CONSTRAINING DARK MATTER PROPERTIES BY ORBITAL PRECESSION AROUND THE GALACTIC SUPERMASSIVE BLACK HOLE

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PRECESSION

- General Relativity predicts that orbital precession exists

$$\frac{d^2u}{d\phi^2} + u = \frac{GM_\odot}{L^2} + 3\frac{GM_\odot}{c^2}u^2$$

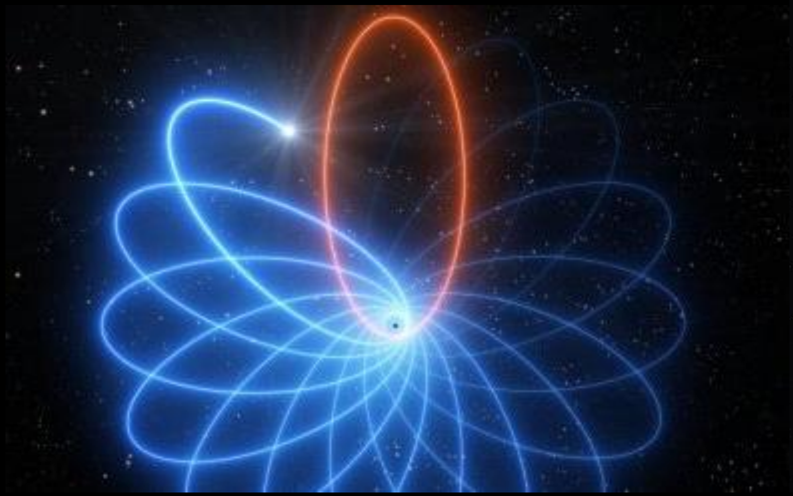
- In our solar system:

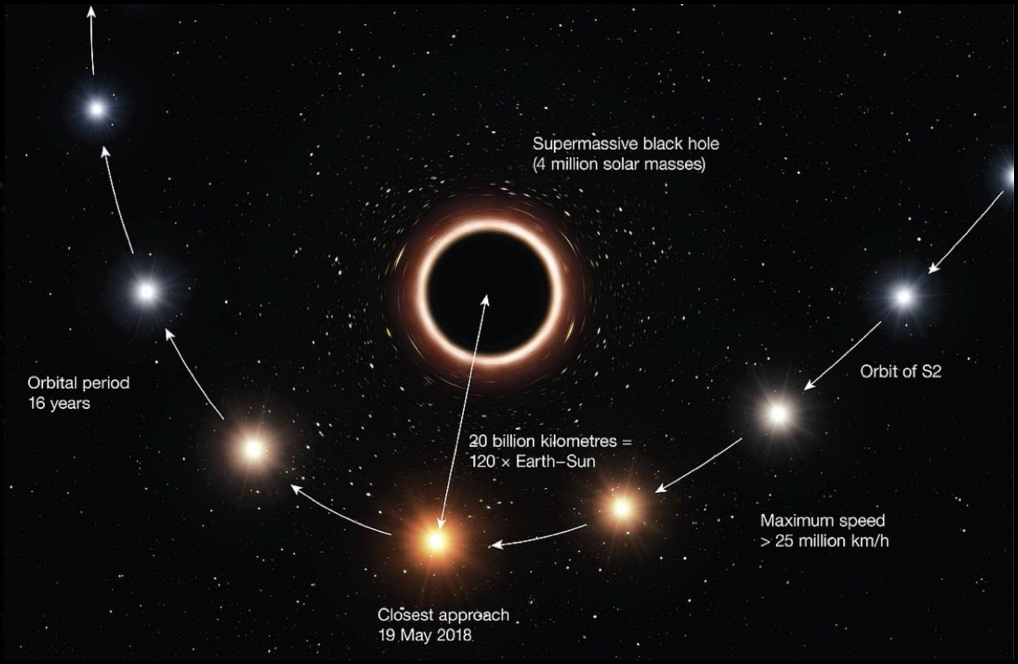
Planet	General relativistic term	Observed
Mercury	42.9799	$42.9799^{+0.0030}_{-0.0006}$
Venus	8.628	8 ± 5
Earth	3.841	5 ± 1
Mars	1.351	1.3624 ± 0.0005
Jupiter	0.0623	0.070 ± 0.004
Saturn	0.0137	0.014 ± 0.002

Precession angle (in arcsec per century) other than Newtonian's effects

PRECESSION

- Orbital precession can also be found in stars orbiting about the supermassive black hole (SMBH) at the Galactic Centre
- The S2 star orbiting the SMBH shows a precession angle of $\approx 0.2°$ per period, which is close to the General Relativity prediction (GRAVITY Collaboration, A&A 636 L5 (2020)).





Credit: https://arstechnica.com/science/2020/04/star-dancing-around-a-supermassive-black-hole-confirms-einstein-yet-again/

S2 ORBIT

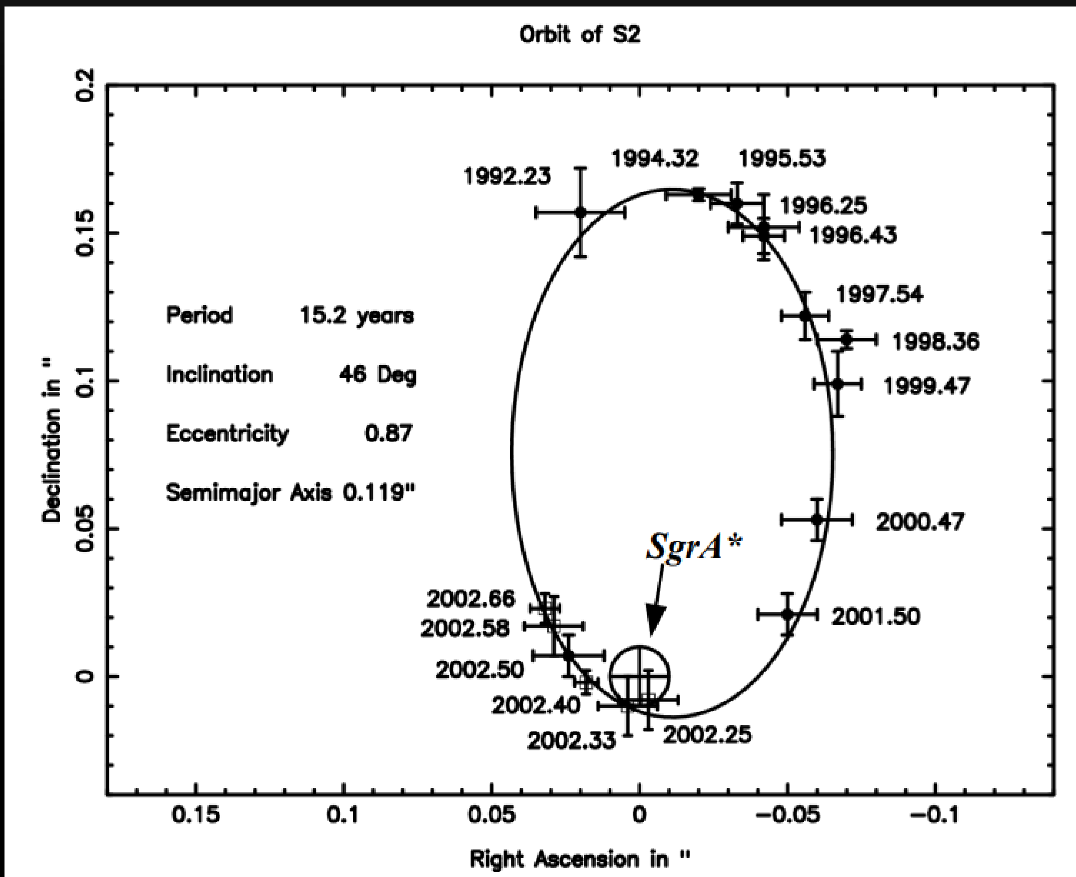


Table 1: Parameters of the S2 orbit

Mass of S2 star	14-15$M_\odot$ [3][4]
M_{BH}	$(4.154 \pm 0.014) \times 10^6 M_\odot$ [1]
Orbit period	15.9-16.0 yrs [13]
Eccentricity (e)	0.88466 $\pm$ 0.00018 [1]
Schwarzschild radius ($r_s = 2GM_{\mathrm{BH}}/c^2$)	0.08 AU [38][39]
Pericentre	120 AU [1]
Apocentre	1820 AU [1]
Velocity at pericentre	7650 km/s [38]
Inclination i	$134.3533°$ [23]
Argument of pericentre ω	$66.7724°$ [23]
Ascending node Ω	$228.024°$ [23]

Schödel et al. (2002). *Nature* 419, 694.

Nobel Prize in Physics 2020:
Reinhard Genzel's group

EXTENDED MASS DISTRIBUTION

- However, it is suggested that some other mass distributions (e.g. neutron stars, dark matter) exist around the SMBH
- Question: how could these distributions affect the orbital precession of the S2 star or other S-cluster stars?
- Assuming the mass profile: $M_{\mathrm{ex}}(r) = K r^n$
- The total dynamical mass distribution is: $M(r) = M_{BH} + M_{ex}(r)$

EXTENDED MASS DISTRIBUTION

- Therefore, following the perturbation method, the equation of motion becomes

$$\frac{d^2u}{d\phi^2} + u = \frac{GM(u)}{L^2} + 3\frac{GM_{BH}}{c^2}u^2$$

where $u = \frac{1}{r}$ and we have assumed $M_{ex} \ll M_{BH}$.

- Putting the extended mass power-law model, we have

$$\frac{d^2u}{d\phi^2} + u = \frac{GM_{BH}}{L^2} + \frac{GK}{L^2}u^{-n} + 3\frac{GM_{BH}}{c^2}u^2$$

EXTENDED MASS DISTRIBUTION

- Let $u = u_0 + \Delta u$ with $\Delta u \ll u_0$. Expanding all the terms up to the first order of $\Delta u/u_0$, we have:

$$\frac{d^2}{d\phi^2}\Delta u \approx \left[\frac{GM_{BH}}{L^2} + \frac{GK}{L^2 u_0^n} + 3\frac{GM_{BH}}{c^2}u_0^2 - u_0\right] - \left[1 - \frac{6GM_{BH}u_0}{c^2} + \frac{GKn}{L^2 u_0^{n+1}}\right]\Delta u$$

- Choose the arbitrary u_0 such that the first term vanishes. Then we get

$$\frac{d^2}{d\phi^2}\Delta u = -(1-\alpha)^2 \Delta u$$

with

$$(1-\alpha)^2 \approx (1-2\alpha) = \left[1 - \frac{6GM_{BH}u_0}{c^2} + \frac{GKn}{L^2 u_0^{n+1}}\right]$$

EXTENDED MASS DISTRIBUTION

- Therefore, the orbital precession per period is

$$\Delta\phi = 2\pi\alpha = \frac{6\pi G M_{BH}}{a(1-e^2)c^2} - \frac{\pi K n}{M_{BH}}[a(1-e^2)]^n$$

- The first term is the GR term while the second term is the precession term due to the extended mass distribution.
- The GR term is prograde while the precession due to the extended mass distribution is retrograde.
- This analytic formula can help constrain the extended mass distribution if we have the observed precession angle.

INVESTIGATIONS

- For small eccentricity case (e.g. Mercury), we can safely use the analytic formula for investigation:

$$\Delta\phi = 2\pi\alpha = \frac{6\pi G M_{BH}}{a(1-e^2)c^2} - \frac{\pi K n}{M_{BH}}[a(1-e^2)]^n$$

- The analytic formula can reveal the behavior of precession angle due to the extended mass distribution. Some of the parameters could be constrained analytically.

- For large eccentricity case (e.g. S2 star), we can solve the equation numerically:

$$\frac{d^2u}{d\phi^2} + u = \frac{GM(u)}{L^2} + 3\frac{GM_{BH}}{c^2}u^2$$

TESTING DIFFERENT MASS DISTRIBUTION

- We can generally test the following suggested mass distribution around the SBMH:

1. Dark matter spike model
2. Fuzzy dark matter model
3. Neutron star distribution model

- Besides, some modified gravity model can write the apparent dark matter term in terms of the power-law mass distribution:

1. MOND
2. Emergent gravity
3. Modified Gravity (MOG) theory

1. DARK MATTER DENSITY SPIKE MODEL

- Dark matter (DM) surrounding a SMBH would redistribute to form a density spike
- The density of DM is given by

$$\rho_{\rm DM}(r) = \rho_0 \left(\frac{r_0}{r}\right)^{\gamma}$$

- The cusp index can range from 1.5 to 2.5.
- If we assume a benchmark value 7/4 (i.e. The Bahcall-Wolf cusp model), the optimal parameters for ρ_0 and r_0 are 2.24x10^{-11} kg/m^3 and 0.012 pc respectively.
- The extended DM mass distribution is given by

$$M_{\rm DM}(r) = \int_{4GM_{\rm BH}/c^2}^{r} \rho_{\rm DM}(r')4\pi r'^2 dr'$$

2. NORMAL STELLAR CLUSTER MODEL

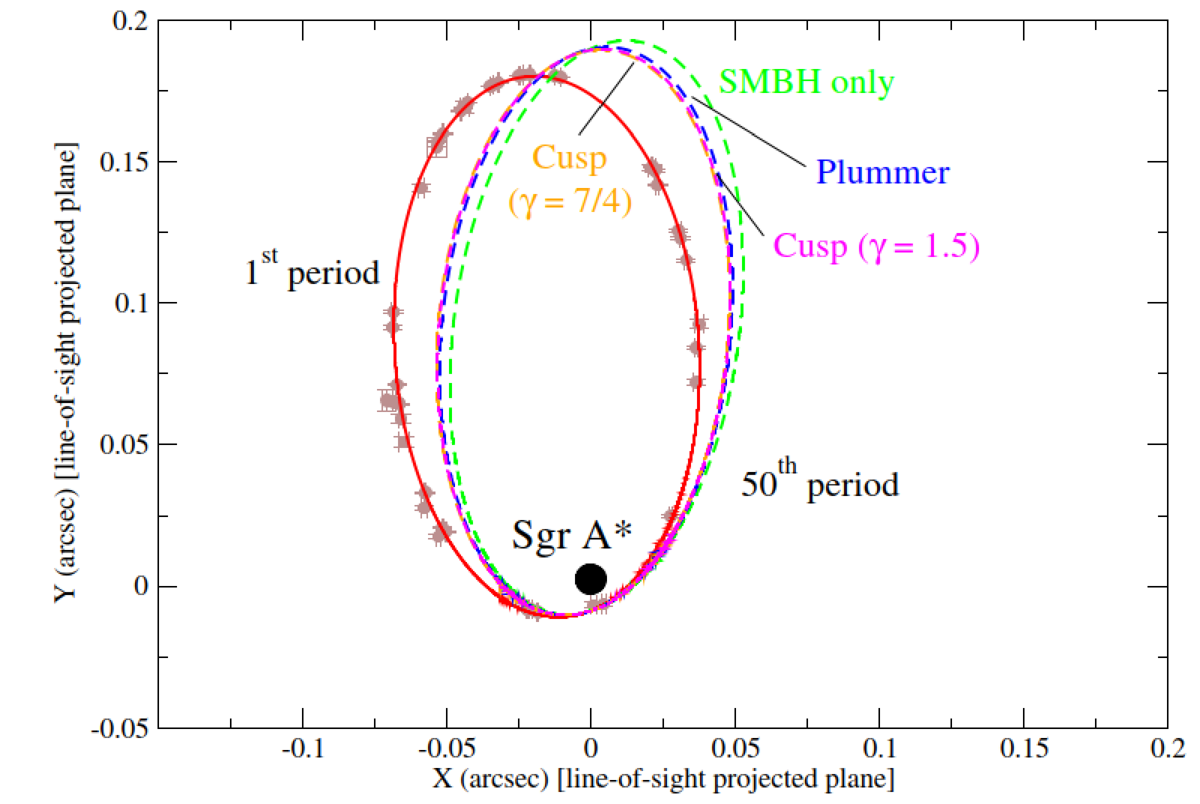
- We may also test the normal stellar cluster model.
- The stellar cluster model at the Galactic Centre is usually modelled by a Plummer model:

$$\rho_{stellar}(r) = \rho_0 \left(1 + \frac{r^2}{r_0^2}\right)^{-5/2}$$

- The optimal parameters for ρ_0 and r_0 are 2.88×10^{-11} kg/m^3 and 0.012 pc respectively

PRECESSIONS

- We transform the orbits to the line-of-sight projected plane
- We plot the 50th period for clearer illustration



For the SMBH only scenario:
precession angle = $0.2077°$

For the Cusp model:
Precession angle = $0.1515°$

For the Plummer model:
Precession angle = $0.1654°$

Chan, Lee & Yu (2022), *Scientific Reports*, 12, 15258.

IMPLICATIONS

- Therefore, the effect of extended mass distribution can be quite significant.
- The precession angle can help differentiate the extended mass distribution model.
- The observed precession angle can constrain the relevant parameters.
- Observed precession angle of S2: $\Delta\phi = (0.22 \pm 0.04)°$ (GRAVITY Collaboration)
- Therefore, the above two optimal scenarios are ruled out.

DM ANNIHILATION

- The precession angle can also help constrain the annihilation parameters
- The DM annihilation would decrease the central DM density, which gives a smaller cusp index

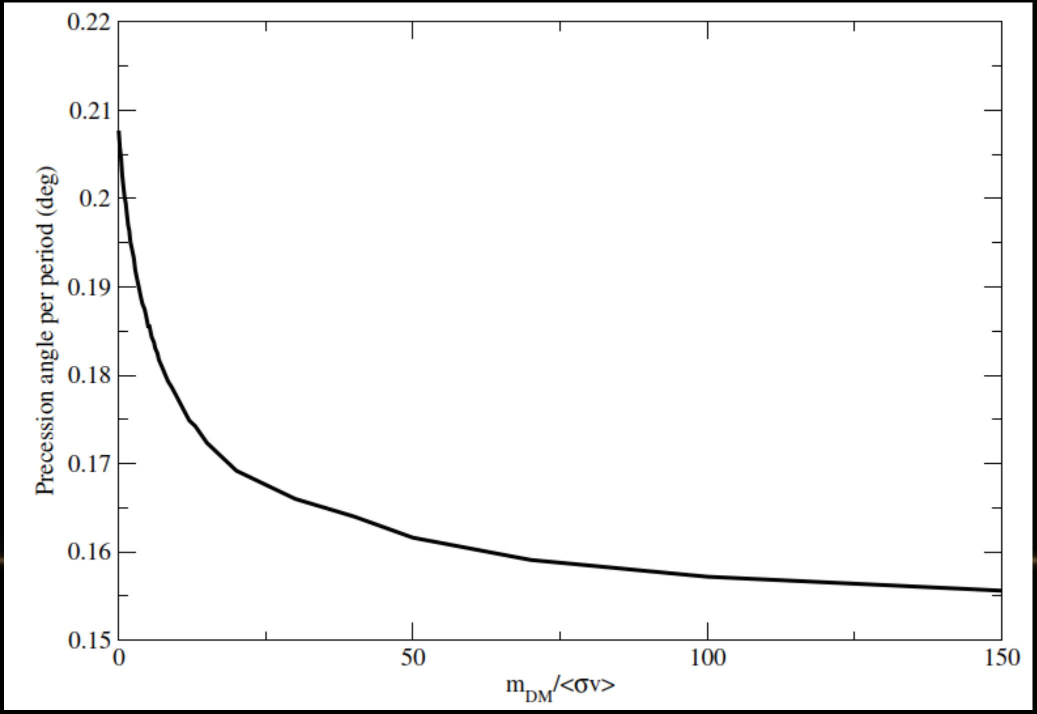
$$\rho_{\mathrm{DM,ann}}(r) = \frac{\rho_{\mathrm{DM}}(r)\rho_{\mathrm{in}}(t,r)}{\rho_{\mathrm{DM}}(r) + \rho_{\mathrm{in}}(t,r)}$$

with

$$\rho_{\mathrm{in}}(t,r) = \left(\frac{m_{\mathrm{DM}}}{\langle \sigma v \rangle t}\right)\left(\frac{r}{r_{\mathrm{in}}}\right)^{-0.5}$$

DM ANNIHILATION

- The precession angle is a function of dark matter mass and annihilation cross section
- More independent data (gamma-ray detection) are required to break the degeneracy



3. FUZZY DM

- The precession angle analysis can also help constrain the fuzzy DM
- Numerical simulations show that fuzzy DM can form a soliton core at the halo centre (Schive et al. 2014):

$$\rho_{\rm DM}(r) = \frac{\rho_0}{(1 + Ar^2)^8}$$

where $A = 9.1 \times 10^{-2} r_c^{-2}$

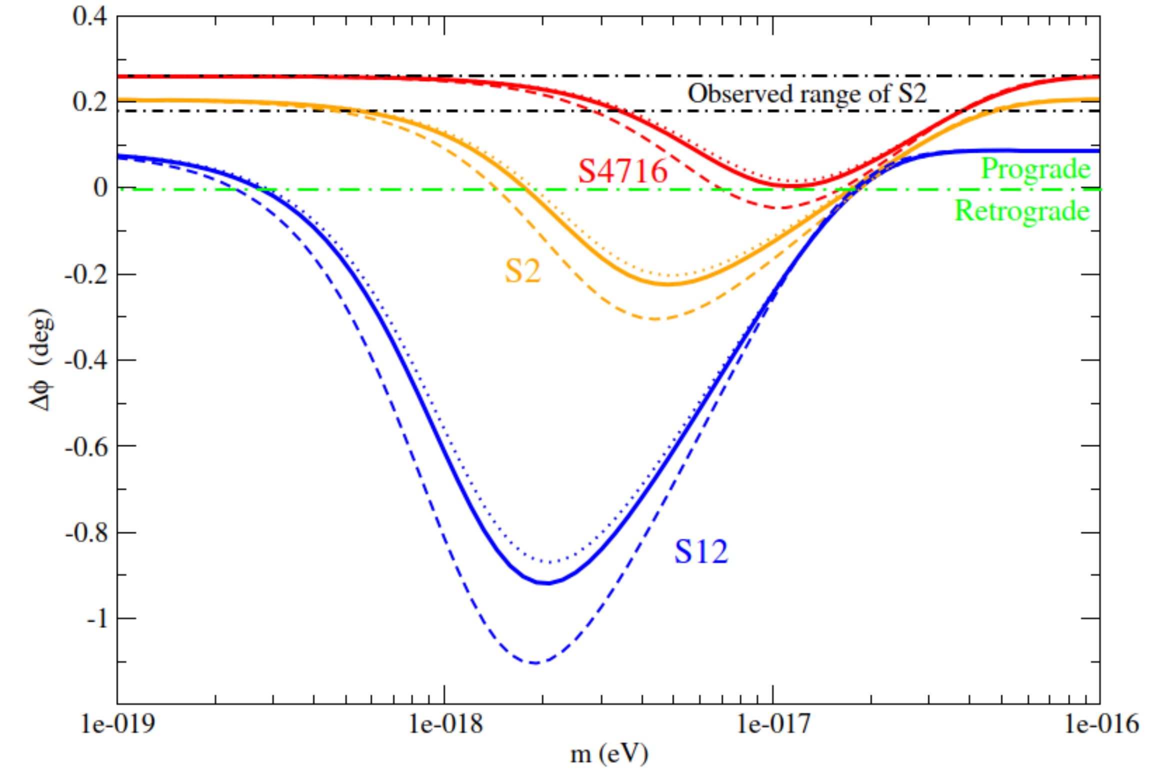
$$\rho_0 = 1.9 \left(\frac{m}{10^{-23}\ \rm eV} \right)^{-2} \left(\frac{r_c}{\rm kpc} \right)^{-4} M_\odot\ \rm pc^{-3}$$

$$r_c = 16 \left(\frac{m}{10^{-23}\ \rm eV} \right)^{-1} \left(\frac{M_h}{10^9 M_\odot} \right)^{-1/3} \rm kpc$$

Recent data from the Gaia satellite constrain the halo mass $M_h = 1.08 \times 10^{12} M_\odot$ (Cautun et al. 2020).

FUZZY DM

- Predictions of the precession angles for three different S-cluster stars (S2, S4716, S12) near the SMBH:



For some ranges of DM mass, the orbital precession could be retrograde in direction

Solid lines: $M_h = 1.08 \times 10^{12} M_\odot$

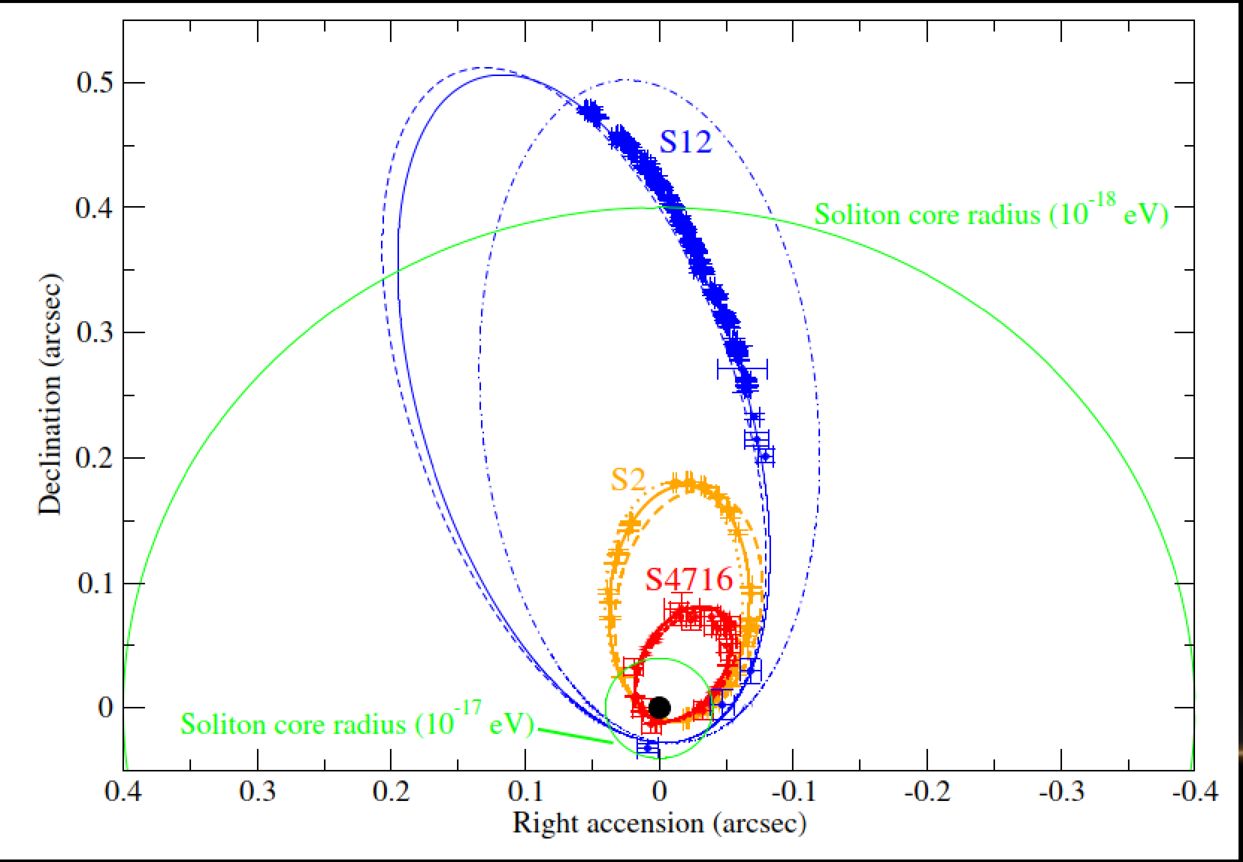
Dotted lines: $M_h = 1.0 \times 10^{12} M_\odot$

Dashed lines: $M_h = 1.4 \times 10^{12} M_\odot$

FUZZY DM

- The predicted orbital precessions after 20 periods

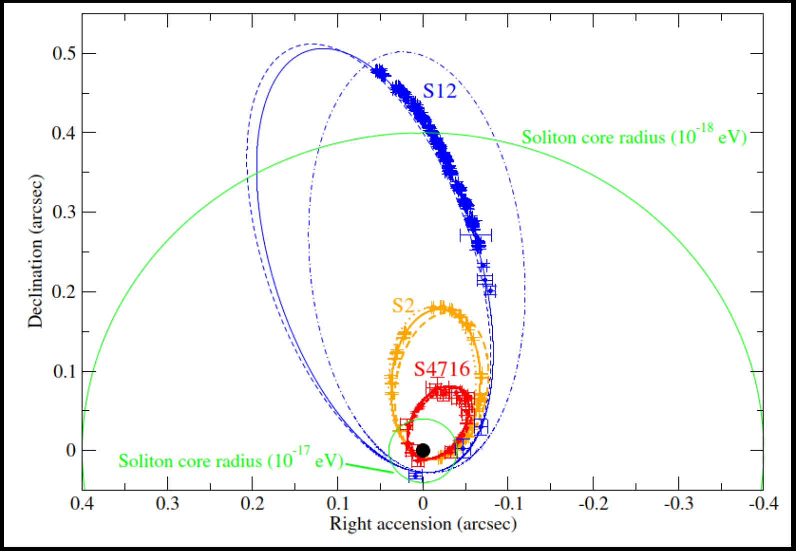
Star	Semimajor axis a (AU)	Eccentricity e	$\Delta\phi_{GR}$ (deg)
S2	1002	0.886	0.21
S12	2390	0.888	0.088
S4716	398	0.756	0.26



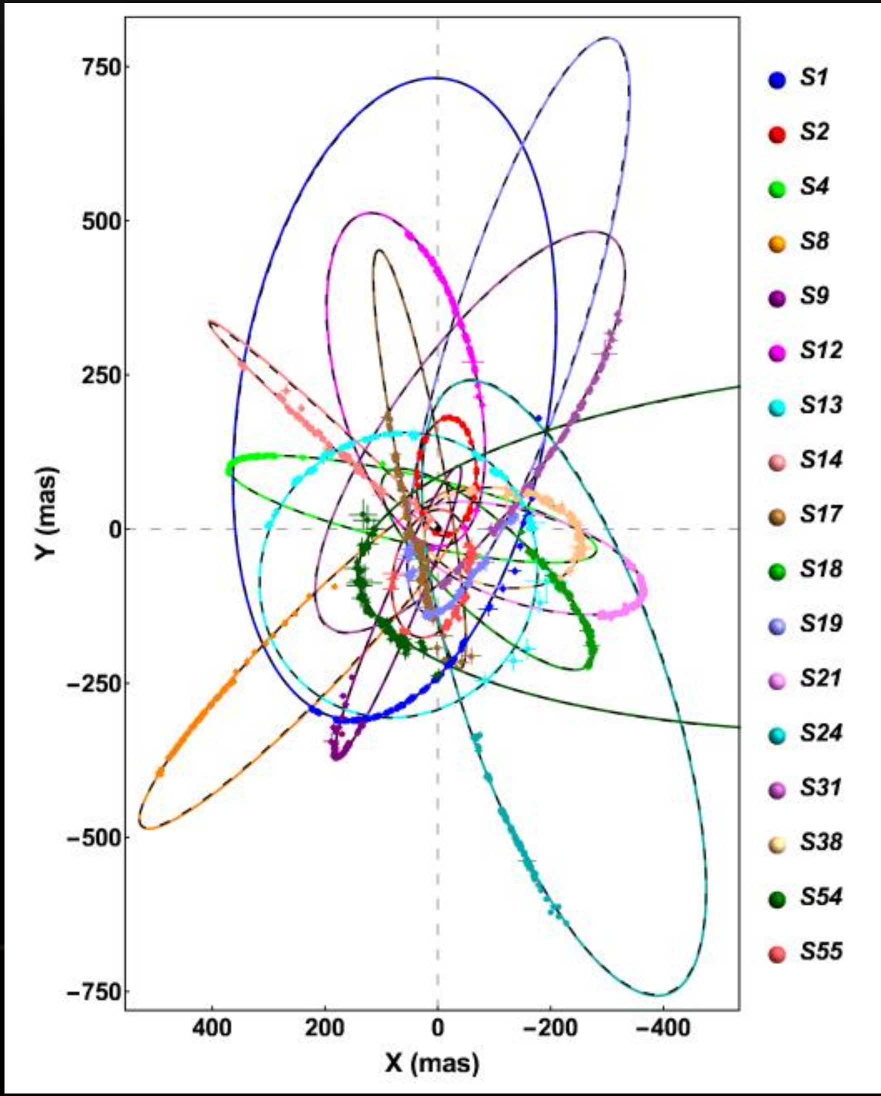
Solid lines: Current orbits
Dotted lines: 10^{-17} eV
Dot-dashed lines: 10^{-18} eV
Dashed lines: 10^{-19} eV

FUZZY DM

- The S12 and S4716 data can give very good constraints (observed precession angles are unknown)
- The S12 star can constrain the small mass regime while the S4716 can constrain the large mass regime
- The period of S4716 is only 4 years (just discovered recently) while the period of S12 is 56 years (already observed for 30 years)



THE S-STAR CLUSTER



Becerra-Vergara et al. (2021). MNRAS 505, L64.

4. MODIFIED GRAVITY MODELS

- The precession angle analysis can also constrain the parameters in modified gravity models
- For example:

MOND:

The gravitational acceleration is modified when the acceleration is small:

$$g = \nu(g_b/a_0)g_b$$

a_0 is the acceleration scale in MOND

The interpolating function can be generally given by:

$$\nu(x) = \left[\frac{1 + (1 + 4x^{-p})^{1/2}}{2}\right]^{1/p}$$

$$\begin{aligned} \nu(x) &= [1 - \exp(-x^{\delta/2})]^{-1/\delta} \\ &\approx 1 + \frac{1}{\delta}\exp(-x^{\delta/2}) \quad \text{for } x \gg 1 \end{aligned}$$

MODIFIED GRAVITY MODELS

- These modified gravitational accelerations can be transformed to an extra apparent mass term in the equation of motion:

$$M_{\rm ex}(r) \approx \frac{1}{p}\left(\frac{GM_\odot}{a_0}\right)^{-p} M_\odot r^{2p}$$

$$\begin{aligned} M_{\rm ex} &= \frac{1}{\delta}\exp(-x^{\delta/2})M_\odot \\ &= \frac{1}{\delta}\sum_{k=0}^{\infty}\frac{1}{k!}\left[-\left(\frac{GM_\odot}{a_0}\right)^{\delta/2}\frac{1}{r^\delta}\right]^k M_\odot \end{aligned}$$

- We can use the precession data to constrain the relevant parameters
- Mercury precession data is preferred because the effects of MOND is relatively larger for Mercury orbit

REFERENCES

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Q&A!

Thanks!

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