Compact objects in Weyl geometric gravity

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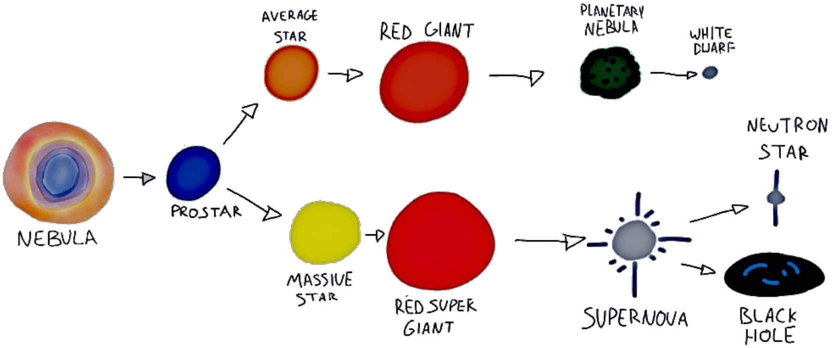
Table of content

A STAR LIFE CYCLE



When the GR effects become important?

- In the limit of a weak gravitational field

$$g_{00} = 1 + \frac{2\Phi}{c^2}$$

where

$$\frac{2\Phi}{c^2} = -\frac{G\,M}{c^2\,r} \approx \begin{cases} -10^{-6}, & \text{at the surface of the sun.} \\ -10^{-4}, & \text{at the surface of a white dwarf.} \\ -10^{-1}, & \text{at the surface of a neutron star.} \end{cases}$$

Some fascinating properties of neutron stars:

- The densest objects this side of an event horizon, with a mean density $\approx 10^{15} g/cm^3$.
- The largest surface gravity, about $10^{14} cm/s^2$, or 100 billion times Earth's gravity.
- The fastest spinning macroscopic objects. A pulsar, PSR J1748-2446ad in the globular cluster Terzan 5, has a spin rate of $714 Hz$, and its surface velocity at the equator is about $c/4$.

Some fascinating properties of NS:

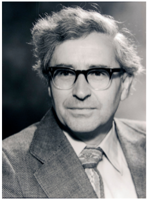
- The largest magnetic field strength, of order $10^{15} G$.
- The highest temperatures, outside the Big Bang, exist at birth or in merging neutron stars, about 700 billion K.
- Neutron stars at birth or in matter from merging neutron stars are the only places in the universe, apart from the Big Bang, where neutrinos become trapped and must diffuse through high density matter to eventually escape.

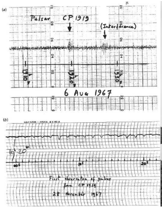
Some important discoveries about NS:

- 1932 Chadwick discovers the neutron.
- 1934 W. Baade and F. Zwicky suggest that neutron stars are the end product of supernovae.
- 1939 Oppenheimer and Volkoff find that general relativity predicts a maximum mass for neutron stars.
- 1964 Hoyle, Narlikar and Wheeler predict that neutron stars rotate rapidly.

Some important discoveries about NS:

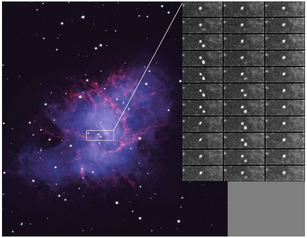
- 1967 Hewish and Bell at Cambridge University discovered a source of rapid, sharp, intense, and extremely regular pulses of radio radiation. The pulses arrived precisely every 1.33728 seconds. (Hewish receives 1974 Nobel Prize).



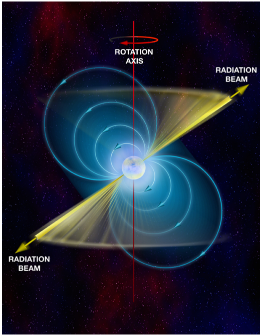


Some important discoveries about NS:

- 1968 Crab pulsar discovered and pulse period found to be increasing, characteristic of spinning stars but not binaries or vibrating stars.



https://noirlab.edu/public/images/noao-03036/



https://earthsky.org

Some important discoveries about NS:

- 1971 Accretion powered X-ray pulsars discovered by the Uhuru satellite.
- 1974 The first binary pulsar, PSR 1913+16, discovered by Hulse and Taylor (Nobel Prize 1993). Its orbital decay is the first observation proving the existence of gravitational radiation.
- 1982 The first millisecond pulsar, PSR B1937+21, discovered by Backer et al..

Some important discoveries about NS:

- 1996 Discovery of the closest neutron star RX J1856-3754 by Walter et al..
- 1998 Kouveliotou discovers the first magnetar.

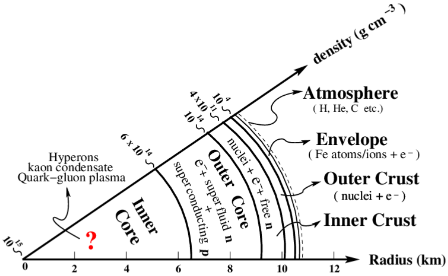
https://www.nasa.gov

NS in general relativity

- Tolman, Oppenheimer and Volkof obtained the equation of hydrostatic equilibrium of static, spherically symmetric compact objects using GR equations of motion.
- The Chandrasekhar limiting mass of white dwarfs is $M_{Ch} \approx 1.4 M_{\odot}$.
- Numerical investigation of TOV equation led to the limiting maximum mass of neutron stars.
- Rhoades and Ruffini (1974) derived a theoretical limit of $3.2 M_{\odot}$ for the maximum mass of neutron stars. This result was obtained by using the principle of causality, the maximally stiff equation of state $p = \rho c^2$, and Le Chatelier's principle, and it is valid even if the equation of state of matter is unknown in a limited range of densities.

NS in general relativity

- The value of the maximum mass depends on the equation of state of the neutron star.
- We do not know the equation of state of the neutron star because of its high density.



arxiv:1212.5410

NS in general relativity

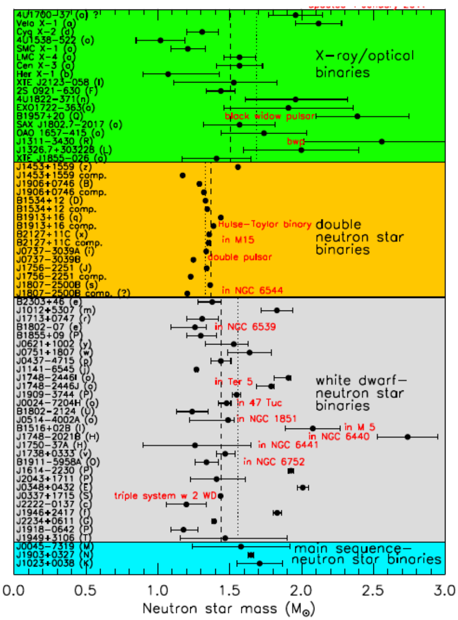
- For a long time, theoretical arguments and observational evidence showed that neutron stars must have a mass distribution centered on a value of the order $1.4M_\odot$.

- The corresponding radius of a $1.4M_\odot$ mass neutron star should be of the order of $10 - 15km$, and its average density is of the order of $6 \times 10^{14} g/cm^3$.

- Using combined electromagnetic and gravitational wave information on the binary neutron star merger GW170817, an upper limit of $M_{max} \leq 2.17M_\odot$ for the maximum mass of a neutron star was found.

NS in general relativity

- From the gravitational event GW190425, that indicated the total mass of the binary neutron star to be of the order of $3.4M_{\odot}$.
- From GW190408, that showed the possible existence of a neutron star with mass $2.5 - 2.65M_{\odot}$, merging with a large black hole with mass $26M_{\odot}$



AIP Conference Proceedings 2127, 020001 (2019)

How can interpret these new data?

- What is the equation of state of the neutron star?
- Is there any mysterious matter in that high density?
- Is there any way to consider these masses with the use of baryonic matters?
- Maybe general relativity can not handle such compact sources and strong gravitational fields.

- In Einstein theory of gravity, the gravitational effects are an intrinsic property of space-time structure.
- Historically, the first proposal to generalize Riemann geometry, was due to Weyl. Weyl was motivated in this generalization by the intention of solving one of the most important problems of theoretical physics, namely, the unification of the gravitational and electromagnetic forces. H. Weyl, Sitzungsber. Preuss. Akad. Wiss. 465, 1 (1918).
- Another generalization of the Riemann geometry was introduced by Cartan, based on the concept of torsion. É. Cartan, C. R. Acad. Sci. (Paris) 174, 593 (1922)

Weyl Geometry

- In Weyl geometry, there are two fundamental principles. The first is the possibility of the variation of the length of a vector during its parallel transport. Secondly, Weyl postulated that the laws of nature must be conformally invariant.
- The physical interpretation of Weyl's geometry was strongly disapproved by Einstein.
- But Weyl's geometry has many beautiful characteristics, and it opens the way for the full implementation of the conformal invariance of physical laws.
- One of the interesting attempts for the reconsideration of Weyl's theory from a physical point of view was due to Dirac.
- Weyl's geometry is at the origin of the gauge theory, which has become the fundamental theoretical tool in particle physics.

Weyl geometric gravity theory

- Weyl geometry is constructed from $\{g_{\mu\nu}(x), \omega_\alpha(x)\}$ that satisfy the nonmetricity condition:

$$\tilde{\nabla}_\lambda g_{\mu\nu} = -n\alpha\omega_\lambda g_{\mu\nu},$$

$$\tilde{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + \alpha\frac{n}{2}\left(\delta^\lambda_\mu\omega_\nu + \delta^\lambda_\nu\omega_\mu - \omega^\lambda g_{\mu\nu}\right),$$

  where n is the Weyl charge and α is the Weyl gauge coupling.

- The Weyl gauge transformations

$$\tilde{g}_{\mu\nu} = \Omega^n(x) g_{\mu\nu}, \quad \tilde{\omega}_\mu = \omega_\mu - \frac{n}{\alpha}\frac{\partial_\mu\Omega(x)}{\Omega(x)},$$

- The field strength $\tilde{F}_{\mu\nu}$ of the Weyl vector field

$$\tilde{F}_{\mu\nu} = \tilde{\nabla}_\mu \omega_\nu - \tilde{\nabla}_\nu \omega_\mu = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu .$$

- The curvatures in Weyl geometry

$$\tilde{R}^\lambda_{\mu\nu\sigma} = \partial_\nu \tilde{\Gamma}^\lambda_{\mu\sigma} - \partial_\sigma \tilde{\Gamma}^\lambda_{\mu\nu} + \tilde{\Gamma}^\lambda_{\rho\nu} \tilde{\Gamma}^\rho_{\mu\sigma} - \tilde{\Gamma}^\lambda_{\rho\sigma} \tilde{\Gamma}^\rho_{\mu\nu},$$

$$\tilde{R}_{\mu\nu} = \tilde{R}^\lambda{}_{\mu\lambda\nu}, \quad \tilde{R}_{\mu\nu} - \tilde{R}_{\nu\mu} = 2\tilde{F}_{\mu\nu},$$

$$\tilde{R} = g^{\mu\sigma} \tilde{R}_{\mu\sigma} .$$

- The Weyl scalar can be expressed in terms of Riemannian geometric quantities as

$$\tilde{R} = R - 3n\alpha \nabla_\mu \omega^\mu - \frac{3}{2} (n\alpha)^2 \omega_\mu \omega^\mu .$$

The action

$$S = \int \left[\frac{1}{4!\xi^2} \tilde{R}^2 - \frac{1}{4} \tilde{F}^2_{\mu\nu} \right] \sqrt{-g} d^4 x + S_m,$$

$$S_m = \beta \int \mathcal{L}_m \sqrt{-g} d^4 x,$$

where β and ξ are constant parameters, and the effective matter Lagrangian $\mathcal{L}_m$,

$$\mathcal{L}_m = \mathcal{L}_m(L_m, \omega^2, \psi)$$

in general can depend on the ordinary matter Lagrangian L_m, on the Weyl vector through $\omega^2 = \omega^\mu \omega_\mu$, and on the matter fields ψ and their couplings.

arXiv:2312.13384 (2023).

- The linearization in the Ricci scalar via the introduction of an auxiliary scalar field ϕ,

$$\tilde{R}^2 \equiv 2\phi^2\tilde{R} - \phi^4.$$

- Substituting $\tilde{R}^2 \to 2\phi^2\tilde{R} - \phi^4$ into the geometric part of the action

$$S = \int \frac{1}{4!\xi^2}\tilde{R}^2\sqrt{-g}d^4x = \int \frac{1}{4!\xi^2}\left(2\phi^2\tilde{R} - \phi^4\right)\sqrt{-g}d^4x.$$

- The variation of the action with respect to ϕ leads to the equation

$$\phi\left(\tilde{R} - \phi^2\right) = 0, \to \phi^2 = \tilde{R}.$$

- The geometrical part of the action

$$S_{geom} = \int \left[\frac{1}{4!\xi^2} \tilde{R}^2 - \frac{1}{4} \tilde{F}^2_{\mu\nu} \right] \sqrt{-g} d^4x +,$$

is invariant under the gauge transformations,

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \hat{\omega}_\mu = \omega_\mu - \frac{1}{\alpha} \partial_\mu \ln(\Omega^2), \quad \hat{\phi} = \frac{\phi}{\Omega}.$$

- The matter part of the action, should not be necessarily gauge invariant, but its variation must be.

- The variation of the matter action (assuming $\delta L_m/\delta\psi = 0$)

$$\delta S_m = -\frac{1}{2}\int T^{(tot)}_{\mu\nu}\delta g^{\mu\nu}\sqrt{-g}d^4x + \int G^{\mu}\delta\omega_{\mu}\sqrt{-g}d^4x$$

$$T^{(tot)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\left(L_m,\omega^2\right)\right)}{\delta g^{\mu\nu}},$$

$$G^{\mu} = \frac{\delta\mathcal{L}_m\left(L_m,\omega^2\right)}{\delta\omega_{\mu}},$$

- Using the gauge transformations

$$\delta g^{\mu\nu} = 2\frac{\delta\Omega}{\Omega}g^{\mu\nu}, \qquad \delta\omega_{\mu} = \frac{2}{\alpha}\nabla_{\mu}\left(\frac{\delta\Omega}{\Omega}\right),$$

- For the variation of the matter action we obtain the condition,

$$\delta S_m = -\int \left[T^{(tot)}_{\mu\nu} g^{\mu\nu} + \frac{2}{\alpha} \nabla_\mu G^\mu \right] \frac{\delta \Omega}{\Omega} \sqrt{-g} d^4 x = 0.$$

- The consistency (trace) condition of the energy momentum tensor

$$T^{(tot)} = -\frac{4}{\alpha} \nabla_\mu \left(\omega^\mu \frac{\partial \mathcal{L}_m \left(L_m, \omega^2 \right)}{\partial \omega^2} \right),$$

- When $\mathcal{L}_m = L_m$, this constraint leads to the familiar form $T^{(m)} = 0$, which means that the only conformally invariant matter has a traceless energy-momentum tensor.

- Final form of the action

$$S = \beta \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R - 6\alpha^2\omega_\mu\omega^\mu - 12\xi^2\bar{\phi}^2\right)\bar{\phi}^2 - \frac{1}{4}F_{\mu\nu}^2 + \mathcal{L}_m\right],$$

where $\bar{\phi} = \phi/\xi$ and $\kappa^2 = 6\beta$.

- To have a canonical kinetic term for the Weyl vector, we have used the redefinitions $\omega_\mu \to \sqrt{\beta}\omega_\mu$ and $\alpha \to \alpha/\sqrt{\beta}$.
- The gauge condition is

$$\nabla_\mu\omega^\mu = 0.$$

- With the use of the gauge condition, the trace condition becomes

$$T^{(tot)} = -\frac{4}{\alpha}\omega^\mu\nabla_\mu\left[\frac{\partial\mathcal{L}_m\left(L_m, \omega^2\right)}{\partial\omega^2}\right].$$

Gravitational field equations

- The form of effective matter Lagrangian

$$\mathcal{L}_m = L_m + \gamma\omega^2,$$

where γ is a constant. The supplementary term added to the matter action is necessary to assure the conformal invariance of the theory, and to satisfy the trace condition.

- The field equation of ω_μ,

$$\nabla_\nu F^{\mu\nu} + \frac{3\alpha^2}{2\kappa^2}\,\bar{\phi}\,\omega^\mu - 2\gamma\,\omega^\mu = 0.$$

- Due to the antisymmetric property of the Weyl field strength $F^{\mu\nu}$ we have

$$\nabla_\sigma F_{\mu\nu} + \nabla_\mu F_{\nu\sigma} + \nabla_\nu F_{\sigma\mu} = 0.$$

- The metric field equation

$$\bar{\phi}^2 G_{\mu\nu} - \kappa^2 \left(T_{\mu\nu} + F_\mu{}^\alpha F_{\nu\alpha} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g_{\mu\nu} \right) + 6\xi^2 \bar{\phi}^4 g_{\mu\nu}$$
$$+ \left(\gamma\kappa^2 - \frac{3}{4}\alpha^2\bar{\phi}^2 \right) \left(2\omega_\mu\omega_\nu - g_{\mu\nu}\omega^2 \right) + g_{\mu\nu}\Box\bar{\phi}^2 - \nabla_\nu\nabla_\mu\bar{\phi}^2 = 0.$$

- The field equation of the scalar field becomes

$$\bar{\phi}^2 = \frac{1}{48\xi^2} \left(2R - 3\alpha^2\omega^2 \right).$$

- The constraint equation takes the form,

$$T^{(m)} = -2\gamma\omega^2.$$

Cosmological evolution of the model

- Assume an isotropic and homogeneous Universe

  arXiv:2405.04129 [gr-qc]

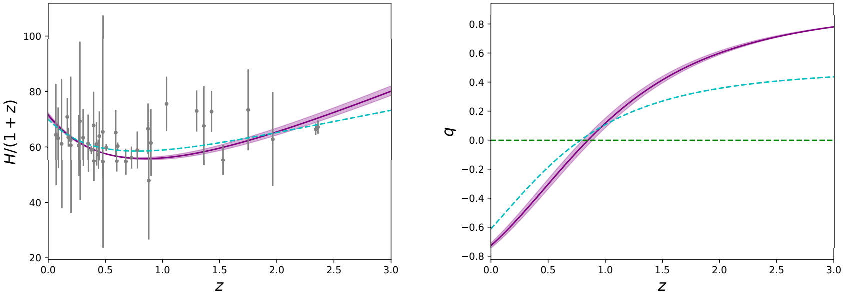
$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right).$$

- The perfect fluid energy-momentum tensor

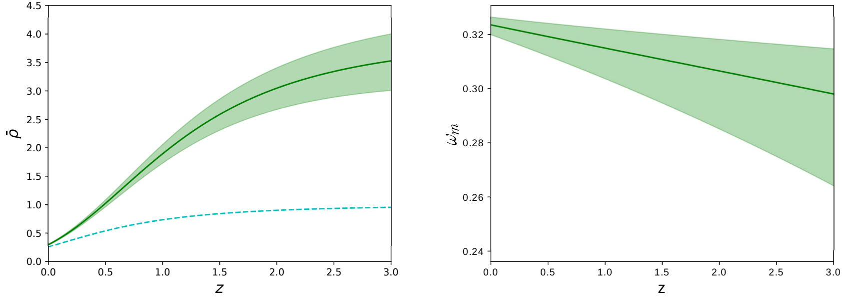
$$T^{\mu}_{\ \nu} = \mathrm{diag}(-\rho, p, p, p), \quad p = \omega_m \rho$$

- From the matter conservation equation, one can obtain

$$\omega_m = -1 - \frac{\dot{\rho}}{3H\rho}.$$



The shaded area denotes the 1σ error.
The dashed line represents the ΛCDM model.



The shaded area denotes the 1σ error.
The dashed line represents the ΛCDM model.

Static spherically symmetric field equations

- The interior line element of static and spherically symmetric compact object

$$ds^2 = -e^{-2f(r)}dt^2 + \frac{1}{g(r)}dr^2 + r^2 d\Omega^2,$$
$$g(r) = 1 - \frac{2\,m(r)}{r},$$

$m(r)$ has the physical interpretation as the total (effective) mass of the star.

- The vector field ω_μ, $\omega^\mu = e^{f(r)}h(r)\delta^\mu_t$.
- The gauge condition, $\nabla_\mu\omega^\mu = 0$, is automatically satisfied.

arXiv:2303.10339 [gr-qc].

- The Lagrangian and energy-momentum tensor of the ordinary matter

$$T^{m}_{\mu\nu} = (p+\rho)\, u_\mu u_\nu + p\, g_{\mu\nu}.$$

- The non-zero component of the Weyl vector field equation

$$h'' + h'\left(\frac{2}{r} + \frac{g'}{2g} - f'\right) - h\left(f'' + \frac{2f'}{r} - \frac{2\gamma}{g} + \frac{f'g'}{2g} + \frac{3\alpha^2}{2\kappa^2 g}\bar{\phi}^2\right) = 0.$$

- The (00) component of the metric field equation is given by,

$$\frac{1}{r^2}\left(1 - rg' - g\right)\bar{\phi}^2 = 3\bar{\phi}^2(2\xi^2\bar{\phi}^2 + \frac{1}{4}\alpha^2 h^2) + 2g\left[\bar{\phi}\left(\frac{2\bar{\phi}'}{r} + \bar{\phi}''\right) + \bar{\phi}'^2\right]$$
$$+ \kappa^2\left[\frac{1}{2}h^2\left(gf'^2 - 2\gamma\right) - \frac{1}{2}gh'(2h\,f' - h')\right] + g'\bar{\phi}\bar{\phi}' + \kappa^2\rho.$$

- The spatial components of the metric field equation

$$\frac{1}{r^2}\left(1-g+2rgf'\right)\bar{\phi}^2 = -\frac{2}{r}g\left(rf'-2\right)\bar{\phi}\bar{\phi}' - \kappa^2 p$$
$$+3(2\xi^2\bar{\phi}^2-\frac{1}{4}\alpha^2h^2)\bar{\phi}^2+\kappa^2\left[\frac{1}{2}g\left(h'-hf'\right)^2+\gamma h^2\right],$$

and

$$g\bar{\phi}^2f''+\frac{1}{2r}\left(rf'-1\right)\left(g'-2gf'\right)\bar{\phi}^2 = 3\bar{\phi}^2(2\xi^2\bar{\phi}^2-\frac{1}{4}\alpha^2h^2)$$
$$+2g\left[\bar{\phi}\bar{\phi}'\left(\frac{1}{r}-f'\right)+\bar{\phi}'^2+\bar{\phi}\bar{\phi}''\right]+g'\bar{\phi}\bar{\phi}'-\kappa^2p$$
$$+\kappa^2\left[\gamma h^2-\frac{1}{2}g\left(h'-hf'\right)^2\right].$$

- The trace constraint equation

$$3p-\rho=2\gamma h^2.$$

- The scalar field equation

$$24\xi^2\bar{\phi}^2 + g'\left(\frac{2}{r} - f'\right) - \frac{2}{r^2} + \frac{2g}{r^2}\left(rf' - 1\right)^2 2gf'' - \frac{3}{2}\alpha^2 h^2 = 0.$$

- The conservation (balance) equation of the energy momentum tensor is given by

$$\begin{aligned}
&\kappa^2 p' + \frac{3\alpha^2}{2}h\bar{\phi}^2\left(h' - hf'\right) + \bar{\phi}\bar{\phi}'\left[g'\left(f' - \frac{2}{r}\right) - 24\xi^2\bar{\phi}^2\right] \\
&+ \bar{\phi}\bar{\phi}'\left[g\left(2f'' - \frac{2\left(rf' - 1\right)^2}{r^2}\right) + \frac{3\alpha^2 h^2}{2} + \frac{2}{r^2}\right] \\
&- \frac{1}{2}\kappa^2\left[2f'\left(-2\gamma h^2 + p + \rho\right) + g'\left(h' - hf'\right)^2 + 4\gamma hh'\right] \\
&- \frac{\kappa^2}{r}g\left(hf' - h'\right)\left[hrf'' + f'\left(rh' + 2h\right) - rh'' - 2h'\right] = 0.
\end{aligned}$$

- The dimensionless parameters and variables for the geometrical and physical quantities are defined as

$$\bar{p} = \frac{p}{\rho_c}, \quad \bar{\rho} = \frac{\rho}{\rho_c}, \quad \bar{m} = \sqrt{\rho_c} m, \quad \eta = \sqrt{\rho_c}\, r,$$

$$\bar{\gamma} = \frac{\gamma}{\rho_c}, \quad \bar{\alpha} = \frac{\alpha}{\sqrt{\rho_c}}, \quad \bar{\xi} = \frac{\xi}{\sqrt{\rho_c}}.$$

The parameter ρ_c is the central density.

Quark stars

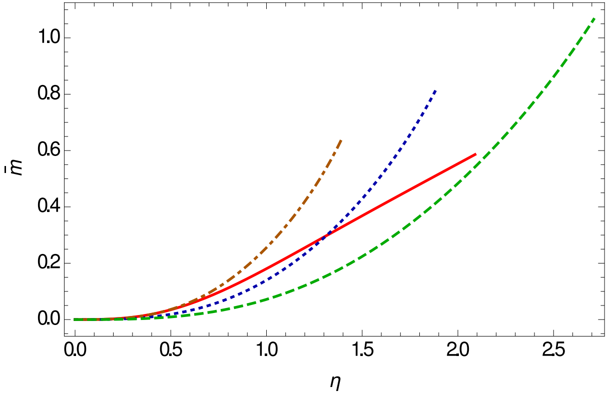
- The MIT bag equation of state

$$3p = \rho - 4B.$$

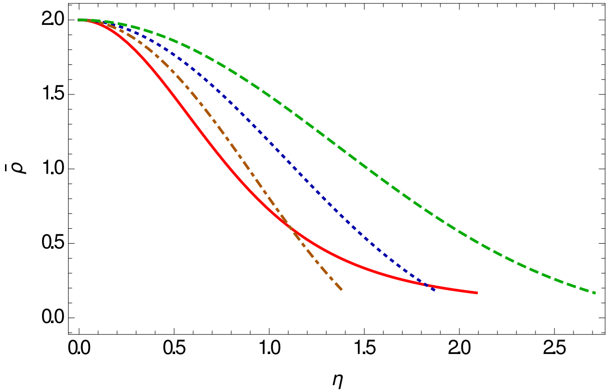
- B is the bag constant and we set $B = 1.03 \times 10^{15}\ \mathrm{g/cm^3}$.
- Using the consistency condition

$$3p - \rho = 2\gamma h^2, \quad \Rightarrow \quad h^2 = -\frac{2B}{\gamma}.$$

- The coupling constant γ should be negative to have positive values of h^2.



The central density is $\rho_c = 4.9 \times 10^{15}\ \mathrm{g/cm}^3$.
The dashed curve is for $\bar{\alpha} = 0.1$, $\bar{\xi} = 0.15$ and $\bar{\gamma} = -0.06$.
The dotted curve is for $\bar{\alpha} = 0.15$, $\bar{\xi} = 0.39$ and $\bar{\gamma} = -0.2$.
The dot-dashed curve is for $\bar{\alpha} = 0.2$, $\bar{\xi} = 0.98$ and $\bar{\gamma} = -0.4$.
The solid curve represents the standard general relativistic mass and density profile for MIT quark stars.



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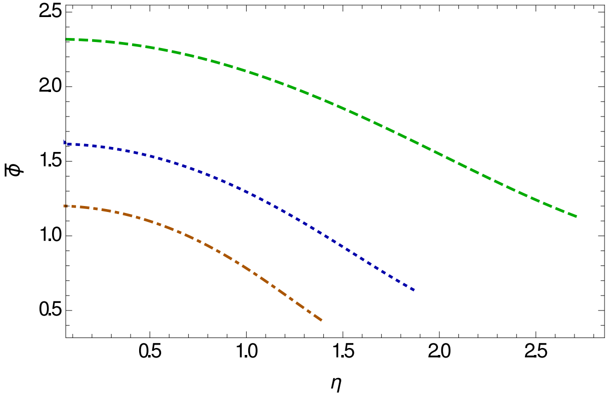

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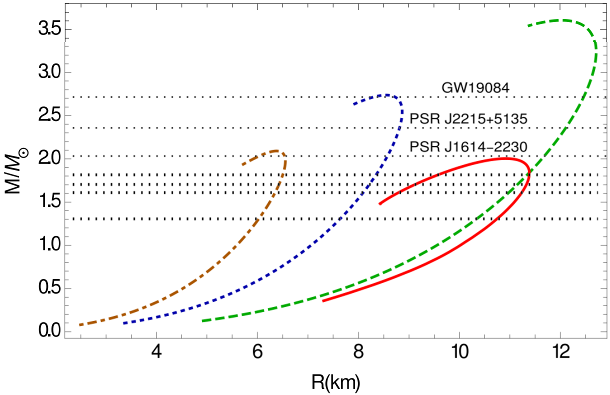


The range of central density is $2.2 \times 10^{15}\ \mathrm{g/cm^3} \leq \rho_c \leq 6.74 \times 10^{15}\ \mathrm{g/cm^3}$.
The dashed curve is for $\bar{\alpha} = 0.1$, $\bar{\xi} = 0.15$ and $\bar{\gamma} = -0.06$.
The dotted curve is for $\bar{\alpha} = 0.15$, $\bar{\xi} = 0.39$ and $\bar{\gamma} = -0.2$.
The dot-dashed curve is for $\bar{\alpha} = 0.2$, $\bar{\xi} = 0.98$ and $\bar{\gamma} = -0.4$.
The solid curve represents the standard general relativistic mass and density profile for MIT quark stars.

The maximum masses, and the corresponding radii and central densities for the MIT bag model quark stars in Weyl geometric gravity.

$\bar{\alpha}$	0.15	0.10	0.20
$\bar{\xi}$	0.39	0.15	0.98
$\bar{\gamma}$	-0.20	-0.06	-0.40
$M_{max}/M_{\odot}$	2.73	3.61	2.09
$R\,(\mathrm{km})$	8.55	12.03	6.38
$\rho_c \times 10^{-15}\,(\mathrm{g/cm^3})$	10.2	12.6	7.69

In standard general relativity, the $M_{max} = 2M_{\odot}$, with a radius of $10.92\,\mathrm{km}$, corresponding to a central density $\rho_c = 1.98 \times 10^{15}\ \mathrm{g/cm^3}$.

Bose-Einstein Condensate stars

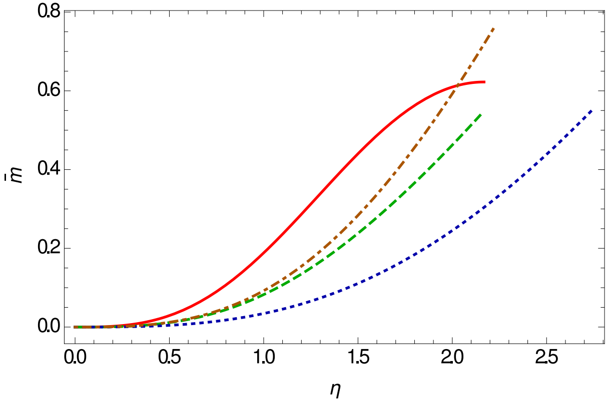
- The equation of state of a Bose-Einstein Condensate is given by

$$p = k\rho^2.$$

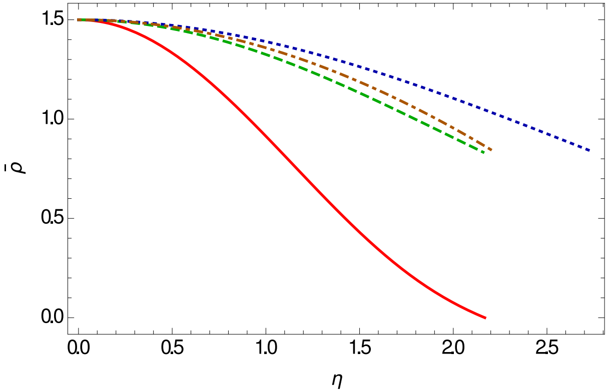
- The Weyl consistency condition can be written as

$$h^2 = \frac{1}{2\gamma}\left(3k\rho - 1\right)\rho \geq 0.$$

- For positive values of γ, we set the stop point in the numerical integration at $\rho = 1/3k$.
- We consider $\bar{k} = \rho_c k = 0.4$, and thus the stop point in the numerical integration is $2.04 \times 10^{15}\ g/cm^3$.



The the central density is $\rho_c = 3.67 \times 10^{15}\ \mathrm{g/cm^3}$.
The dashed curve is for $\bar{\alpha} = 0.074$, $\bar{\xi} = 0.19$ and $\bar{\gamma} = 0.03$.
The dotted curve is for $\bar{\alpha} = 0.07$, $\bar{\xi} = 0.05$ and $\bar{\gamma} = 0.06$.
The dot-dash curve is for $\bar{\alpha} = 0.12$, $\bar{\xi} = 0.34$ and $\bar{\gamma} = 0.10$.
The solid curve represents the standard general relativistic mass and density profile for Bose-Einstein condensate stars.



The the central density is $\rho_C = 3.67 \times 10^{15}$ g/cm^3.
The dashed curve is for $\bar{\alpha} = 0.074$, $\bar{\xi} = 0.19$ and $\bar{\gamma} = 0.03$.
The dotted curve is for $\bar{\alpha} = 0.07$, $\bar{\xi} = 0.05$ and $\bar{\gamma} = 0.06$.
The dot-dash curve is for $\bar{\alpha} = 0.12$, $\bar{\xi} = 0.34$ and $\bar{\gamma} = 0.10$.
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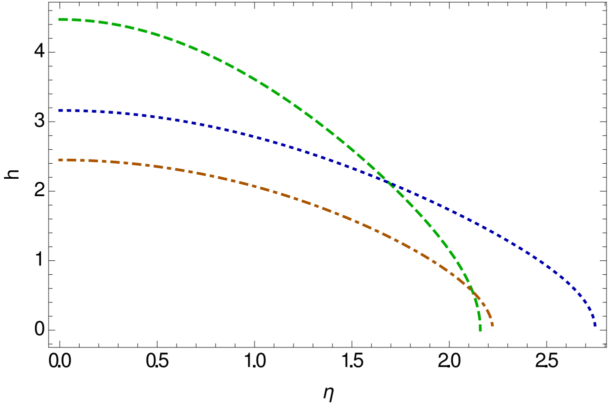
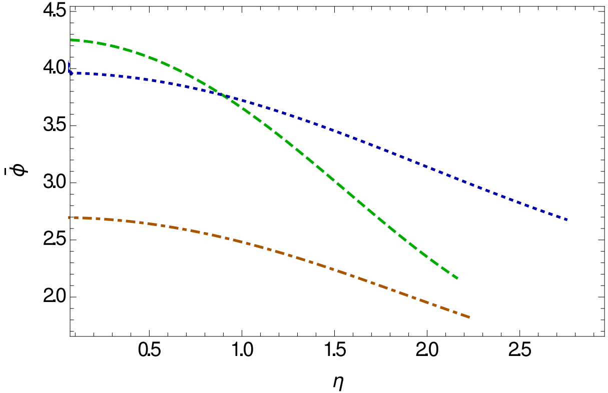


The the central density is $\rho_C = 3.67 \times 10^{15}\ \mathrm{g/cm^3}$.
The dashed curve is for $\bar{\alpha} = 0.074$, $\bar{\xi} = 0.19$ and $\bar{\gamma} = 0.03$.
The dotted curve is for $\bar{\alpha} = 0.07$, $\bar{\xi} = 0.05$ and $\bar{\gamma} = 0.06$.
The dot-dash curve is for $\bar{\alpha} = 0.12$, $\bar{\xi} = 0.34$ and $\bar{\gamma} = 0.10$.



The the central density is $\rho_c = 3.67 \times 10^{15}$ g/cm^3.
The dashed curve is for $\bar{\alpha} = 0.074$, $\bar{\xi} = 0.19$ and $\bar{\gamma} = 0.03$.
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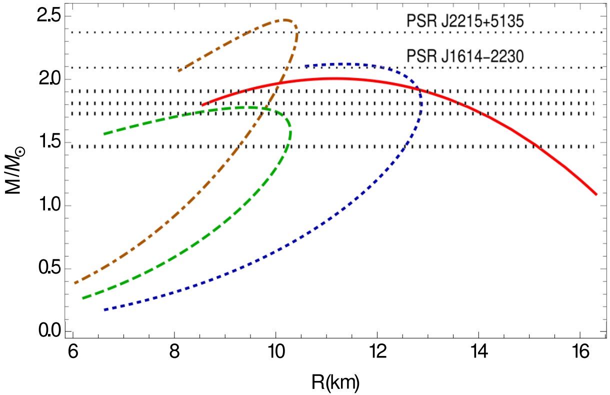

The range of central density is $2.2 \times 10^{15}\ \mathrm{g/cm^3} \leq \rho_c \leq 6.74 \times 10^{15}\ \mathrm{g/cm^3}$.
The dashed curve is for $\bar{\alpha} = 0.074$, $\bar{\xi} = 0.19$ and $\bar{\gamma} = 0.03$.
The dotted curve is for $\bar{\alpha} = 0.07$, $\bar{\xi} = 0.05$ and $\bar{\gamma} = 0.06$.
The dot-dash curve is for $\bar{\alpha} = 0.12$, $\bar{\xi} = 0.34$ and $\bar{\gamma} = 0.10$.
The solid curve represents the standard general relativistic mass and density profile for Bose-Einstein condensate stars.

The maximum masses and the corresponding radii and central densities for the Bose-Einstein Condensate stars in Weyl geometric gravity.

$\bar{\alpha}$	0.07	0.074	0.12
$\bar{\xi}$	0.05	0.19	0.34
$\bar{\gamma}$	0.06	0.03	0.10
$M_{max}/M_{\odot}$	2.12	1.70	2.47
$R\,(\mathrm{km})$	11.36	9.44	10.18
$\rho_c \times 10^{-15}\,(\mathrm{g/cm^3})$	5.12	4.35	4.17

The maximum mass of the standard general relativistic BEC stars is $M = 2M_{\odot}$, with radius $R = 11.17\,\mathrm{km}$, with a central density $\rho_c = 2.58 \times 10^{15}\,\mathrm{g/cm^3}$.

Summary

- We have considered the Weyl geometric gravity theory.
- The action is linearized in the Weyl scalar by introducing an auxiliary scalar field.
- To keep the theory conformally invariant the trace condition is imposed on the matter energy-momentum tensor.
- The field equations are obtained by imposing the variational principle.
- This model can explain the cosmological evolution of the Universe.
- This model can explain the high mass observed compact objects.

Thanks for your attention