Relativity forever?

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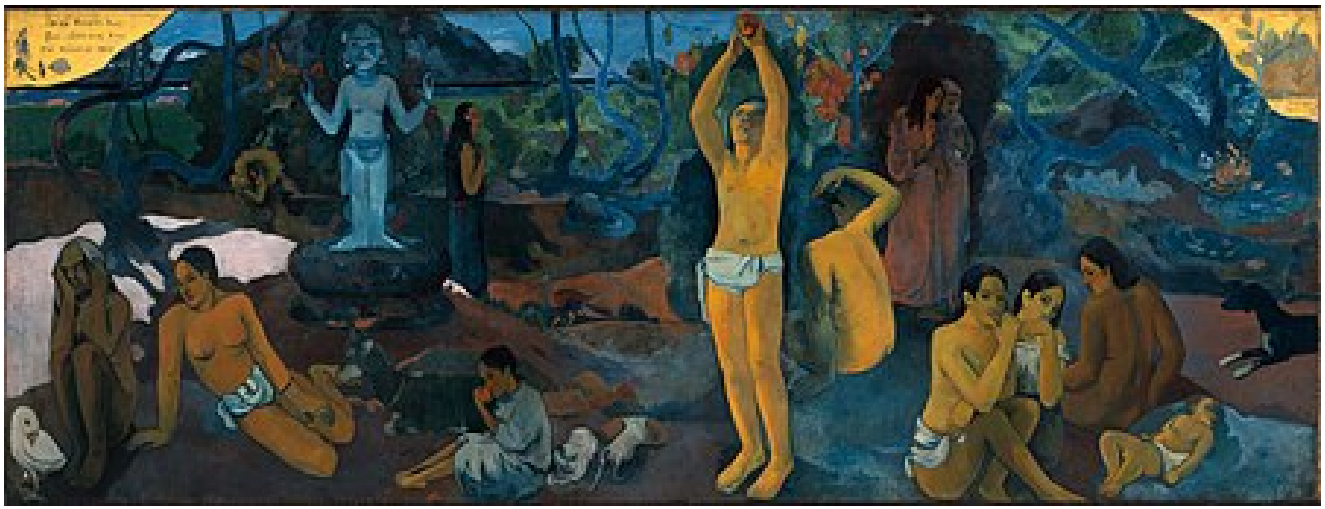
Details can be found in the papers

- O. I. Chashchina, Z. K. Silagadze, Breaking the light speed barrier. Acta Physica Polonica B 43 (2012), 1917-1952.
- A. E. Sagaydak, Z. K. Silagadze, On Finslerian extension of special relativity, Mod. Phys. Lett. A 37N17 (2022), 2250106.
- O. I. Chashchina, Z. K. Silagadze, Relativity 4-ever?, MDPI Physics 4(2) 2022, 421-439.
- A. Alizzi, A. Sen, Z. K. Silagadze, Do moving clocks slow down? Eur. J. Phys. 43 (2022), 065601.



"In the center, Gauguin meditates on what we are. Here are two women, talking about destiny (or so he described them), a man looking puzzled and half-aggressive, and in the middle, a youth plucking the fruit of experience....it is humanity's innocent and natural desire to live and to search for more life. A child eats the fruit, overlooked by the remote presence of an idol - emblem of our need for the spiritual..." Text from "Sister Wendy's American Masterpieces"

How not to become an idolater

"Let us keep our minds open by all means, as long as that means keeping our sense of perspective and seeking an understanding of the forces which mould the world. But don't keep your minds so open that your brains fall out! There are still things in this world which are true and things which are false; acts which are right and acts which are wrong, even if there are statesmen who hide their designs under the cloak of high-sounding phrases".

From the speech of Walter Kotschnig delivered at Smith College on November 8, 1939.

Plan of the talk

- Where are we now
  - Experimental basis of SR
  - Status of superluminarity
- Where did SR come from
  - Disadvantages of the traditional approach
  - Spacetime picture from the very beginning
- Where to go
  - Finsler spacetime
  - Area metric
- Instead of a conclusion

Disadvantages of the traditional approach to SR

- Einstein's postulates are formulated in Newtonian terms, which are relativized and change their meaning in the course of the presentation. Such presentation lends a paradoxical flavor to our most fundamental and well-established theory.
- Typical response of a student: "To take as a postulate that the speed of light is constant relative to changes in reference frame is to assume an apparent absurdity. It goes against common sense. No wonder, thinks a student, that we can derive other absurdities, such as time dilation and length contraction, from the premises" R. Baierlein, Two myths about special relativity, Am. J. Phys. **74** (2006) 193.
- For this reason, the standard approach to teaching special relativity is similar to what Synge calls the "cuckoo process", and this process is not particularly effective.
- "Often the result is to destroy completely the confidence of the student in perfectly sound and useful concepts already acquired" (J. S. Bell, 2004).

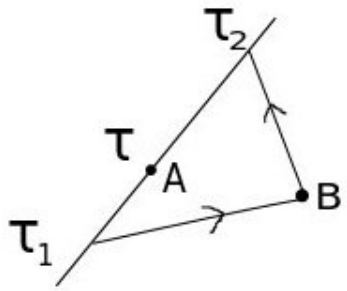
Spacetime picture from the very beginning

- The root of the conceptual problems students have with the relativistic concept of time is that they confuse coordinate time with what they consider to be “true time".
- The only “true time" in relativity is the proper time. “One can often define a global time, but matter-energy doesn’t listen to it. Restricted to the clocks to which the physics listens, the ideal master clock of classical physics has shattered into a huge plurality of miniature ideal clocks in relativity, one for every time-like path" (C. Callender, 2017).
- The great implication of general relativity is that coordinates play a secondary, albeit important (in practical calculations), role compared to the internal geometrical structure of spacetime.
- Minkowski’s four-dimensional formalism makes it possible to practice the same philosophy in the special theory of relativity.

Time in special relativity: proper, private and public

- An approach to special relativity that emphasizes from the outset the role of absolute rather than relative concepts was initiated by Alfred Robb (1911).
- The light-cone structure defines a partial conical order in spacetime introducing absolute concepts *before* and *after*.
- To make this order quantitative, the concept of an ideal clock is introduced, which measures proper time along its time-like world line.
- To extend the concept of time beyond observer's world line, we need the concept of *simultaneity*, which must be defined by a convention.
- Following Milne (1948), we distinguish between the private time of a particular observer and the public time used and shared by a set of privileged observers.
- In inertial coordinate systems, privileged observers have parallel world lines and public time essentially coincides with private time.
- The Milne model, in which the world lines of privileged inertial observers are not parallel, but intersect in one event (the Big Bang event) illustrates the difference between private and public times.

Robb-Geroch's definition of relativistic interval



- Event A is simultaneous with B if it corresponds to the proper time $\tau = \frac{1}{2}(\tau_1 + \tau_2)$.
- The distance between simultaneous events A and B is $l = \frac{1}{2}(\tau_2 - \tau_1)$, and in this case $l^2 = (\tau_2 - \tau)(\tau - \tau_1)$.
- Relativistic interval is just a generalization of the concept of "distance" (Robb-Geroch): $s^2 = (\tau_2 - \tau)(\tau - \tau_1)$.
- Only proper times are used in Robb-Geroch's definition of relativistic interval.
- Requiring observer-independence of this definition, it can be shown that the standard Minkowski geometry does follow (details in A. Alizzi, A. Sen, Z. K. Silagadze, Do moving clocks slow down? Eur. J. Phys. 43 (2022), 065601.)

Finslerian generalization

- Robb-Geroch's definition of a relativistic interval admits a simple generalization leading to a Finsler geometry.
- the distance l between simultaneous events will not change, if we define it as follows

$$l^2 = \left(\frac{\tau - \tau_1}{\tau_2 - \tau}\right)^b (\tau_2 - \tau)(\tau - \tau_1),$$

- It can be elevated to the definition of the relativistic interval

$$s^2 = \left(\frac{\tau - \tau_1}{\tau_2 - \tau}\right)^b (\tau_2 - \tau)(\tau - \tau_1) = (\tau_2 - \tau)^{1-b}(\tau - \tau_1)^{1+b},$$

- Leads to the Finsler geometry with Bogoslovsky metric

$$ds^2 = (n_\lambda dx^\lambda)^{2b}(\eta_{\mu\nu} dx^\mu dx^\nu)^{1-b}.$$

- Details in A. E. Sagaydak, Z. K. Silagadze, On Finslerian extension of special relativity, Mod. Phys. Lett. A 37N17 (2022), 2250106.

Area metric

- It is believed that concept of length is no longer meaningful below the Planck scale, and classical ideas about gravity and space-time cease to be valid.
- Metric measures tangent vectors, an area metric is a fourth rank tensor field, which measures two-dimensional tangent areas.
- Metric-induced area metric $G_{abcd} = g_{ac}g_{bd} - g_{ad}g_{bc}$.
- All area metrics in three dimensions are metric-induced because in 3D areas are dual to their normal vectors. Not true in higher dimensions.
- Is it possible that we do not have concept of metric but the concept of area metric is still valid?
- Drummond and Hathrell one-loop effective action for photon propagation in a Ricci flat Lorentzian space-time background

$$S = \int d^4x\sqrt{-g}\left(g^{a[c}g^{d]b} + \frac{\alpha}{90\pi m^2}C^{abcde}\right)F_{ab}F_{cd}.$$

P. M. Ho, T. Inami, Geometry of area without length, Prog. Theor. Exper. Phys. **2016**, Issue 1, 013B03.

Experimental basis of SR

- Experimental confirmations of special relativity are ubiquitous.
- Special relativity is the most precisely tested theory.

http://math.ucr.edu/home/baez/physics/Relativity/SR/experiments.html

$$\delta\mathcal{L} \sim \vec{B}^2 \qquad \left|1 - \frac{c_\gamma}{c}\right| < 3 \times 10^{-22}$$

$\frac{|c_L - c_R|}{c} < 10^{-32}$ (M. Goldhaber, V. Trimble, Limits on the Chirality of Interstellar and Intergalactic Space, J. Astrophys. Astr. 17 (1996) , 17-21).

$$\delta\mathcal{L} \sim \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} A_\lambda s_\sigma = \vec{A} \cdot \vec{B} s_0 + (\vec{A} \times \vec{E}) \cdot \vec{s}$$

$\frac{|\vec{s}|}{m} \approx 4 \times 10^{-38}$ (S. Coleman, S. L. Glashow, Cosmic Ray and Neutrino Tests of Special Relativity, Phys. Lett. B405(1997), 249-252).

Golden shine of the theory of relativity

- Every time you enjoy glitter of gold, you meet special relativity. P. Pyykkö, Relativistic effects in structural chemistry, Chem. Rev. 88 (1988), 563-594.
- The melting point of Hg is $-39^{\circ}C$, while Cd (the non-relativistic analog of mercury) is $+321^{\circ}C$. J. Autschbach, Perspective: Relativistic effects, J. Chem. Phys. 136 (2012), 150902.
- 10 out of 12 volts in a lead-acid car battery come from relativistic effects. R. Ahuja, A. Blomqvist, P. Pyykkö, P. Zaleski-Ejgierd, Relativity and the Lead-Acid Battery, Phys. Rev. Lett. 106 (2011), 018301.
- We also meet relativity, both special and general, every time we use GPS system for navigation, because without these theories GPS would not function properly. N. Ashby, Relativity in the Global Positioning System, Living Review Relativity 6 (2003), 1.

Why Are We Interested in relativity?

"The principal reason for concentrating our attention specifically on the theory of relativity is, of course, not any sympathy with ignorant attacks on this theory, but the circumstance that its basic postulates touch upon the deepest foundations of the physics – geometry of space-time".

D.I. Blokhintsev

There are also psychological reasons: special relativity assumes a light speed barrier, while a human being cannot tolerate any limitations due to the rebellious nature of human mind.

For the same reasons, we refused to believe that the light speed barrier was really impossible to overcome in the material world. And we set our hearts on inventing a way, which had never been imagined before, to overcome it.

Experimental status of light-speed barrier

The results were overwhelmingly convincing: it is impossible to overcome light speed barrier on a horseback.

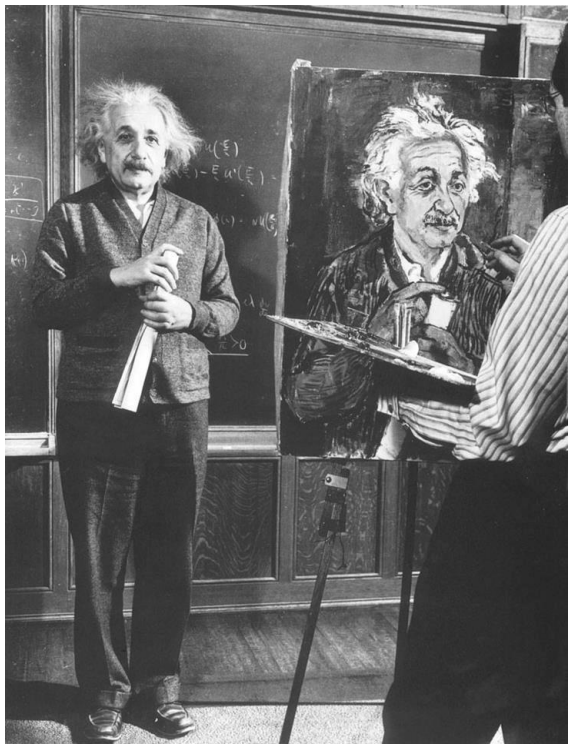
The horse was somewhat embarrassed by this result

Why the Light Speed Barrier?

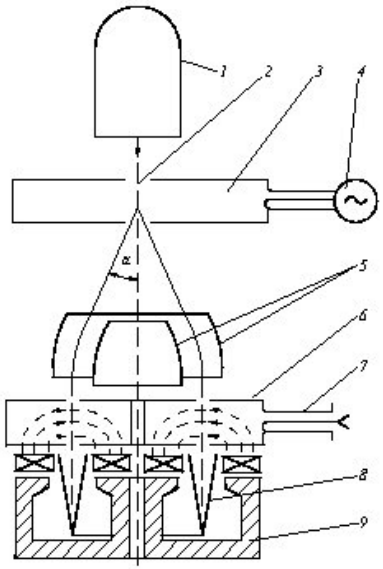
$$W = mc^2 \left\{ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right\}.$$

"Thus, when $v = c$, W becomes infinite. Velocities greater than that of light have, as in our previous results, no possibility of existence"

A. Einstein, Annalen Phys. 17, 891-921 (1905).



For Light Spots, There Is No Light Speed Barrier



- The initial idea of a Gyrocon — M.S. Neyman (1940), P.V. Hartley (1945), J. W. McRae (1946).
- Harmonic used for the generation had phase velocity 1.84c.
  Output power of the Gyrocon in the continuous mode reached 5 MW.
  The wavelength range from 30 cm to 1.1 m.
  The efficiency of the beam energy conversion into radiation of about 80%. (G.I. Budker group, Novosibirsk, 1979).

Does Somebody Live to the North of the Himalayas?

"As analogy we may consider demographers studying the population patterns of the Indian Subcontinent. They would find that there are a lot of people in India but as you approached the Himalayas there are fewer and fewer people. Climbing the giddying heights is a difficult task and it is well-nigh impossible to climb it let alone live there. Suppose a demographer calmly asserts that there are no people north of the Himalayas since none could climb over the mountain ranges. That would be an absurd conclusion.People of Central Asia are born there and live there: they did not have to be born in India and cross the mountain ranges" (E.C.G. Sudarshan).

Tachyons

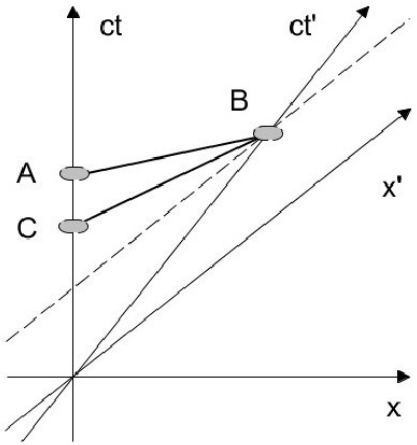
O. M. P. Bilaniuk, V. K. Deshpande and E. C. G. Sudarshan, Am. J. Phys. 30 (1962), 718-723.

G.B. Malykin, V.S. Savchuk, E.A. Romanets, Lev Yakovlevich Strum and the hypothesis of the existence of tachyons. Phys.-Uspekhi 55(2012), 1134-1139.

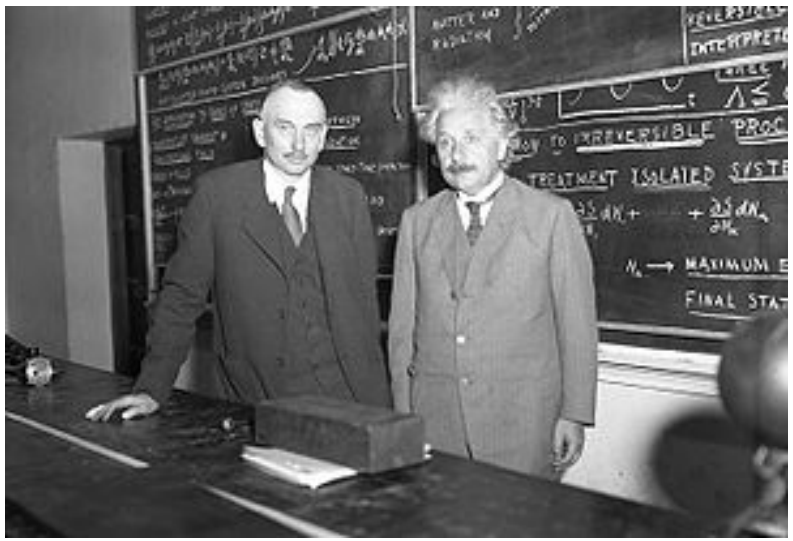
Lev Yakovlevich Strum

- Was a head of Theoretical Physics Department at Kiev University from 1932 to 1936 (replaced by Nathan Rosen).
- His love for philosophy played a tragic role in his story, as it put him in touch with the philosopher Semyon Semkovsky, Lev Trotsky's cousin.
- Shortly after Semkovsky was arrested as Trotsky's relative, Shtrum was arrested too.
- He was tortured, forced to sign self-incriminations, and executed with a group of other Ukrainian scientists.
- The Stalinist government tried to annihilate him spiritually as well: his works were removed from Soviet libraries and destroyed.
- As a result, the tachyon hypothesis fell into oblivion for many years.

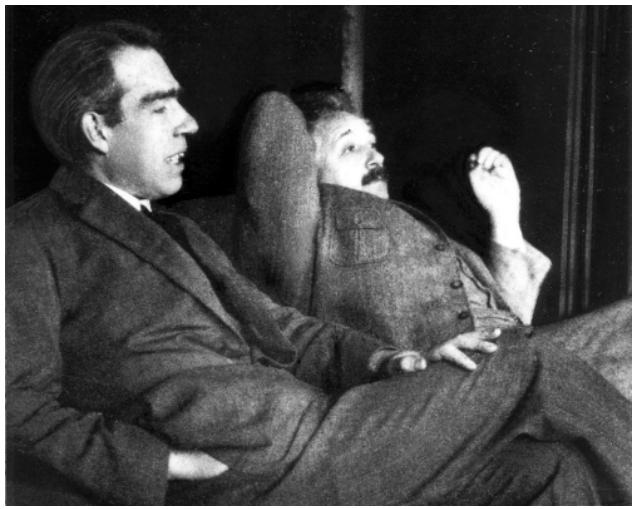
Tolman's Antitelephone Paradox



Einstein 1907, Einstein and Sommerfeld 1910, Tolman 1917



How does quantum mechanics fit in with special relativity?



"Imagine a particle moving on average at very nearly the speed of light, but with an uncertainty in position, as required by quantum theory. Evidently, there will be some probability for observing this particle to move a little faster than average, and therefore faster than light, which special relativity won't permit. The only known way to resolve this tension involves ..."

F. A. Wilczek, Asymptotic Freedom: From Paradox to Paradigm (Nobel lecture), http://arxiv.org/abs/hep-ph/0502113

Wightman propagator

$$D(x-y) =< 0|\phi(x)\phi(y)|0 >= \int \frac{d\vec{p}}{(2\pi)^3} \frac{e^{-ip\cdot(x-y)}}{2\sqrt{\vec{p}^2+m^2}}$$

If $x-y=(0,\vec{r})$ is purely in the spatial direction,

$$D(x-y) = \frac{m}{4\pi^2 r} K_1(mr)$$

and

$$K_1(x) \approx \sqrt{\frac{\pi}{2x}}\, e^{-x}, \quad \text{if} \quad x \gg 1.$$

Causality and relativistic quantum theory

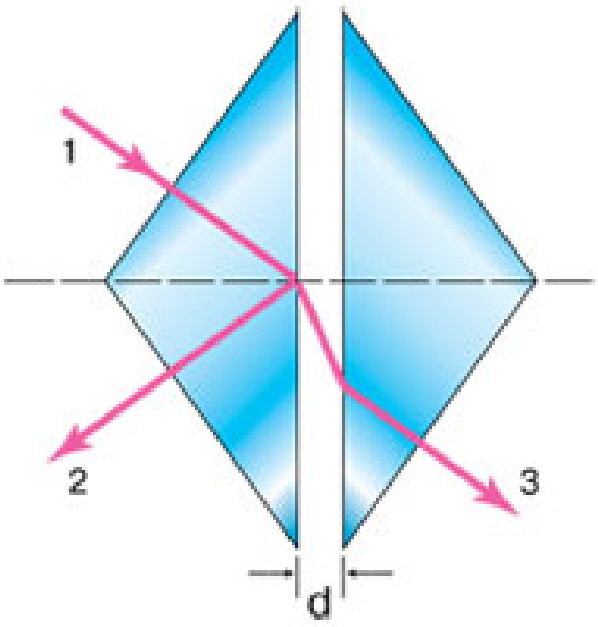
"Relativistic causality – formulate it as you like! – is a subtle matter in relativistic quantum theories. (J. Butterfield, 2007.

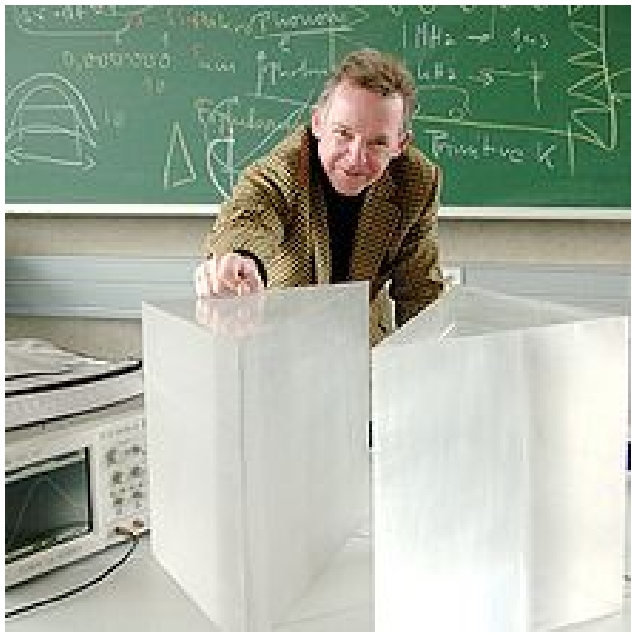
- Hegerfeldt's theorem – initially localized quantum system immediately develops an infinite tail.
- Entanglement and mutual correlations can be generated at space-like separated points, in spirit of old Einstein, Podolsky and Rosen paradox.
- Hartman effect – the transmission time across a potential barrier becomes independent of barrier thickness for very thick barriers.
- Scharnhorst effect – light signals travel a bit faster than c in the Casimir vacuum between two closely spaced conducting plates.
- Drummond-Hathrell effect – photons may propagate with superluminal velocity in certain gravitational backgrounds.

Neither of these strange effects allows information to be transmitted faster than c, and thus, they are compatible with special relativity, although in a very subtle way.

Frustrated Total Internal Reflection

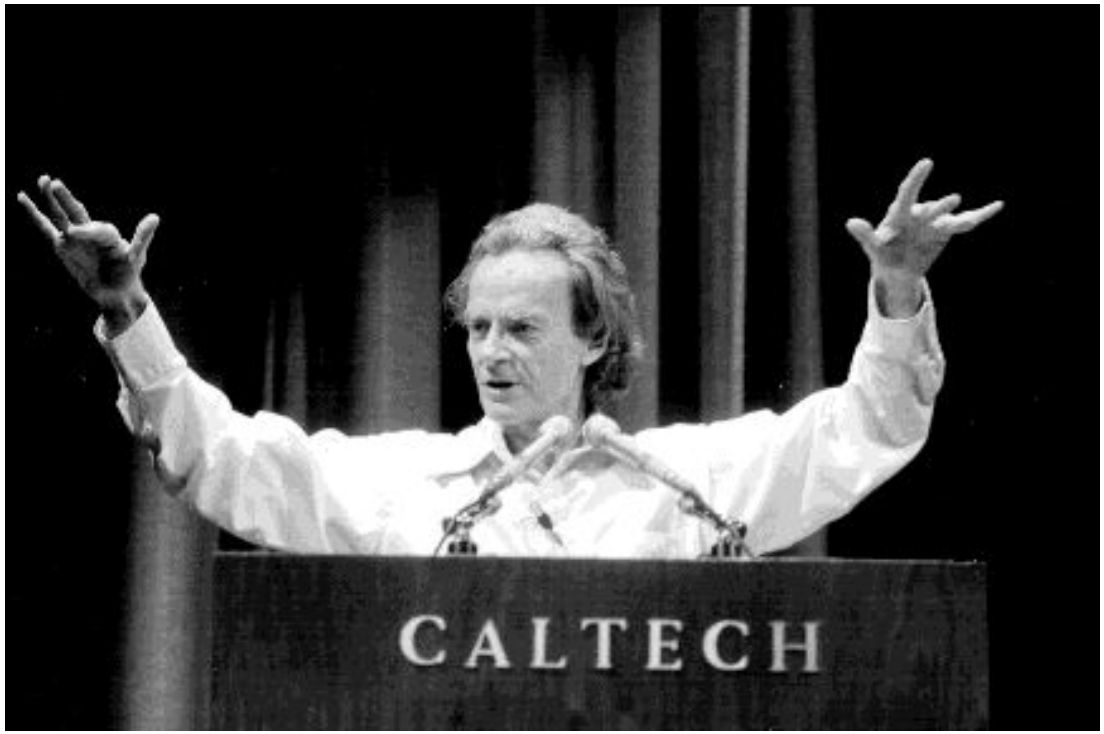
The Hartman effect was experimentally confirmed in frustrated total internal reflection (Günter Nimtz).





M.V. Davidovich, On the Hartman paradox, electromagnetic wave tunneling and supraluminal velocities (comment on the paper "Tunneling of electromagnetic waves: paradoxes and prospects" by A.B. Shvartsburg), Physics Uspekhi 52 (2009), 415-418.

There Are No Paradoxes in Physics

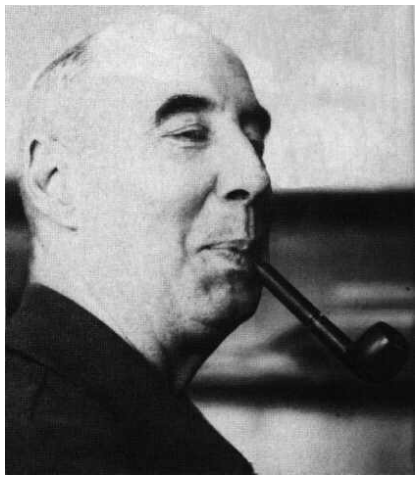


"Of course, in physics there are never any real paradoxes because there is only one correct answer; at least we believe that nature will act in only one way (and that is the right way, naturally). So in physics a paradox is only a confusion in our own understanding" (R. Feynman).

The Idea of Antiparticles

- The above-mentioned phenomena illustrate that there is some tension between quantum theory and special relativity in what concerns superluminality.
- "The only known way to resolve this tension involves introducing the idea of antiparticles" (Frank Wilczek).
- Quantum field theory offers a miraculously clever solution to the superluminal propagation dilemma:
  - It is true that, for a particle propagating superluminally, there is always a reference frame in which the particle propagates backwards in time in apparent violation of causality.
  - However, the particle's energy in this frame is negative.
  - Negative energy particle propagating backward in time can be interpreted as a positive energy antiparticle propagating forward in time.
- The existence of antiparticles is crucial for a cancellation of all acausal terms in commutators of two local observables at spacelike separation.

Stueckelberg – An Unconventional Hero



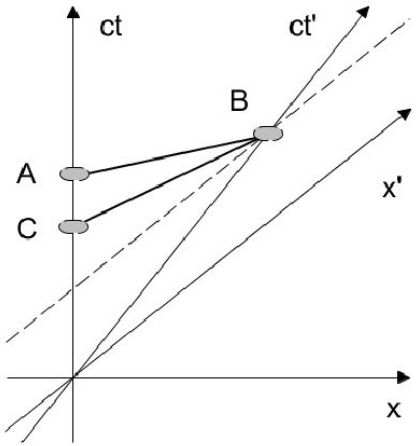
Johann Melchior Ernst Karl Gerlach Stueckelberg, Freiherr von Breidenbach zu Breidenstein und Melsbach

B. Feldman, The Nobel Prize: A History of Genius, Controversy, and Prestige (Arcade Publishing, 2001).

P. Freund, A passion for discovery (World Scientific, 2007). E.C.G.

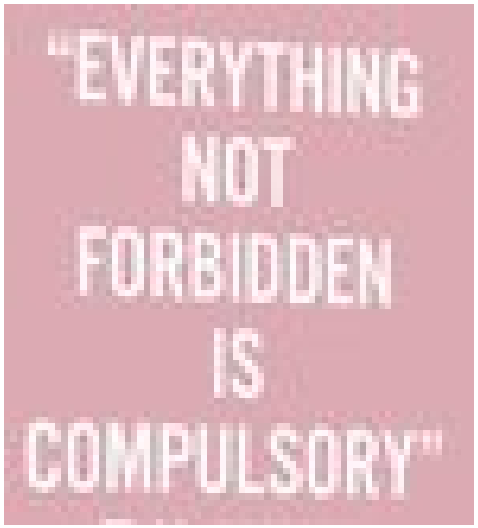
Stueckelberg, An Unconventional Figure of Twentieth Century Physics (Birkhäuser Verlag AG, 2009).

Reinterpretation Principle



- While moving from B to C, the tachyon has a negative energy.
- Therefore, it can be reinterpreted as an antitachyon with a positive energy moving forward in time.
- Not everyone agrees with such a solution to the Tolman's antitelephone paradox.
- "Causality objections against superluminal particles are by far the most subtle, and much room for reflection remains in this regard" O.-M. Bilaniuk and E.C.G Sudarshan.

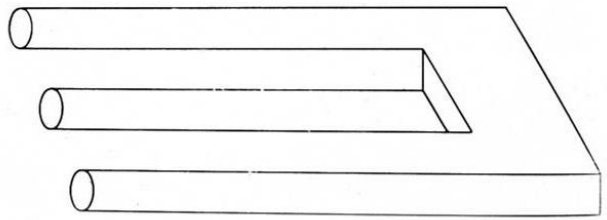
Gell-Mann's Totalitarian Principle



- Ascribed to Murray Gell-Mann.
- In fact, the totalitarian principle first appeared in T.H. White's Arthurian fantasy novel *The Once and Future King*.
- Assuming reinterpretation principle solves Tolman's antitelephone paradox, we can ask: If tachyons are not forbidden by special relativity, why are they not observed?

Tachyons and Quantum Theory

- In classical theory, superluminality is emphasized as a defining property of tachyons.
- This is no longer justified in quantum theory.
- Classical concept of propagation velocity is not well-defined for evanescent modes or virtual photons, as nothing well defined and localized propagates through the tunneling barrier in these cases.
- The notion of particles, borrowed from the classical physics, is not quite adequate in the quantum world.
- it is better to call a quantum particle a quanton, which is neither a particle nor a waive, but a completely new object unknown to classical physics (J.-M. Lévy-Leblond).



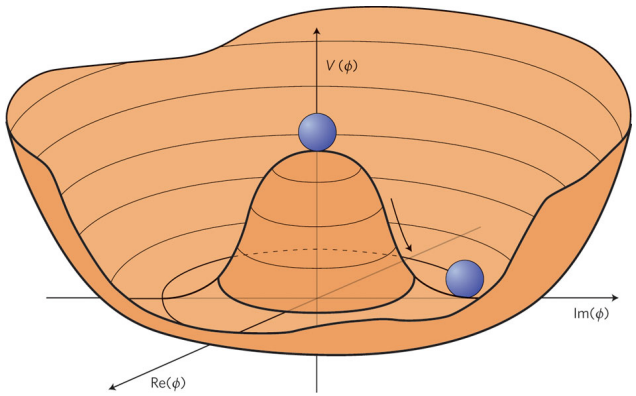
Classification of Quantons

- $P^\mu P_\mu > 0$, s - Bradyons
- $P^\mu P_\mu < 0$, s - Tachyons
- $P^\mu P_\mu = 0$, λ - Luxons
- $P^\mu P_\mu = 0$, continous λ
- $P^\mu = 0$ - Pomeron?

J. Swain, Particles, Fields, Pomerons and Beyond, arXiv:1110.5087 [hep-ph].

Tachyons and Instability

- Usually, for the field ϕ, the squared mass of the corresponding quanton equals the second derivative of the self-interaction potential $V(\phi)$ of this field at the origin $\phi = 0$.
- If the squared mass is negative as for tachyons, the origin cannot be the minimum of the potential.
- As a result, the tachyonic field will roll down towards the true vacuum, acquiring a non-zero vacuum expectation value (tachyon condensation).
- Small excitations of the tachyonic field around the true vacuum will appear as bradyons, not tachyons.



Tachyons in modern physics

If one considers spacelike four-momentum as a defining property of tachyons, then it becomes clear that tachyons play an important role in modern physics:

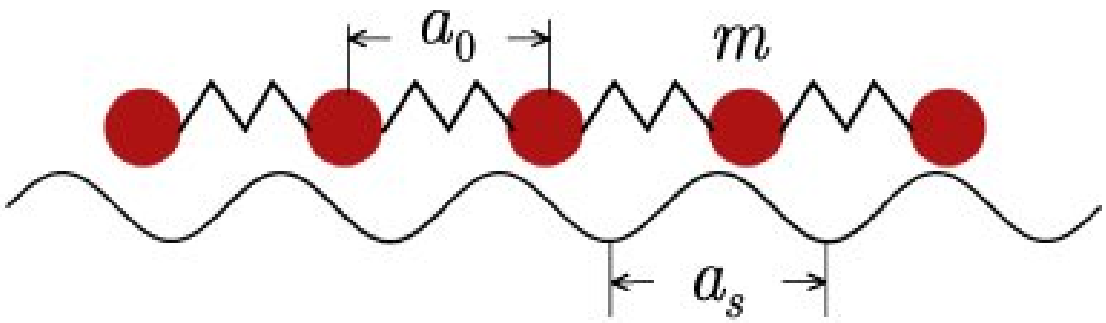
- every quanton can become tachyonic in a virtual state (the four-momentum of a virtual quanton can be spacelike).
- The most notorious example of tachyon condensation is Higgs mechanism in the Standard Model.
- One can say that the Higgs boson is the most famous would-be tachyon.
- Tachyon condensation is important in string theory (A. Sen, String theory and tachyons, Curr. Sci. 81 (2001), 1561-1567).
- Tachyons can play a role in early cosmology (G.W. Gibbons, Thoughts on Tachyon Cosmology, arXiv:hep-th/0301117).

Tachyons and the Light Speed Barrier

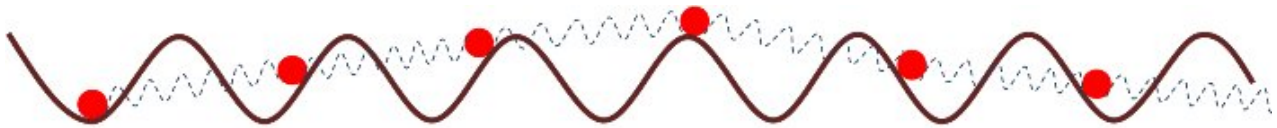
Initially, we were interested in tachyons as superluminal objects, but a disappointment awaits us here:

- Tachyons cannot support the true superluminal propagation, even in a rolling state towards true vacuum.
- In particular, impossibility to construct a stable localized free-tachyon wave-packet implies that one cannot transmit information by tachyon field since the information would be destroyed by the inherent instability of these fields.
- "Contrary to popular prejudice: the tachyon is not a tachyon!" (G.W. Gibbons, Thoughts on Tachyon Cosmology, arXiv:hep-th/0301117).

The Frenkel-Kontorova Model



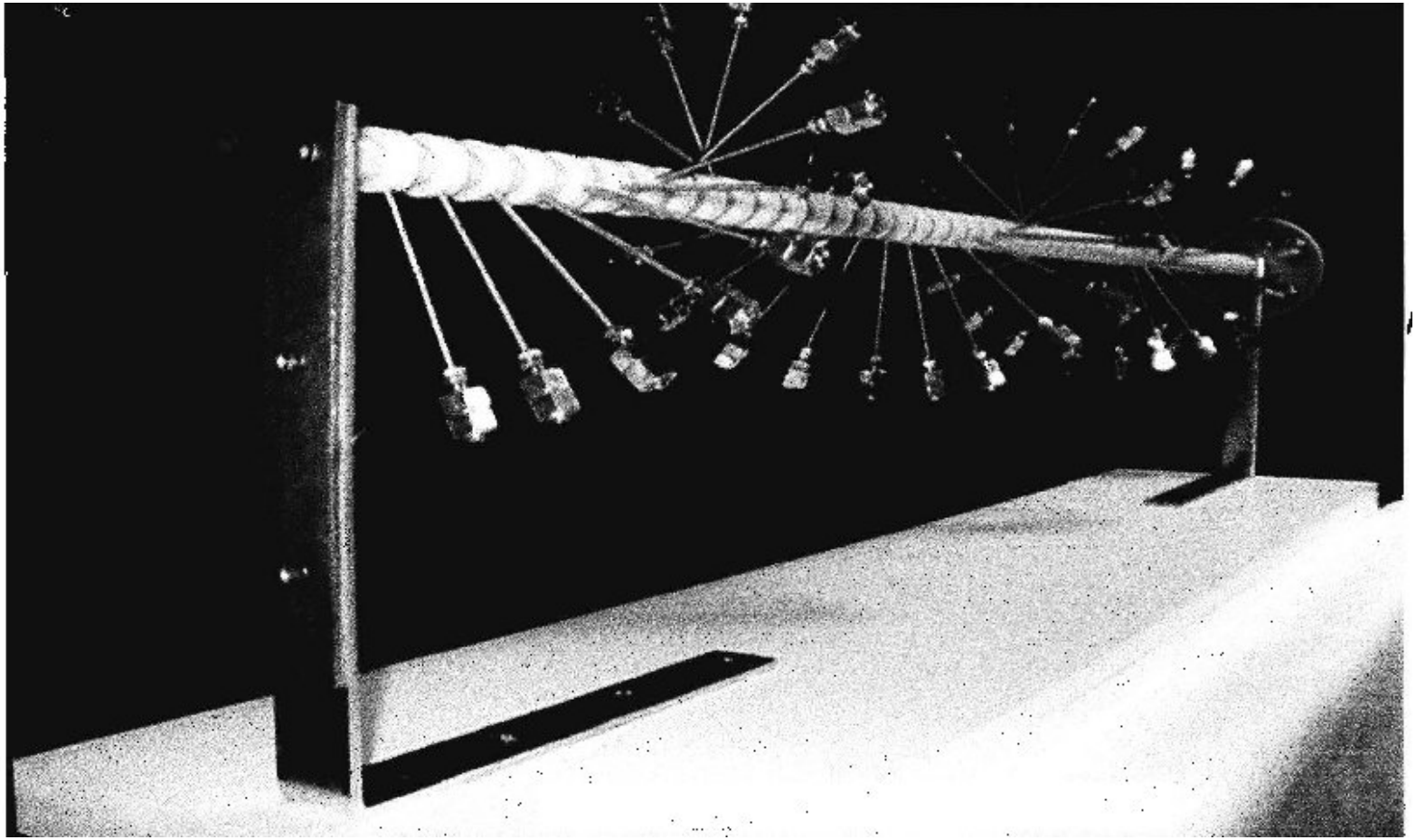
$$\mathcal{L} = \sum_n \left\{ \frac{m}{2}\left(\frac{dx_n}{dt}\right)^2 - \frac{k}{2}(x_{n+1} - x_n - l)^2 - \frac{V_0}{2}\left(1 - \cos\left(\frac{2\pi x_n}{l}\right)\right)\right\}$$



$$\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} - \frac{\partial^2\Phi}{\partial x^2} + \frac{1}{\lambda^2}\sin\Phi = 0$$

$$x = nl, \quad \Phi = \frac{2\pi u}{l}, \quad c = l\sqrt{\frac{k}{m}}, \quad \lambda = \frac{l^2}{\pi}\sqrt{\frac{k}{2V_0}}$$

A mechanical Analogue of the Frenkel-Kontorova Model



$$\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} - \frac{\partial^2\Phi}{\partial x^2} + \frac{1}{\lambda^2}\Phi = 0.$$

Near $\Phi = \pi$ (tachyonic modes):

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2} - \frac{1}{\lambda^2}\varphi = 0, \quad \Phi = \pi - \varphi.$$

Tachyonic Modes are Unstable: do not Allow Supersonic Energy Transfer.

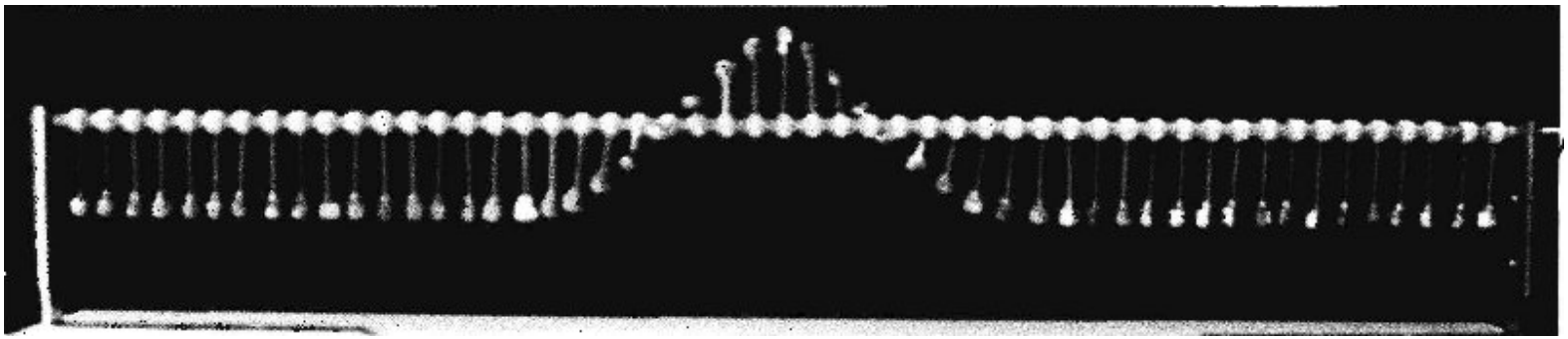
$$\omega = c\sqrt{k^2 - \frac{1}{\lambda^2}}$$

Y. Aharonov, A. Komar, L. Susskind, Superluminal behavior, causality, and instability, Phys. Rev. 182 (1969), 1400-1403.

Frenkel-Kontorova Solitons

$$\Phi(x,t) = 4 \arctan \exp\left[\frac{\sigma}{L}(x - vt)\right]$$

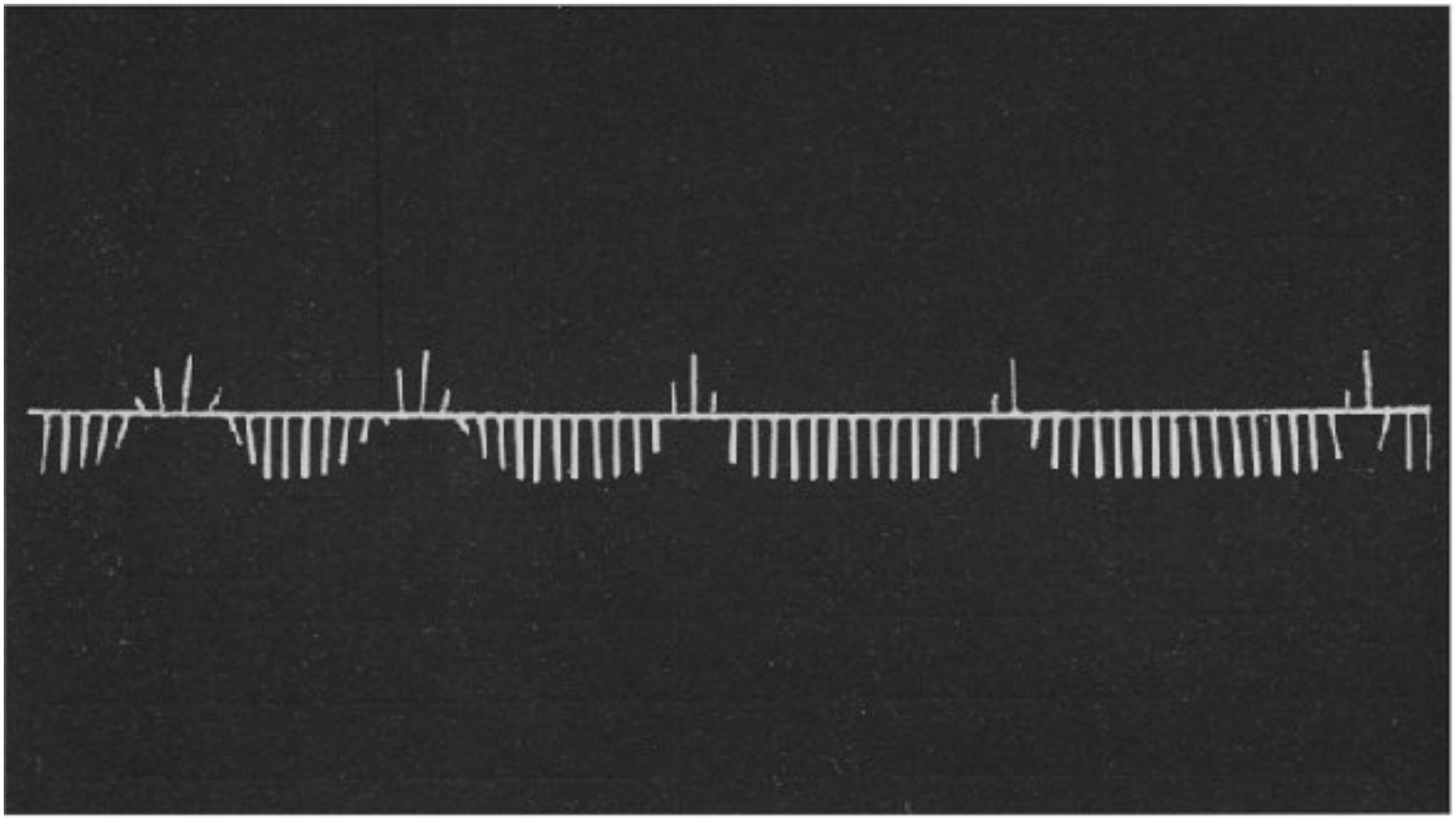
$\sigma = +1$ kink, $\sigma = -1$ anti-kink



$$E = \frac{V_0}{4} \int_{-\infty}^{\infty} \left[\frac{\lambda^2}{c^2}\left(\frac{\partial \Phi}{\partial t}\right)^2 + \lambda^2 \left(\frac{\partial \Phi}{\partial x}\right)^2 + 2(1 - \cos\Phi)\right] \frac{dx}{l}$$

$$L = \lambda\sqrt{1 - \frac{v^2}{c^2}} \equiv \frac{\lambda}{\gamma}, \quad E = Mc^2\gamma, \quad M = 4\,\frac{\lambda}{l}\,\frac{V_0}{c^2}$$

Lorentz-Fitzgerald Length Contraction



A. Barone, F. Esposito, C.J. Magee, A.C. Scott, Theory and applications of the sine-gordon equation, Riv. Nuovo Cim. 1 (1971), 227-267.

Anti-dislocation (T-kink)

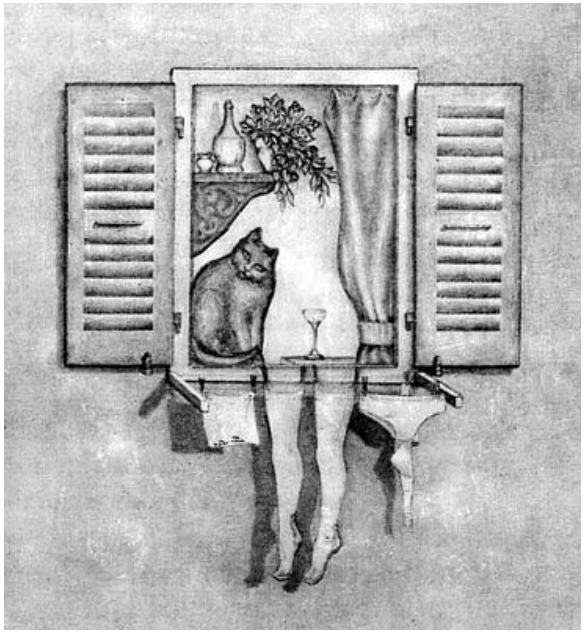
$$\Phi(x,t) = \pi - 4\arctan\exp\left[-\frac{\sigma}{\tilde{L}}(x-vt)\right], \quad \tilde{L} = \lambda\sqrt{\frac{v^2}{c^2}-1}$$

F.C. Frank, J.H. van der Merwe, One-Dimensional Dislocations. IV. Dynamics, Proc. Roy. Soc. Lond. A 201 (1950), 261-268.

- One can visualize infinitely many minimums of the Frenkel-Kontorova external potential as depressions of a infinitely long slate.
- A kink corresponds to an infinite rope which begins in one depression of the slate and ends up in another neighboring depression. One needs an infinite energy to throw the rope from one depression to another one — the kink is stable (can be annihilated only by a counter anti-kink).
- T-kink corresponds to a rope which lays on a slate ridge, then somewhere on the ridge it falls in the depression and raises again to the adjacent ridge. Of course, such configuration cannot be stable.

"Mouth of Flower" - Optical illusion from Mexican artist Octavio Ocampo.

Emergent Phenomena are Quite Frequent



"The Window Opposite" - Optical illusion from Swiss artist Sandro del Prete.

P. W. Anderson, More Is Different: Broken symmetry and the nature of the hierarchical structure of science, Science **177** No.4047 (1972), 393.

Toda Supersonic Solitons

$$\mathcal{L} = \sum_n \left\{ \frac{m}{2} \left(\frac{du_n}{dt} \right)^2 - \frac{k}{b} \left\{ u_n - u_{n-1} + \frac{1}{b} \left[e^{-b(u_n - u_{n-1})} - 1 \right] \right\} \right\}$$

When the velocity of a Toda soliton approaches the sound velocity, its behavior is opposite to that of a tachyon: its energy approaches to zero and its width approaches to infinity. Despite this difference, the final result is the same: Toda solitons, like tachyons,cannot cross the sound barrier and become subsonic.

Kosevich-Kovalev Solitons

Interpolate between the subsonic Frenkel-Kontorova solitons and supersonic Toda solitons, and can cross sound barrier.

$$\mathcal{L} = \sum_n \left\{ \frac{m}{2}\left(\frac{du_n}{dt}\right)^2 - \right.$$

$$\left. \frac{k}{2}(u_{n+1}-u_n)^2\left[1+\frac{\chi}{l^2}(u_{n+1}-u_n)^2\right] - \frac{V_0}{2}\left(1-\cos\left(\frac{2\pi u_n}{l}\right)\right)\right\}$$

$$\chi = \frac{\pi^2}{12}, \quad L^2 = \frac{\lambda^2}{2}\left[1-\frac{v^2}{c^2}+\sqrt{\left(1-\frac{v^2}{c^2}\right)^2+\frac{1}{3}\left(\frac{l}{\lambda}\right)^2}\right].$$

$$L = \frac{l/2\sqrt{3}}{\sqrt{\frac{v^2}{c^2}-1}}, \quad v \gg c; \quad L = \lambda\sqrt{1-\frac{v^2}{c^2}}, \quad v \ll c, \quad l/\lambda \ll 1.$$

A. M. Kosevich, A. S. Kovalev, The supersonic motion of a crowdion. The one-dimensional model with nonlinear interaction between the nearest neighbours, Solid State Commun. 12 (1973), 763-765.

Elvisebrions

Bradyons

Luxons

Tachyons

Elvisebrions

Two linked Frenkek-Kontorova chains

$$\frac{\partial^2 \Phi}{\partial t^2} - \frac{\partial^2 \Phi}{\partial x^2} = -\delta^2 \sin(\Phi - \Psi)$$

$$\frac{\partial^2 \Psi}{\partial t^2} - \epsilon^2 \frac{\partial^2 \Psi}{\partial x^2} = \sin(\Phi - \Psi)$$

$$\delta^2 = \frac{m_2}{m_1} \ll 1, \quad \epsilon = \frac{c_2}{c_1}.$$

S. Yomosa, Soliton excitations in deoxyribonucleic acid (DNA) double helices, Phys. Rev. A **27**, 2120-2125 (1983). ($\epsilon = 1$)

Details in O. I. Chashchina, Z. K. Silagadze, Breaking the light speed barrier, Acta Phys. Pol. B 43 (2012), 1917

Fast and slow solutions

Fast solution ($c_2^2 < v^2 < \frac{m_1 c_1^2 + m_2 c_2^2}{m_1 + m_2}$, or $c_1^2 < v^2 < \infty$)

$$\Phi(x,t) = \frac{2\,\delta^2\left(\frac{v^2}{c^2} - \epsilon^2\right)}{\delta^2\left(\frac{v^2}{c^2} - \epsilon^2\right) + \frac{v^2}{c^2} - 1} \arcsin\left[k\,\mathrm{sn}\left(\frac{\sigma}{\tilde{L}}(x - vt),\ k\right)\right],$$

$$\Psi(x,t) = \frac{2\left(1 - \frac{v^2}{c^2}\right)}{\delta^2\left(\frac{v^2}{c^2} - \epsilon^2\right) + \frac{v^2}{c^2} - 1} \arcsin\left[k\,\mathrm{sn}\left(\frac{\sigma}{\tilde{L}}(x - vt),\ k\right)\right],$$

Slow solution ($0 \le v^2 < c_2^2$, or $\frac{m_1 c_1^2 + m_2 c_2^2}{m_1 + m_2} < v^2 < c_1^2$)

$$\Phi(x,t) = \frac{2\,\delta^2\left(\frac{v^2}{c^2} - \epsilon^2\right)}{\delta^2\left(\frac{v^2}{c^2} - \epsilon^2\right) + \frac{v^2}{c^2} - 1} \arcsin\left[\mathrm{dn}\left(\frac{\sigma}{L}(x - vt),\ k\right)\right],$$

$$\Psi(x,t) = \frac{2\left(1 - \frac{v^2}{c^2}\right)}{\delta^2\left(\frac{v^2}{c^2} - \epsilon^2\right) + \frac{v^2}{c^2} - 1} \arcsin\left[\mathrm{dn}\left(\frac{\sigma}{L}(x - vt),\ k\right)\right],$$

At $k \to 1$, $\epsilon \to 1$, $\delta \to 0$, $\lambda\,\delta \to \lambda_0$, slow solution $\to$ FK-soliton (kink), fast solution $\to$ T-kink.

Higgs portal?

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_{int}, \quad \mathcal{L}_{int} = g\Phi^+\Phi\,\Psi^+\Psi$$

B. Patt and F. Wilczek, Higgs-field portal into hidden sectors, hep-ph/0605188.

G. F. Giudice, M. Raidal and A. Strumia, Lorentz Violation from the Higgs Portal, Phys. Lett. B **690**, 272-279 (2010) [arXiv:1003.2364 [hep-ph]].

$$\mathcal{L}_1 = \frac{1}{c_1^2}\frac{\partial\Phi^+}{\partial t}\frac{\partial\Phi}{\partial t} - \nabla\Phi^+\cdot\nabla\Phi + \mu^2\Phi^+\Phi - \frac{1}{2}\lambda(\Phi^+\Phi)^2$$

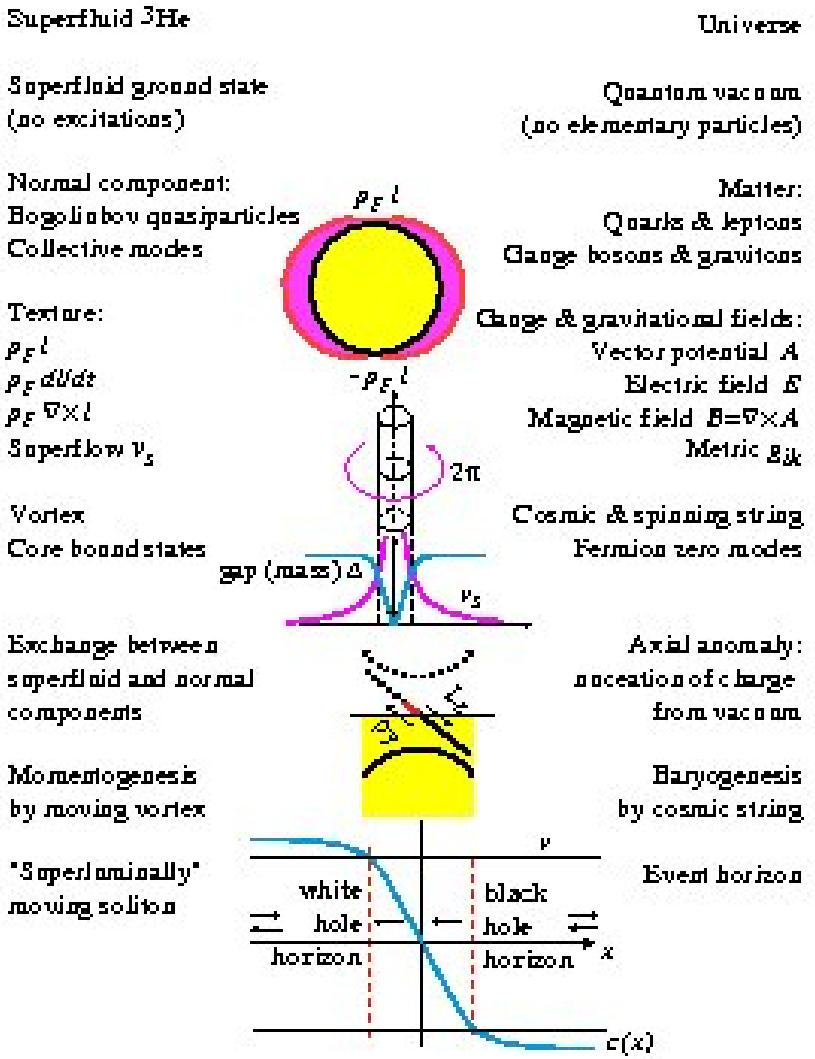
$$\mathcal{L}_2 = \frac{1}{2}\left[\frac{1}{c_2^2}\frac{\partial\Psi}{\partial t}\frac{\partial\Psi}{\partial t} - \nabla\Psi\cdot\nabla\Psi\right]$$

$$\mathcal{L}_{int} = g\,\Phi^+\Phi\,\frac{\partial\Psi}{\partial x}$$

Describes pulses, fronts and oscillations of an elastic rod: J. Lega and A. Goriely, Physica D 132 (1999), 373.

The incomprehensible efficiency of mathematics

"Miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. E. P. Wigner, The Unreasonable Effectiveness of Mathematics in the Natural Sciences, Comm. Pure and Appl. Math. 13 (1959). 1.



G.E. Volovik, Field theory in superfluid ^{3}He: What are the lessons for particle physics, gravity, and high-temperature superconductivity? Proc. Nat. Acad. Sci. 96 (1999) 6042.

Similar Ideas

L. Gonzalez-Mestres, Properties Of A Possible Class Of Particles Able To Travel Faster Than Light, arXiv:astro-ph/9505117.

R. Geroch, Faster Than Light? arXiv:1005.1614 [gr-qc].

"In short, the causal cones of special relativity, from this perspective, have no special place over and above the cones of any other system. This is democracy of causal cones with a vengeance. This, of course, is not the traditional view... On the other hand, it is entirely possible that there exist any number of other systems – not yet observed (or maybe they have been!) – that employ quite different sets of causal cones. And the cones of these «other systems» could very well lie outside the null cones of special relativity, i.e., these systems could very well manifest superluminal signals. None of this would contradict our fundamental ideas about how physics is structured: An initial-value formulation,causal cones governing signals, etc."

How confident are we in the Elvisebrion hypothesis?

- Martin Rees once said that he is sufficiently confident about the Multiverse to bet his dog's life on it.
- He was supported by Andrei Linde who was ready to bet his own life.
- While Steven Weinberg has said that he is confident enough in the Multiverse Hypothesis to bet the lives of both Andrei Linde and Martin Rees's dog.

S. Weinberg, Living in the multiverse, arXiv:hep-th/0511037.

Not very confident

We're not sure enough to put the lives of our beloved animals on the line.

But, on the other hand,

We are confident enough in the hypothesis to stake the lives of both Schrödinger's cat and Wigner's friend!

Instead of a conclusion

"I believe that there is something great in astronomy, in physics, in all the natural sciences that allows the human being to look beyond its present place and to arrive at some understanding of what goes on beyond the insignificant meanness of spirit that so often pervades our existence. There is a Nature; there is a Cosmos; and we walk towards the understanding of it all. Is it not wonderful? There are many charms in the profession; as many charms as in love provided, of course, that they are not in the service of mercantile aims." Martin Lopez-Corredoira, What do astrophysics and the world's oldest profession have in common? arXiv:astro-ph/0310368