rho = \rho_0 \equiv \frac{3\,M_0}{4\,\pi\,R^3}\,\Theta(R-r)$$

with the (Newtonian) proper mass in general given by:

$$M_0 = 4\,\pi \int_0^R r^2\,\rho(r)\,dr$$

and the additional constraint given by the conservation equation

$$p' = -V'\,(\rho + q_p\,p)$$

- Set the couplings to some numerical values to simplify the equations.
- The complexity of the problem requires one to find solutions separately in two regimes:
  - Small and intermediate compactness (stars)
  - Large compactness (black holes)

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# Small and intermediate compactness [Phys.Rev.D 98 (2018) 10, Eur.Phys.J.C 79 (2019) 11]

- An approximate solution:

$$V_s = V_0 + \frac{G_N M_0}{2 R^3} e^{V_R - V_0} r^2$$

(series expansion of the potential around r=0)

  - Odd powers vanish because:

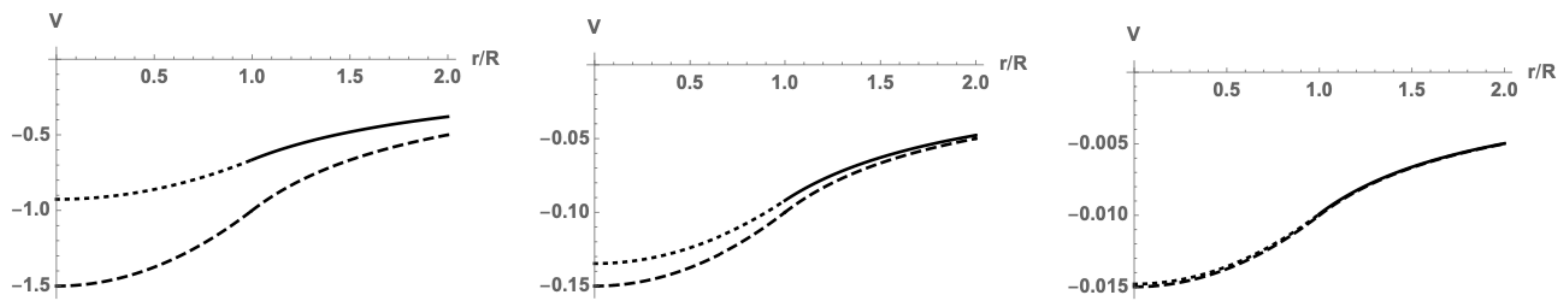
$$V'_{in}(0) = 0$$

- ADM and proper mass relationship:

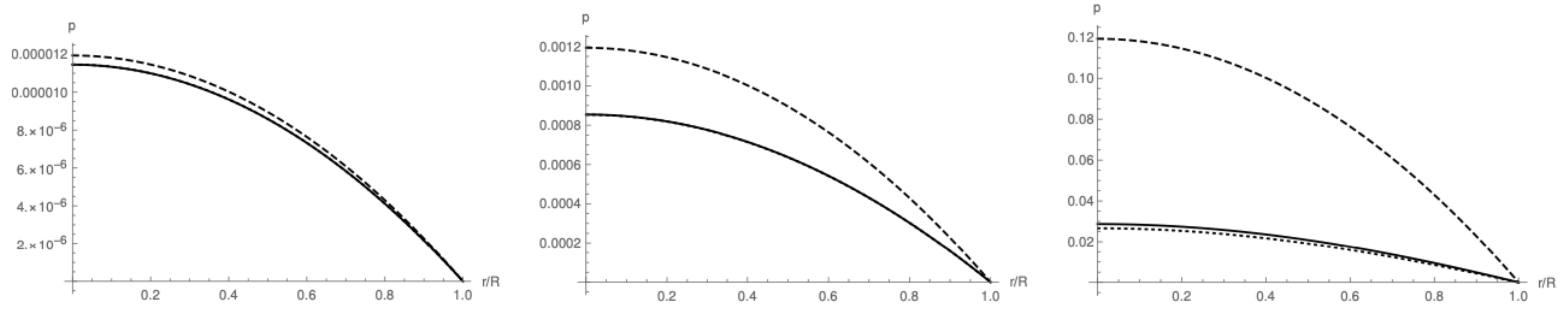
$$M_0 = \frac{M e^{-\frac{\mathcal{X}}{2(1+6\mathcal{X})^{1/3}}}}{(1+6\mathcal{X})^{1/3}}$$

- Potential after using the boundary conditions:

$$V_s = \frac{\left[(1+6\mathcal{X})^{1/3} - 1\right] + 2\mathcal{X}\left[(r/R)^2 - 4\right]}{4(1+6\mathcal{X})^{1/3}}$$




Potential $V_{out}$ (solid line) *vs* approximate solution (dotted line) *vs* Newtonian potential (dashed line), for $\mathcal{X} = 1$ (left panel), $\mathcal{X} = 1/10$ (center panel) and $\mathcal{X} = 1/100$ (right panel).




Pressure (solid line) *vs* numerical pressure (dotted line) *vs* Newtonian pressure (dashed line), for $\mathcal{X} = 1/100$ (left panel), $\mathcal{X} = 1/10$ (center panel) and $\mathcal{X} = 1$ (right panel).

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# Large compactness [Phys.Rev.D 98 (2018) 10, Eur.Phys.J.C 79 (2019) 11]

- Rely fully on comparison methods
  - Start with simpler eq. in terms of $\psi(r; A, B)$
  - The potential is then written:

    $$V_{\rm in} = f(r; A, B)\, \psi(r; A, B)$$

  - Solutions for function $f(r; A, B)$ are not feasible
  - Find constants such that

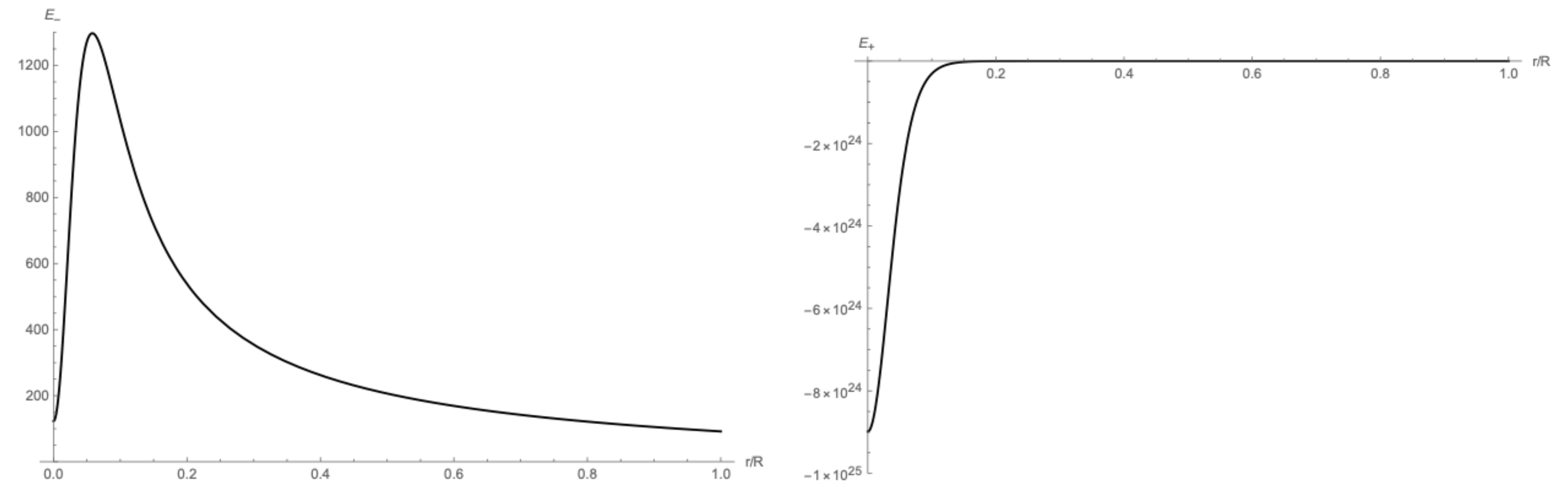
    $$C_- < f(r) < C_+$$

  - And the potential will be bound by

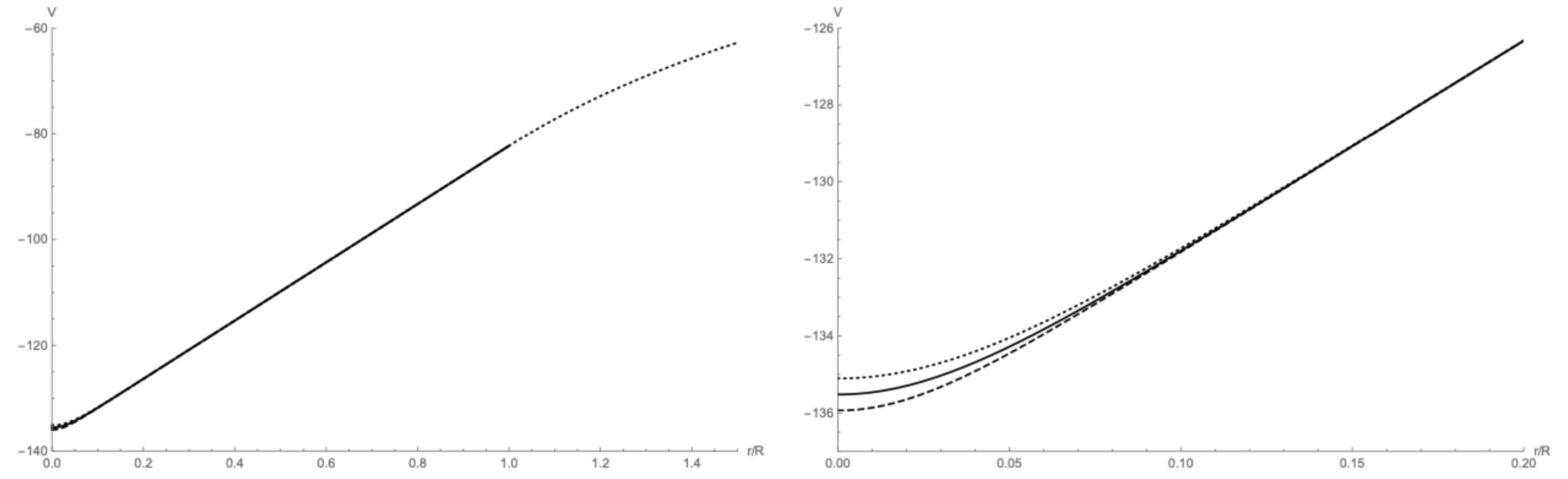
    $$V_\pm = C_\pm\, \psi(r; A_\pm, B_\pm)$$

  - Approximate linear solution:

    $$V_{\rm lin} \simeq V_R + V_R'\,(r - R)$$




Left panel: $E_-$ for $C_- = 1$. Right panel: $E_+$ for $C_+ = 1.6$. Both cases considering $\mathcal{X} = 10^3$.




Left panel: approximate inner potentials $V_-$ (dashed line), $\tilde{V}$ (solid line) and $V_+$ (dotted line) for $0 \le r \le R$ and exact outer potential $V_{\rm out}$ (dotted line) for $r > R$. Right panel: approximate inner potentials $V_-$ (dashed line), $\tilde{V}$ (solid line) and $V_+$ (dotted line) for $0 \le r \le R/5$. Both plots are for $\mathcal{X} = 10^3$.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# Horizon and Buchdahl limit [Phys.Rev.D 98 (2018) 10, Eur.Phys.J.C 79 (2019) 11]

- In general relativity
  - Schwarzschild radius

$$R_H = 2\, G_N\, M$$

  - Buchdahl limit (using TOV-equation)

$$R > (9/8)\, R_H$$

- We assume a Newtonian horizon

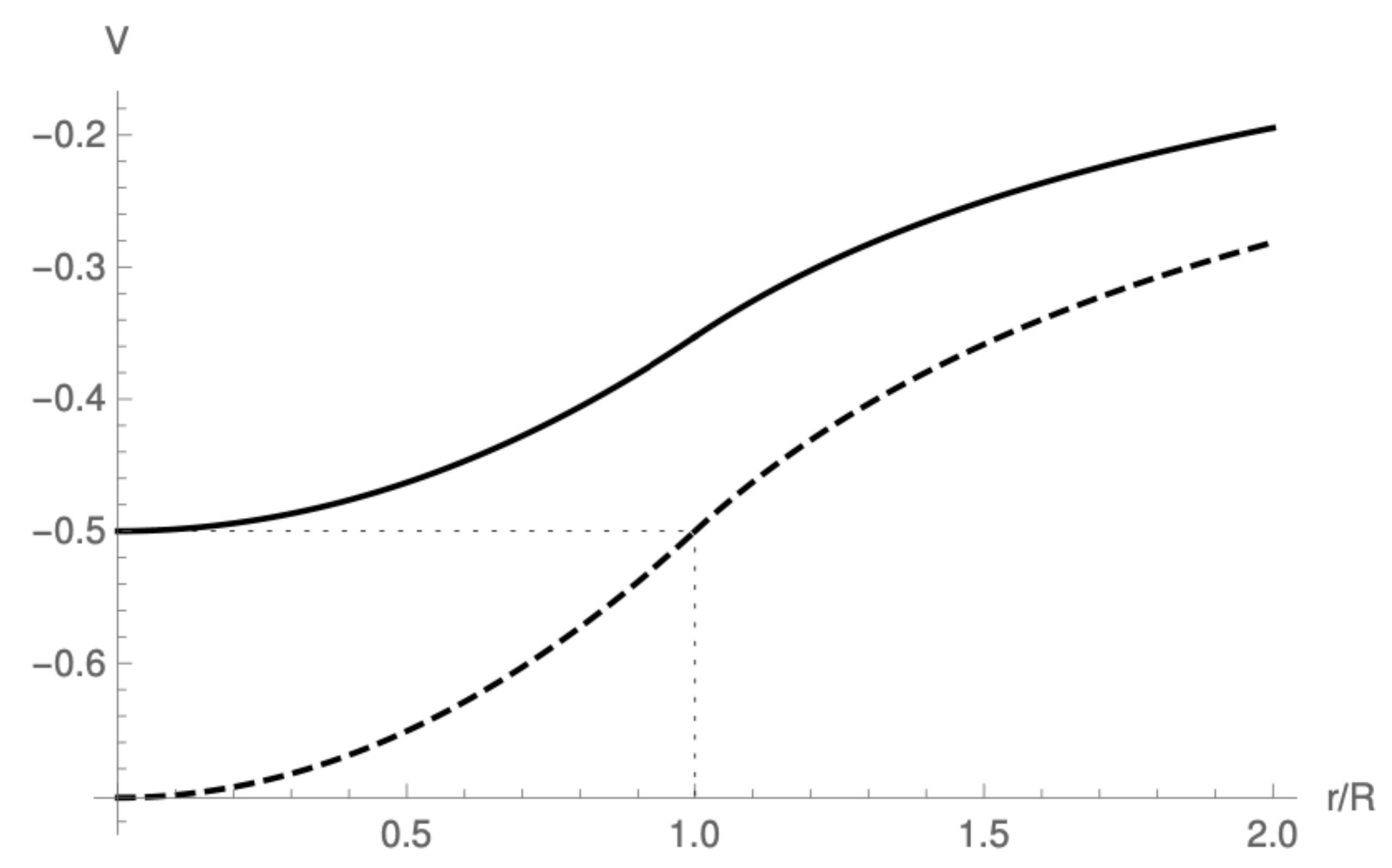
$$2\, V(r_H) = -1$$

  - Horizon inside the source

$$2\, V_{\rm in}(R_H = 0) = -1$$

  - Horizon at the edge of the source

$$2\, V_{\rm in}(R_H = R) = 2\, V_{\rm out}(R) = -1$$




Potentials for $R_{\rm H} = 0$ (solid line) and $R_{\rm H} = R$ (dashed line).

$$\begin{cases} \text{no horizon} & \text{for } \; G_{\rm N}\, M/R \lesssim 0.46 \\ 0 < r_{\rm H} \leq R \simeq 1.4\, G_{\rm N}\, M & \text{for } \; 0.46 \lesssim G_{\rm N}\, M/R \leq 0.69 \\ r_{\rm H} \simeq 1.4\, G_{\rm N}\, M & \text{for } \; G_{\rm N}\, M/R \gtrsim 0.69 \; . \end{cases}$$

- *No Buchdahl limit exists for Bootstrapped Newtonian stars!*

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# Polytropic stars [Phys.Rev.D 102 (2020) 10]

- Polytropic eq. of state:

$$p(r) = \gamma\, \rho^n(r) = \tilde{\gamma}\, \rho_0 \left[\frac{\rho(r)}{\rho_0}\right]^n$$

  - same EOM as before with couplings set to 1:
  - Use conservation eq. and EOS to write EOM

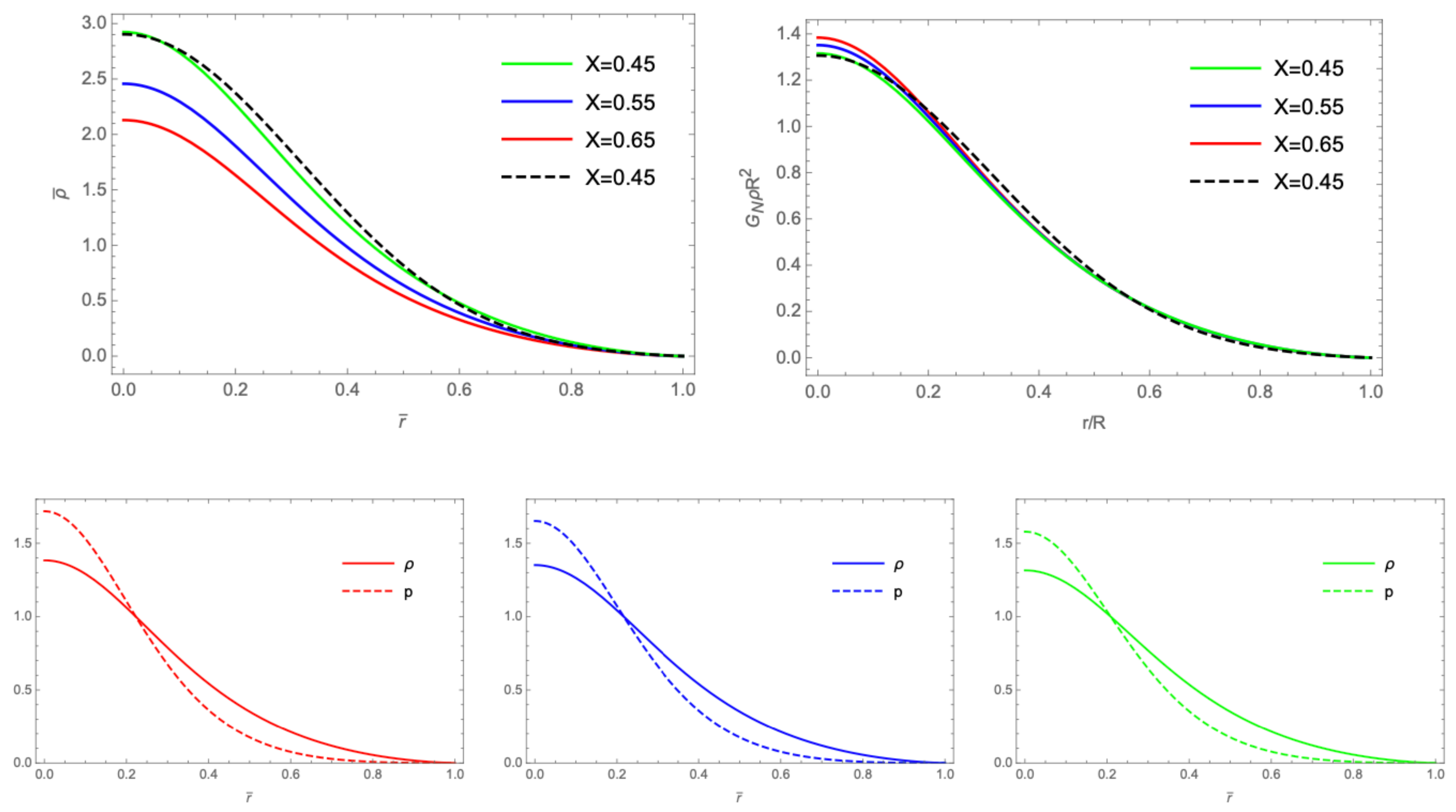
in terms of the density and compactness.

- Therefore, use Gaussian density profiles:

$$\rho = \begin{cases} \rho_0\, e^{-\frac{r^2}{b^2 R^2}}\,, & r \le R \\ 0\,, & r > R\,. \end{cases}$$

- impose a slight discontinuity at:

$$\rho_R \equiv \rho(R) = 0$$

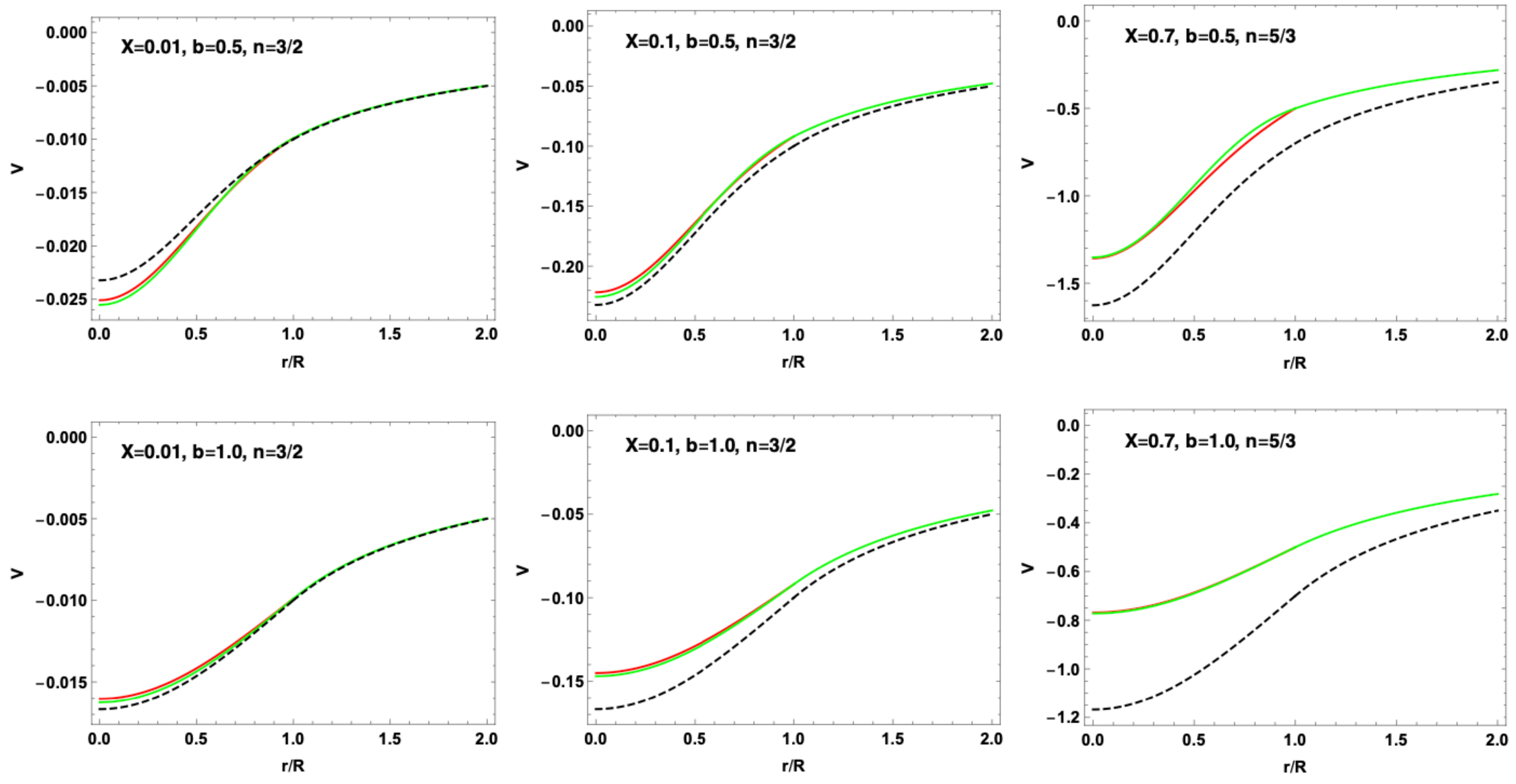



Upper panels: density profile obtained numerically for $\tilde{\gamma} = 1$, $n = 5/3$ (solid lines) and Gaussian approximation (dashed line) for the smallest compactness (left panel: dimensionless quantities; right panel: dimensionful quantities). Lower panels: density (solid lines) and pressure (dashed lines) for the cases in the upper panels

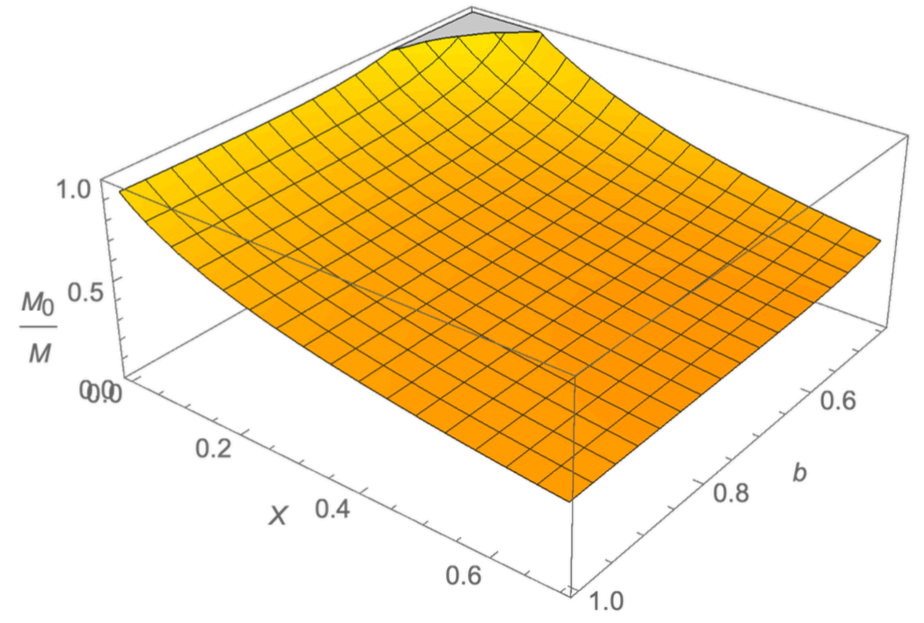
# Polytropic stars [Phys.Rev.D 102 (2020) 10]

...skip intermediary steps. Some conclusions:

- Numerical errors (resulting from solving the EoM) are smaller for larger values of b.
- The Newtonian and bootstrapped Newtonian potentials are more different for more compact objects. The differences becomes insignifiant for smaller densities.
- Newtonian potential generates deeper wells for most cases (all except upper left plot).
- In Newtonian physics $M_0/M=1$, while in the bootstrapped Newtonian model it is (almost) always smaller than one.
- Bootstrapped Newtonian stars can be much more compact than general relativistic ones and can withstand higher pressures.




Bootstrapped potentials for $\mathcal{X} = 0.01$ and $\mathcal{X} = 0.1$ we used $n = 3/2$, while for $\mathcal{X} = 0.7$ we used $n = 5/3$. The dashed black lines represent the Newtonian potential $V_N$ for a Gaussian matter distribution with the same $b$.




$M_0/M$ as a function of $b$ and $\mathcal{X}$.

# On the masses of bootstrapped Newtonian stars [Mod.Phys.Lett.A 35 (2020) 21]

- Generally the ADM mass and the proper mass are different! (In Newtonian physics they are the same)
  - Go back to the simple case of uniform densities!
  - We take a look at the effect of the *higher order term coupling* $q_\rho$ on the relationship between the ADM mass and proper mass, so we set the other couplings to 1.
  - We use the same approximation as before (series expansion of the potential around r=0) and get:

$$V_s \simeq \frac{R^2\left[(1+6\,\mathcal{X})^{1/3}-1\right]+2\,\mathcal{X}\left(r^2-4\,R^2\right)}{4\,R^2\,(1+6\,\mathcal{X})^{1/3}}$$

  - What is most interesting though is that only the ratio of the masses depends on the coupling:

$$\frac{M_0}{M} \simeq \frac{e^{-\frac{\mathcal{X}}{2(1+6\,\mathcal{X})^{1/3}}}\,(1+8\,\mathcal{X})}{(1+6\,\mathcal{X})^{2/3}\left[1-q_\rho+\frac{(1+8\,\mathcal{X})}{(1+6\,\mathcal{X})^{1/3}}\,q_\rho\right]}$$

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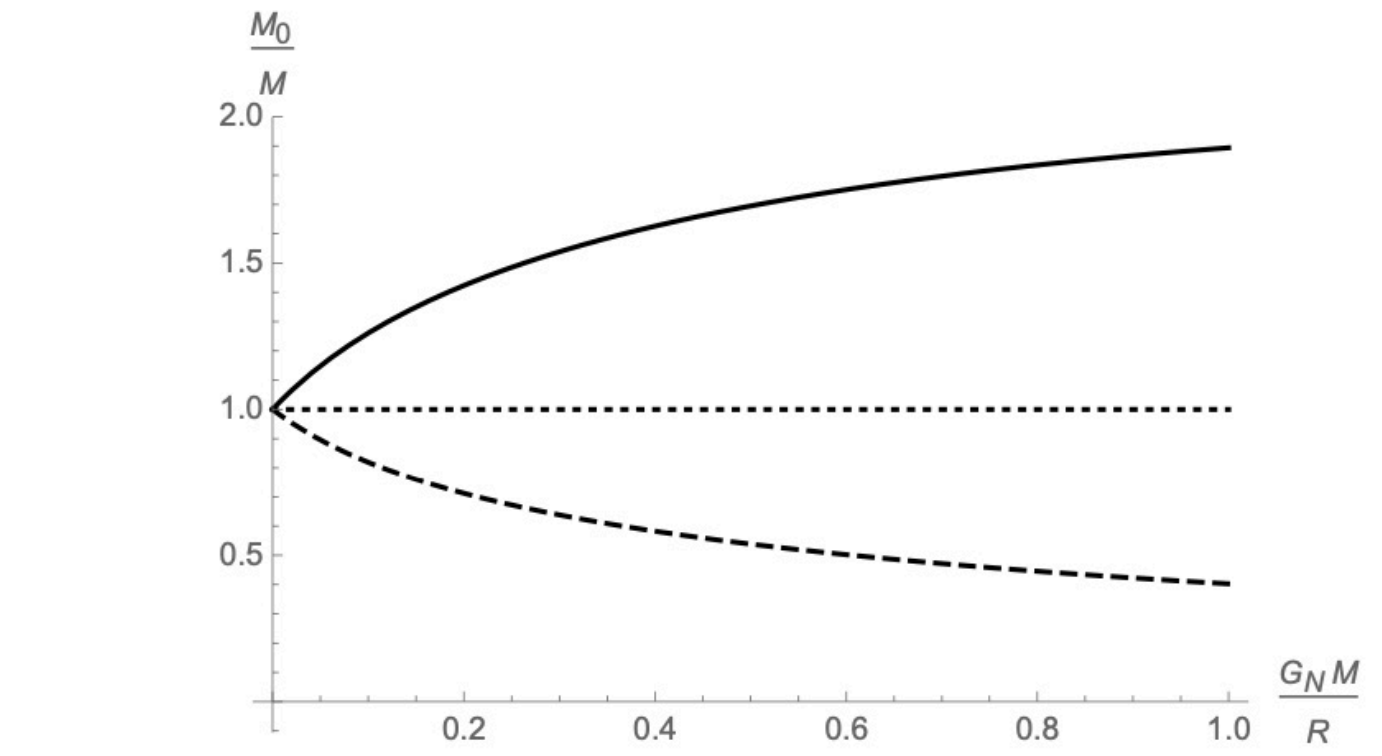
# On the masses of bootstrapped Newtonian stars [Mod.Phys.Lett.A 35 (2020) 21]

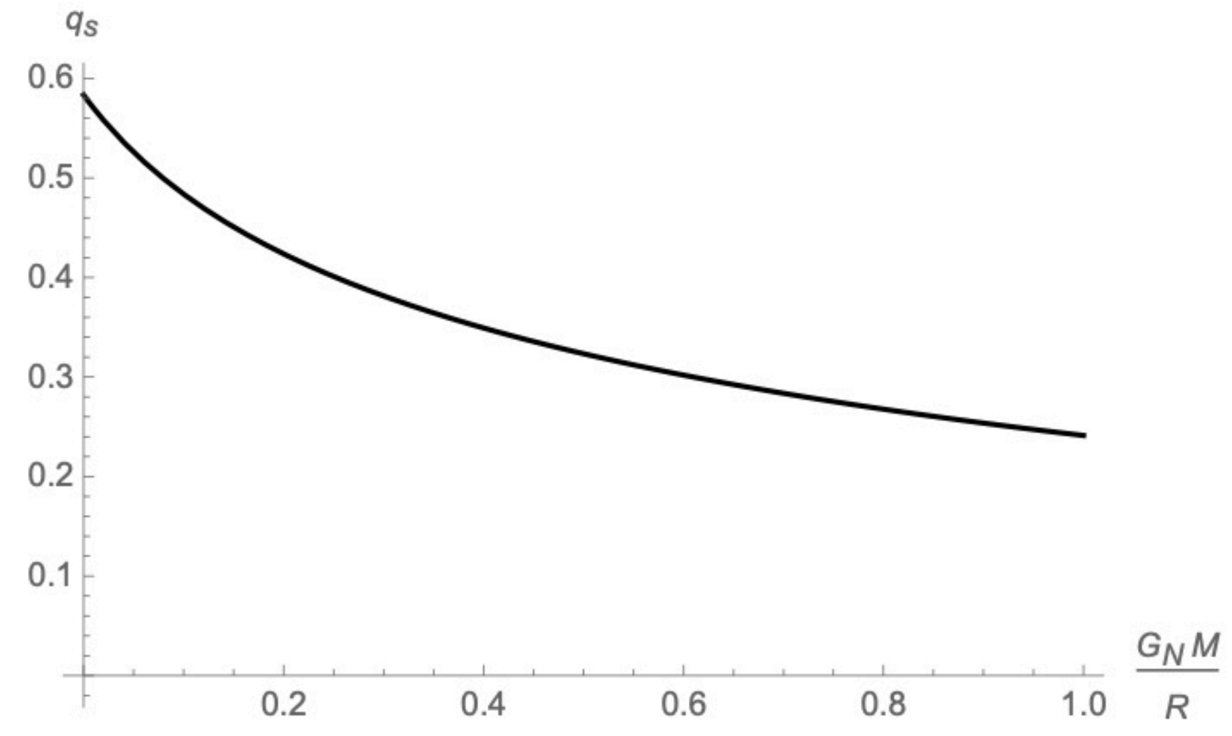
- In the *low compactness limit* the ratio goes to one.
- There is a critical value for which this ratio is equal to one:

$$q_s \simeq \frac{(1+8\,\mathcal{X})\,e^{-\frac{\mathcal{X}}{2\,(1+6\,\mathcal{X})^{1/3}}} - (1+6\,\mathcal{X})^{1/3}}{(1+6\,\mathcal{X})^{1/3}\left[1+8\,\mathcal{X}-(1+6\,\mathcal{X})^{1/3}\right]}$$

  - Below the critical value of the coupling the ratio is greater than one.
  - Above the critical value the ratio is smaller than one.
- A quite similar treatment with similar results was performed for the high compactness regime. (details can be found in the reference above)




Ratio $M_0/M$ for small and medium compactness for $q_\rho = 1$ (dashed line), and $q_\rho = 0$ (solid line). In these two cases $M_0$ is always different from $M$ (dotted line).




Critical value $q_s$ of $q_\rho$ for which $M = M_0$ for small and medium compactness.

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# Binary mergers: mass gap and black hole area law [Phys.Lett.B 834 (2022) 137455]

- This is interesting in the context of the LIGO discovery of gravitational waves.
- We start from the horizon radius:

$$V_{\rm out}(R_H) = -1/2 \quad \rightarrow \quad R_H = \frac{6\, q_V\, G_N\, M}{(1+2\, q_V)^{3/2} - 1}$$

& ADM - proper mass relation, which reads:

low compactness

$$M_0 = \frac{M}{(1+6\, q_V\, \mathcal{X})^{1/3}} \simeq (1 - 2\, q_V\, \mathcal{X})\, M$$

high compactness (black hole limit and beyond):

$$M_0 \simeq \frac{M}{q_V^{1/3}\, \mathcal{X}^{1/3}}$$

And following constraints:

- the *amount of ejected mass cannot have an arbitrarily small value*. This imposes a *lower bound on the ejected mass during the merger*, quantity which is a function of the masses and radii of the initial stars (or black holes).
- as they *increase in size black holes become less and less compact*. So, when black holes merge they likely transform into other heavier and less dense black holes.

# Binary mergers: mass gap and black hole area law [Phys.Lett.B 834 (2022) 137455]

- For instance in case of the coalescence of two stars we have

$$M_0^{(f)} = M_0^{(1)} + M_0^{(2)} - \delta M_0$$

$$M_{(f)} \simeq \left(1+6q_V \mathcal{X}_{(f)}\right)^{1/3} \left[ \frac{M_{(1)}}{\left(1+6q_V \mathcal{X}_{(1)}\right)^{1/3}} + \frac{M_{(2)}}{\left(1+6q_V \mathcal{X}_{(2)}\right)^{1/3}} - \delta M_0 \right]$$

and for the the merger of two stars (of low compactness) we also expect to have

$$\delta M \simeq M_{(1)} + M_{(2)} - M_{(f)} \geq \delta M_0$$

Separate cases:

★ Stars merging into stars

★ Stars merging into a black hole

★ Star merging with a black hole

★ Black holes merging into a black hole

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# Binary mergers: mass gap and black hole area law [Phys.Lett.B 834 (2022) 137455]

- *Stars merging into stars*:

$$\delta M_0 \gtrsim \left(1 - \frac{\mathcal{X}_{(1)}}{\mathcal{X}_{(f)}}\right) M_{(1)} + \left(1 - \frac{\mathcal{X}_{(2)}}{\mathcal{X}_{(f)}}\right) M_{(2)}$$

$$\mathcal{X}_{(f)} \lesssim \frac{\mathcal{X}_{(1)}\, M_{(1)} + \mathcal{X}_{(2)}\, M_{(2)}}{M_{(1)} + M_{(2)} - \delta M_0}$$

  - Constrains the increase of the compactness by the amount of proper mass/energy emitted

- *Stars merging into a black hole*:

$$\mathcal{X}_{(f)} \lesssim \frac{1}{q_V} + 6\, \frac{\mathcal{X}_{(1)}\, M_{(1)} + \mathcal{X}_{(2)}\, M_{(2)}}{M_{(1)} + M_{(2)} - \delta M_0}$$

  - RHS must be greater than one, since the first term is greater than one.

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# Binary mergers: mass gap and black hole area law [Phys.Lett.B 834 (2022) 137455]

- *Star merging with a black hole*:

$$\mathcal{X}_{(f)}^{1/3} \lesssim \frac{q_V^{-1/3}\left(M_{(1)} + M_{(2)} - \delta M_0\right)}{M_{(1)}/\left(q_V^{1/3}\,\mathcal{X}_{(1)}^{1/3}\right) + \left(1 - 2\,q_V\,\mathcal{X}_{(2)}\right) M_{(2)} - \delta M_0}$$

- *Merger of two black holes*:
  - When black holes merge, it is assumed that *no proper mass is emitted* !

$$\mathcal{X}_{(f)} \lesssim \left(\frac{M_{(1)} + M_{(2)}}{M_{(1)}\,\mathcal{X}_{(2)}^{1/3} + M_{(2)}\,\mathcal{X}_{(1)}^{1/3}}\right)^3 \mathcal{X}_{(1)}\,\mathcal{X}_{(2)}$$

  - If $\quad \mathcal{X}_{(1)} \simeq \mathcal{X}_{(2)} \equiv \mathcal{X}_{(i)} \quad \rightarrow \quad \delta M \simeq \left(M_{(1)} + M_{(2)}\right)\left(1 - \frac{\mathcal{X}_{(f)}^{1/3}}{\mathcal{X}_{(i)}^{1/3}}\right) \quad \rightarrow \quad \mathcal{X}_{(f)} \lesssim \mathcal{X}_{(i)}$

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# Binary mergers: mass gap and black hole area law [Phys.Lett.B 834 (2022) 137455]

- *Area law and black hole thermodynamics*

  - Suppose a black hole of mass $M$ absorbs a star of a much smaller mass $\delta M$, and no significant amount of proper mass is radiated away. Also, assume for simplicity that $\mathscr{X}_{(f)} \simeq \mathscr{X}_{(1)} \equiv \mathscr{X} \geq 1$ and $\mathscr{X}_2 \ll 1$. The *black hole area* $\mathscr{A} = 4\pi R_H^2$ changes as:

$$\frac{\Delta \mathcal{A}}{\mathcal{A}} \simeq 2\,\frac{M_{(f)} - M}{M} \simeq 2\, q_V^{1/3}\, \mathcal{X}^{1/3} \left(1 - 2\, q_V\, \mathcal{X}_{(2)}\right) \frac{\delta M}{M}$$

  - *Entropy*:

    - The temperature is: $T = \dfrac{\kappa}{2\,\pi}, \qquad \kappa = a(r)\big|_{r=R_H} = \dfrac{G_N\, M}{R_H^2}\left(1 + 6\, q_V\, \dfrac{G_N\, M}{R_H}\right)^{-1/3}$

    or $$T \simeq \frac{\beta(q_V)}{8\,\pi\, G_N\, M}$$

    which leads to the entropy: $dS = \dfrac{dM}{T} \qquad \rightarrow \qquad S = \dfrac{4\,\pi\, G_N\, M^2}{\beta(q_V)} = \beta(q_V)\,\dfrac{\mathcal{A}}{4\, G_N}$

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# Binary mergers: mass gap and black hole area law [Phys.Lett.B 834 (2022) 137455]

- The entropy can be used to impose more constraints on the result of a *two black holes collision*
  - no proper matter energy is emitted during the process
  - entropy is an additive quantity
  - entropy must increase in such a collision
  - for simplification purposes assume initial black holes have roughly the same compactness

$$\mathcal{X}_{(f)}^{2/3}\left(M_{(1)}+M_{(2)}\right)^2 \geq \mathcal{X}_{(i)}^{2/3}\left(M_{(1)}^2+M_{(2)}^2\right)$$

  - Along with the previous constraint obtained for this case we get

$$\left[\frac{M_{(1)}^2+M_{(2)}^2}{\left(M_{(1)}+M_{(2)}\right)^2}\right]^{3/2} \lesssim \frac{\mathcal{X}_{(f)}}{\mathcal{X}_{(i)}} \lesssim 1$$

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# Binary mergers: mass gap and black hole area law [Phys.Lett.B 834 (2022) 137455]

- GW150914 signal observed by LIGO

$$E_{\rm GW} = \delta M \simeq M_{(1)} \left(1 - \frac{\mathcal{X}_{(f)}^{1/3}}{\mathcal{X}_{(1)}^{1/3}}\right) + M_{(2)} \left(1 - \frac{\mathcal{X}_{(f)}^{1/3}}{\mathcal{X}_{(2)}^{1/3}}\right)$$

- The final black hole mass is computed as:

$$M_{(f)} \simeq \mathcal{X}_{(f)}^{1/3} \left[\frac{M_{(1)}}{\mathcal{X}_{(1)}^{1/3}} + \frac{M_{(2)}}{\mathcal{X}_{(2)}^{1/3}}\right] \quad \to \quad 62 \simeq 29 \left(\frac{\mathcal{X}_{(f)}}{\mathcal{X}_{(1)}}\right)^{1/3} + 36 \left(\frac{\mathcal{X}_{(f)}}{\mathcal{X}_{(2)}}\right)^{1/3}$$

- Since initial masses are similar, we assume similar compactness values and find

$$\frac{\mathcal{X}_{(f)}}{\mathcal{X}_{(i)}} \simeq 0.87$$

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# Dynamical stability of bootstrapped Newtonian stars

- Newton's second law for a thin shell (considering $q_V = q_p = q_\rho \equiv 1$):

$$(\rho\, dr)\, \ddot{r} = -\left[(\rho + p)\, V' + p'\right] dr \qquad \text{or} \qquad \ddot{r} = -\frac{\rho + p}{\rho}\, V' - \frac{1}{\rho}\, p'$$

  - When the acceleration is null:

$$p' = -(\rho + p)\, V'$$

- Homologous adiabatic perturbations: $p = p_0 \left(\frac{\rho}{\rho_0}\right)^\gamma$

$$dm_0 \to constant$$

$$r_0 \to r_0 \left(1 + \frac{\delta r}{r_0}\right)$$

$$\frac{\delta \rho}{\rho_0} = -3\, \frac{\delta r}{r_0} \qquad \frac{\delta p}{p_0} = \gamma\, \frac{\delta \rho}{\rho_0} \equiv -3\, \gamma\, \frac{\delta r}{r_0}$$

$$dm_0\, \ddot{r} = -\left(1 + p_0\, \frac{\rho^{\gamma - 1}}{\rho_0^\gamma}\right) V'\, dm_0 - 4\, \pi\, r^2\, dp$$

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# Dynamical stability of bootstrapped Newtonian stars

- Homogeneous stars:

(after performing some simple algebra)

$$\ddot{\delta r} = -\frac{\mathcal{X}\left[(3\,\gamma - 1)\,\rho_0 + 2\,p_0\right]}{R^2\,\left(1 + 6\,\mathcal{X}\right)^{1/3}\,\rho_0}\,\delta r$$

With solution of the type:

$$\delta r = C_+\,e^{i\,\omega\,t} + C_-\,e^{-i\,\omega\,t}$$

where

$$\omega = \sqrt{\frac{\mathcal{X}\left[(3\,\gamma - 1)\,\rho_0 + 2\,p_0\right]}{R^2\,\left(1 + 6\,X\right)^{1/3}\,\rho_0}}$$

-> positive values under the $\sqrt{\ }$ $\rightarrow$ oscillatory behaviour and the star is dynamically stable;
-> negative values under the $\sqrt{\ }$ $\rightarrow$ the star is unstable.

# Dynamical stability of bootstrapped Newtonian stars

- Polytropic stars:

$$\rho = \begin{cases} \rho_0 \, e^{-\frac{r^2}{b^2 R^2}} \, , & r \leq R \\ 0 \, , & r > R \, . \end{cases}$$

equations are much more involved, but can be brought to the simple form:

$$\ddot{\delta r} = -f(\mathcal{X}, r, R, \gamma, n, b) \, \delta r \equiv -f(r) \, \delta r$$

-> one can plot f(r) for various parameters sets.

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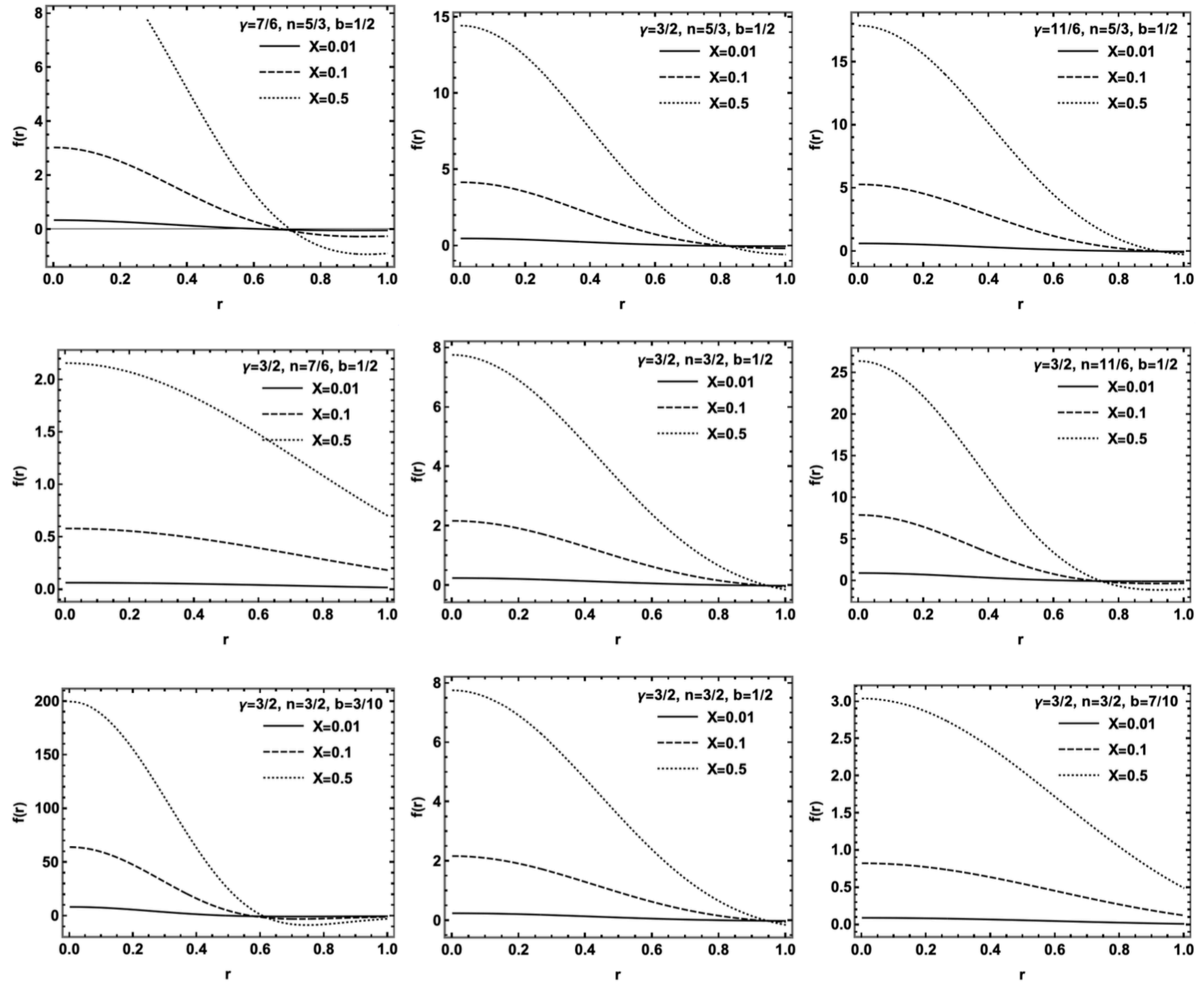
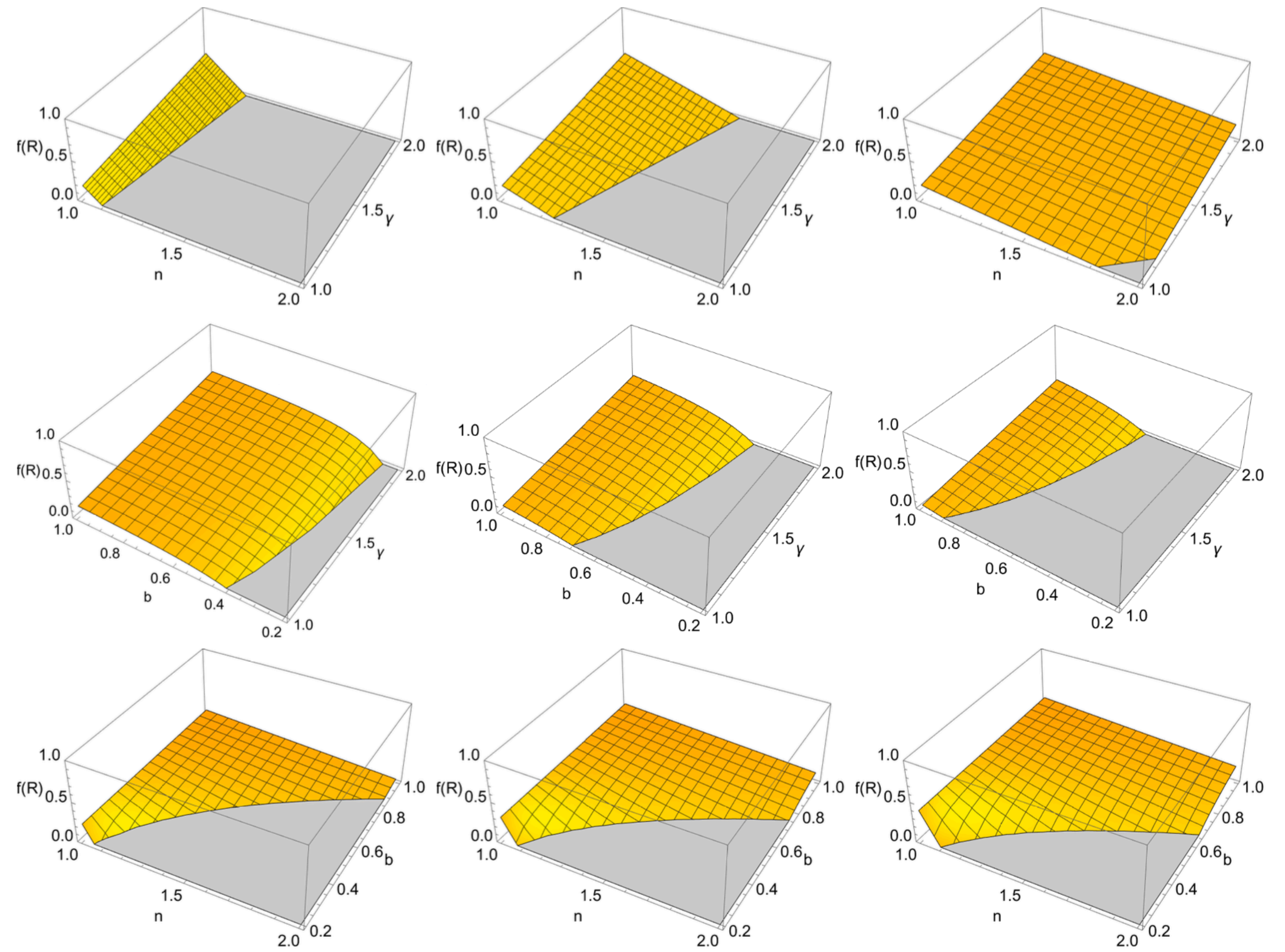
# Dynamical stability of bootstrapped Newtonian stars

- Polytropic stars:




Plots of $f(r)$ as a function of $r$ for $R = 1$. Top panels: the adiabatic index increases from left to right. Middle panels: the polytropic index $n$ increases from left to right. Bottom panels: the gaussian width $b$ varies, increasing from left to right. Note the different ranges on the vertical axis.




Top panels: 3D plots of f(R) for $b = 0.3$ (left), $b = 0.5$ (center) and $b = 0.9$ (right). Middle panels: 3D plots of f(R) for $n = 7/6$ (left), $n = 3/2$ (center) and $n = 11/6$ (right). Bottom panels: 3D plots of f(R) for $\gamma = 7/6$ (left), $\gamma = 3/2$ (center) and $\gamma = 11/6$.

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# Bootstrapped Newtonian Gravity Discussion

- One of the most features of the model is the absence of a Buchdahl limit, which means that the (matter) source can be held in equilibrium by a large enough (and finite) pressure for any (finite) compactness value;
- For compactness values of about $X \simeq 0.46$, a horizon appears within the source. The horizon radius becomes equal to the radius of the source when the compactness value reaches $X \simeq 0.69$;
- For polytropic stars, the matter density can be well approximated by a Gaussian distribution. For flatter distributions we recover the results obtained for uniform distributions (a consistency test);
- In the high compactness regime the bootstrapped picture generates stars that are more compact than the ones resulting from solving the TOV equation. This picture holds following comparisons to General Relativity.
- Bootstrapped Newtonian stars with uniform densities are dynamically stable to holonomous adiabatic perturbations.
- Overall, flatter density distributions seem to be favoured in this model.