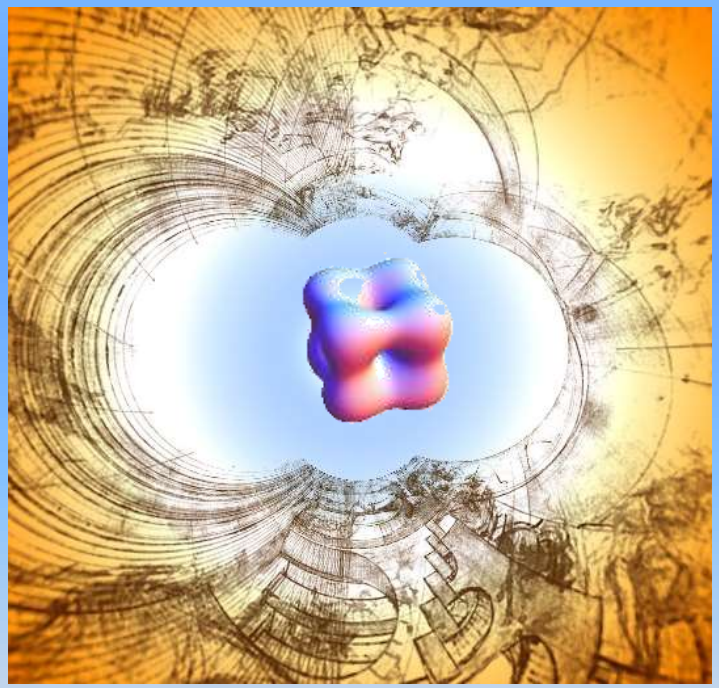
HEP-Cosmo-Astro joint seminar - *15.12.2022*

Curing conical singularities with scalar hair



Eugen Radu

Aveiro University, Portugal

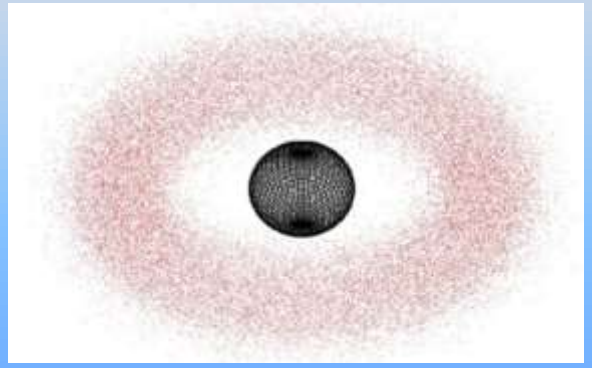
(work done with C. Herdeiro)

the problem we address:

- the ***no hair conjecture*** can be violated

- Schwarschild/Kerr Black Hole + scalar hair:

many interesting solutions

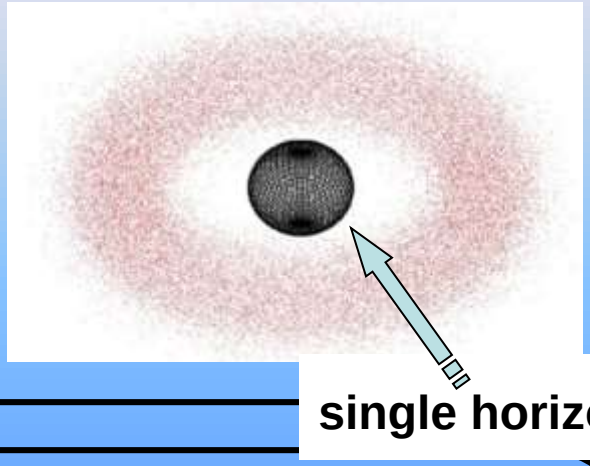


the problem we address:

- the ***no hair conjecture*** can be violated

- Schwarschild/Kerr Black Hole + scalar hair:

many interesting solutions



Q1: what about multi-Black Holes?

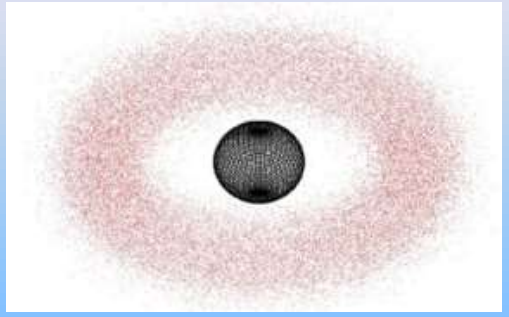
- **electro-vacuum:** N Black Holes *(exact solutions)*
- simplest case N=2: the Bach-Weyl vacuum solution (2Schwarzschild)
- generic presence of *conical singularities*

the scalar hair can balance the 2BH system

the problem we address:

- the ***no hair conjecture*** can be violated
- Schwarschild/Kerr Black Hole + scalar hair:

many interesting solutions



Q1: what about multi-Black Holes?

- electro-vacuum, four dimensions: N Black Holes *(exact solutions)*
- simplest case N=2: the Bach-Weyl vacuum solution (2Schwarzschild)
- generic presence of *conical singularities*

the scalar hair can balance the system

Q2: what about higher dimensions?

D=5: the static Emparan-Reall vacuum Black Ring

- it possesses conical singularities

static, regular Black Rings with scalar hair

a warning:

- *this is rather a mathematical problem*
  (we do not claim <u>any contact</u> with observations)
- *toy model to test the validity of some results in vacuum GR*
- *asymptotically flat case, only*

also

- *all configurations are static*
- *no results on stability*
- *main issue: technical – find the proper approach*

results in C. Herdeiro, E.R: e-Print: 2004.00336
Y. Brihaye, C. Herdeiro, E.R., e-Print: 2207.13114
C. Herdeiro, E.R., *to appear*.

general remarks on numerics:

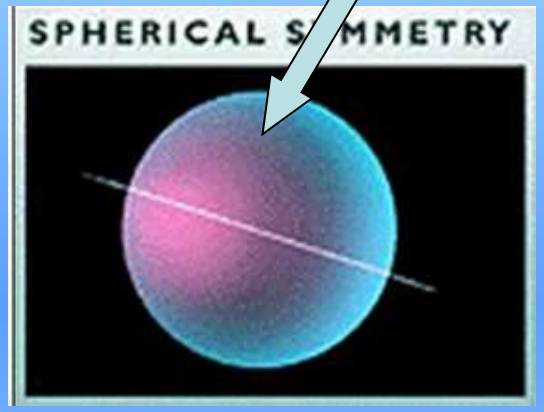
- ***the solutions we report are found by solving numerically the field equations with boundary conditions***
  (no evolution/time dependence)
- ***the problem results in a set of elliptic PDEs + constraints***
- ***the physical properties are derived from the numerical output***
- ***balance (****i.e no conical singularities****) is achieved for a critical set of input parameters***

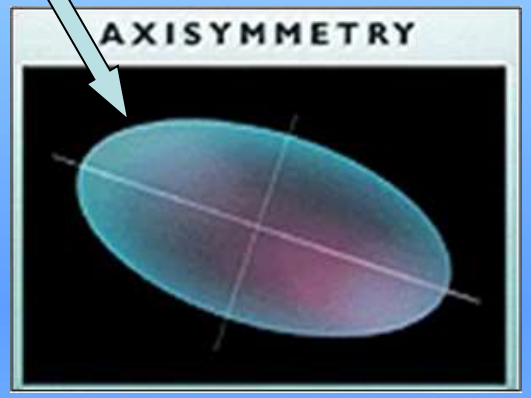
gravity + electromagnetism

Schwarzschild/Kerr Black Hole

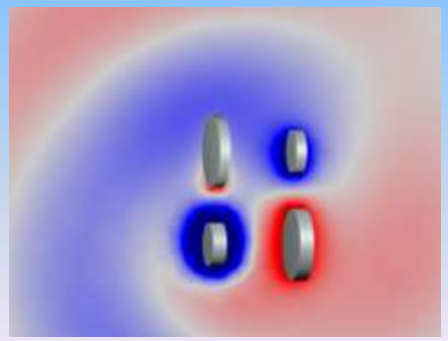
add electric charge **Q**

Reissner-Nordström/Kerr-Newmann Black Hole





Q: are there more general electro-vacuum BHs?



and:

What abour the other matter fields ?

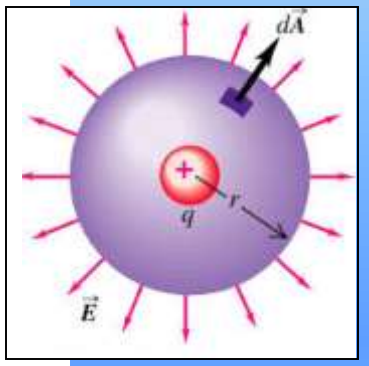
uniqueness theorems for Black Hole

UNIQUENESS OF KERR'S SOLUTION

- Kerr's solution describes *all* black holes without electric charge
- More generally,

  "BLACK HOLES HAVE NO HAIR"

- No-hair theorem: All traces of the matter that formed a BH disappear except for:



MASS

ANGULAR MOMENTUM

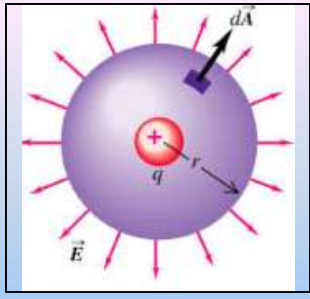
CHARGE

not important In astrophysics

*"**hair**" is a metaphor for any messy/ complicated details* (other fields, multipoles etc)

{ however, ***no general proof*** }

{ uniqueness and no-hair theorems: ***case by case study*** }



this seminar:

- **the case of a *scalar field***
- ***a no-hair* theorem**
- **(single) BHs with scalar hair**
- **more general case: two Black Holes with scalar hair**

(the issue of conical singularities)

a (classic) no-hair theorem: (J. Bekenstein 1972)

no (static) scalar field around a Black Hole

Klein-Gordon equation

$$\nabla^2\phi = \frac{\partial V(\phi)}{\partial \phi} \quad \Leftrightarrow \quad \phi\nabla^2\phi = \phi\frac{\partial V(\phi)}{\partial \phi}$$

Identity:

$$\phi\nabla^2\phi = \nabla(\phi\nabla\phi) - (\nabla\phi)^2$$

a no-hair theorem:

$$\nabla^2\phi = \frac{\partial V(\phi)}{\partial \phi}$$

$$\Leftrightarrow$$

$$\phi\nabla^2\phi = \phi\frac{\partial V(\phi)}{\partial \phi}$$

Identity:

$$\phi\nabla^2\phi = \nabla(\phi\nabla\phi) - (\nabla\phi)^2$$

$$\nabla(\phi\nabla\phi) - (\nabla\phi)^2 = \phi\frac{\partial V(\phi)}{\partial \phi}$$

$$\Leftrightarrow$$

$$\nabla(\phi\nabla\phi) = (\nabla\phi)^2 + \phi\frac{\partial V(\phi)}{\partial \phi}$$

a no-hair theorem:

$$\nabla^2\phi = \frac{\partial V(\phi)}{\partial\phi} \quad \Longleftrightarrow \quad \phi\nabla^2\phi = \phi\frac{\partial V(\phi)}{\partial\phi}$$

$$\int d^3x\,\sqrt{-g}\,\nabla(\phi\nabla\phi) = \int d^3x\,\sqrt{-g}\left[(\nabla\phi)^2 + \phi\frac{\partial V(\phi)}{\partial\phi}\right]$$

$$\underbrace{\oint_\infty (\phi\nabla\phi)}_{=0} - \underbrace{\oint_H (\phi\nabla\phi)}_{=0}$$

$$\int d^3x\,\sqrt{-g}\left[(\nabla\phi)^2 + \phi\frac{\partial V(\phi)}{\partial\phi}\right] = 0$$

≥ 0

e.g. $V(\phi) = \frac{1}{2}m^2\phi^2$

$$\int d^3x\,\sqrt{-g}\left[(\nabla\phi)^2 + m^2\phi^2\right] = 0$$

$\phi=0$

Q.E.D.

Black holes with scalar hair:
simplest case (also used here)

$$\int d^3x\,\sqrt{-g}\left[(\nabla\phi)^2+\phi\frac{\partial V(\phi)}{\partial\phi}\right]=0$$

Bekenstein no-hair theorem:

a key assumption:-- the potential V obeys: Φ(dV/dΦ)≥ 0 everywhere,

violation:

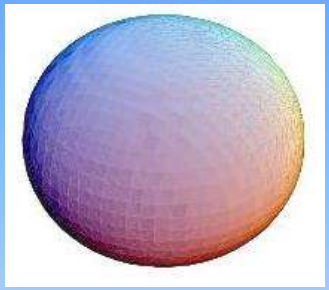
$$\Phi V'<0$$

- *the system may possess particle-like solutions*

  even in a flat space background

- *it requires violation of the energy conditions*

- *all known solutions are unstable*



single black hole

1504.08209 [gr-qc]

"probe limit"

scalar field in Schwarzschild BH background

(Klein-Gordon equation only)

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ .$$

simplest (?) example:
(Coulomb field)

$$\Phi(r) = -\frac{Q_S}{r}$$

$$V(\Phi) = -\lambda\Phi^5 < 0$$

(no backreaction)

$$Q_S = -\left(\frac{2M}{5\lambda}\right)^{1/3} < 0$$

$$(T^S)^t_t = \frac{14M - 5r}{2^{1/3}5^{5/3}r^5}\left(\frac{M}{\lambda}\right)^{2/3}, \qquad E = \frac{3\pi}{2^{1/3}5^{5/3}}(M\lambda^2)^{-1/3}$$

including the backreaction:

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)\right)$$

a convenient metric ansatz:

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

$$N(r) \equiv 1 - \frac{2m(r)}{r}$$

$$S_{eff} = \int_{r_H}^{\infty} dr\ \sigma(r)\left[m' - \left(\frac{1}{2}Nr^2\Phi'^2 + r^2V(\Phi)\right)\right]$$

condition $\Phi V' < 0$ ⇒ Black Holes with scalar hair

- many examples
- unstable configurations

…even exact solutions

an example:

JHEP 04 (2022) 096 - 2107.05656

C. Herdeiro+ER: 1504.08209

the solution:

$$\Phi(r) = -\frac{Q_S}{r}$$

scalar field potential compatible with it:

$$V(\Phi) = -\frac{15}{2}\lambda e^{\frac{\Phi^2}{2}} W + \frac{1}{2Q_S^2}\left((1+2e^{\Phi^2})(\Phi^2-3) + \frac{W^2-27}{\Phi^2-3}\right)$$

$$\Phi V' < 0$$

$$W \equiv 3\Phi + \sqrt{\frac{\pi}{2}} e^{\frac{\Phi^2}{2}} (\Phi^2-3)\mathrm{Erf}(\frac{\Phi}{\sqrt{2}})$$

not natural:

mass function:

$$m(r) = \frac{r^3}{Q_S^2}\left[1 + \frac{Q_S^2}{r^2} + e^{\frac{Q_S^2}{r^2}}(\frac{225}{8}\lambda^2 Q_S^4 - 1)\right.$$
$$\left. - \frac{1}{2}e^{\frac{Q_S^2}{r^2}}\left(\frac{15}{2}\lambda Q_S^2 - \frac{Q_S}{r}e^{-\frac{Q_S^2}{2r^2}} + \sqrt{\frac{\pi}{2}}\mathrm{Erf}(\frac{Q_S}{\sqrt{2}r})\right)^2\right],$$

with $\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t.$

extra metric function:

$$\sigma(r) = e^{-\frac{Q_S^2}{2r^2}}$$

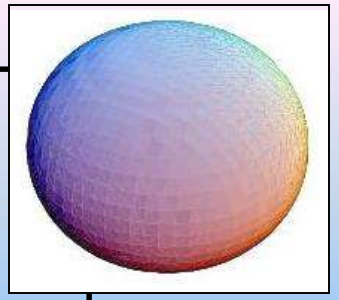
etc...

- large literature on **single Black Holes** with scalar hair

(various directions+interesting solutions)

- what about multi-Black Holes with scalar hair?

((it seems) no work in the literature)

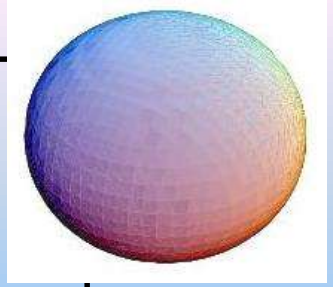


- large literature on **single Black Holes** with scalar hair

(various directions+interesting solutions)

- what about multi-Black Holes?

(no work in the literature)

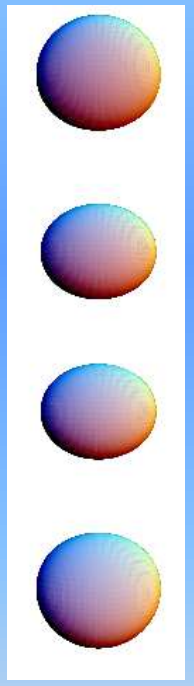


- vacuum: the Israel-Khan solution:

N-coliniar Schwarzschild BHs

- electro-vacuum:

Majumdar-Papapetrou BHs (extremal case—special; not included here)



- all known static, non-extremal solutions contain conical singularities

(asymptotic flatness)

the issue of conical singularities

radial variable *azimuthal angle* *period* $\Delta\varphi$

$$d\sigma^2 = g_{\rho\rho}d\rho^2 + g_{\varphi\varphi}d\varphi^2$$

C: proper circumference

$$\alpha = \left.\frac{dC}{dR}\right|_{R=0} = \lim_{\rho\to 0}\frac{\sqrt{g_{\varphi\varphi}}\Delta\varphi}{\int_0^\rho \sqrt{g_{\rho\rho}}d\rho} = \lim_{\rho\to 0}\frac{\partial_\rho\sqrt{g_{\varphi\varphi}}\Delta\varphi}{\sqrt{g_{\rho\rho}}}$$

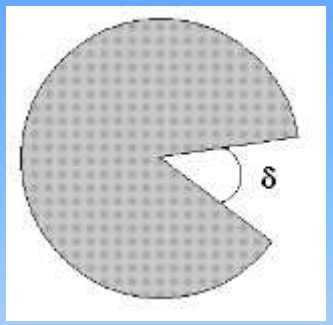
R: proper radius

the presence of a conical singularity
is expressed by means of

$$\delta = 2\pi - \alpha$$

δ>0: deficit

δ<0: excess



vacuum: the **Israel-Khan** solution (1964):
N-coliniar Schwarzschild BHs

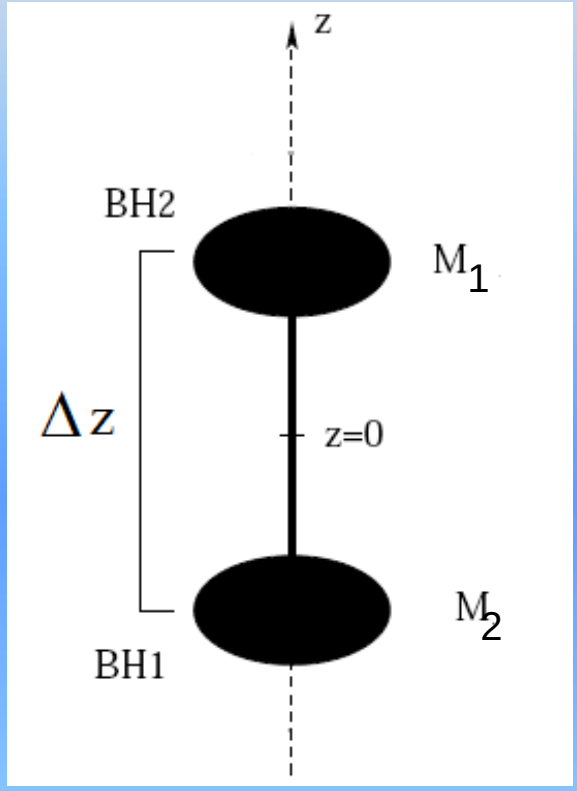
N=2: the **Bach-Weyl** system (1922)

between the BHs (**strut**) $\delta<0$

it provides the extra-force to prevent the collapse

the conical singularity

from the horizons to infinity (**string**) $\delta>0$



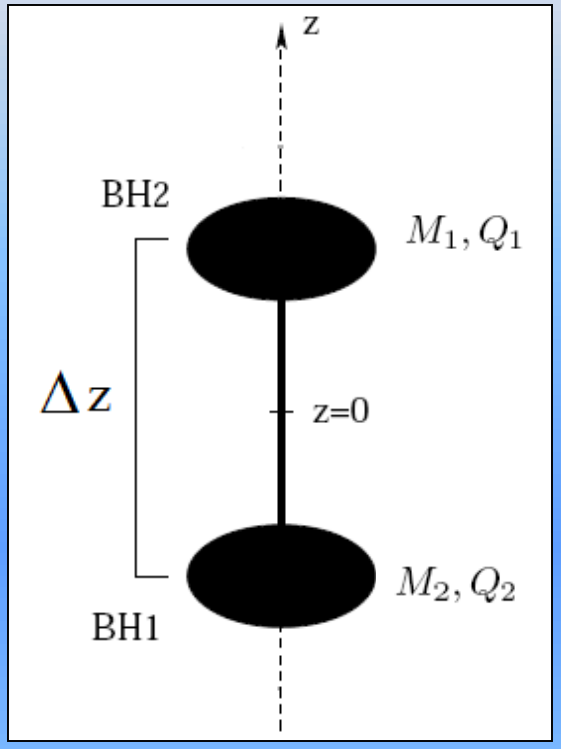
'effective stress energy tensor' associated with the singularity.

extra-source $T_i^j = -\delta_i^j \frac{1}{8\pi}(2\pi - \alpha)\delta_\Sigma$ $(i,j) \neq (\rho,\varphi)$

the presence of conical singularities: ***undesirable feature***

the presence of conical singularities: ***undesirable feature***

the same picture is found for (generic) 2Reissner-Nordstrom Black Holes



however, a consistent thermodynamical description is still possible: $S = A_H/4$

(C. Herderio, E. Radu +.. - arXiv:0912.3386)

Q: what about other matter fields?

Bach-Weyl BHs (2Scwharzschild) with scalar hair: balanced solutions (no conical singularities)

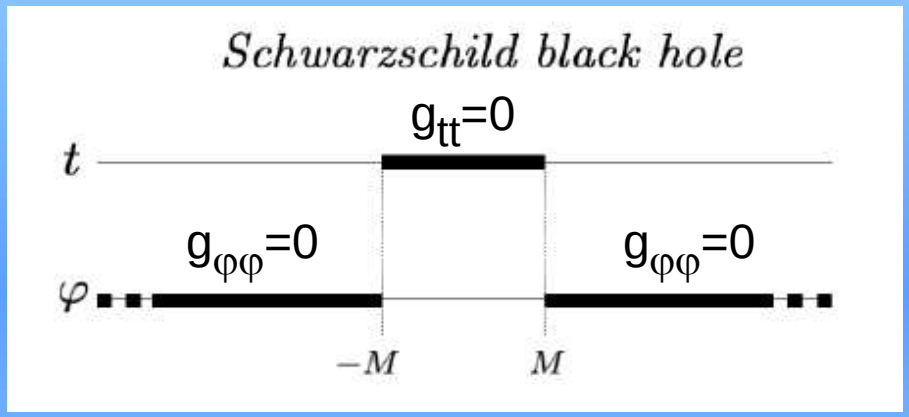
crucial (technical) point:

the use of Weyl-type coordinates+rod structure:

(the z-axis is divided into 3-intervals (called rods of the solution)

$$ds^2 = f_1(\rho,z)(d\rho^2+dz^2) + f_2(\rho,z)d\varphi^2 - f_0(\rho,z)dt^2$$

$$R_{\mu\nu} = 0$$



vacuum

$$f_0(\rho,z) = \frac{r_+ + r_- - 2M}{r_+ + r_- + 2M}$$

$$f_1(\rho,z) = \frac{(r_+ + r_-)^2 - (2M)^2}{4r_+r_-}$$

$$f_2(\rho,z) = \frac{\rho^2}{f_0(\rho,z)}$$

$$r_\pm = \sqrt{\rho^2 + (z \pm M)^2}$$

$$\rho = \sqrt{r^2 - 2Mr}\sin\theta$$
$$z = (r-M)\cos\theta$$

$$ds^2 = \frac{dr^2}{1-\frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - (1-\frac{2M}{r})dt^2$$

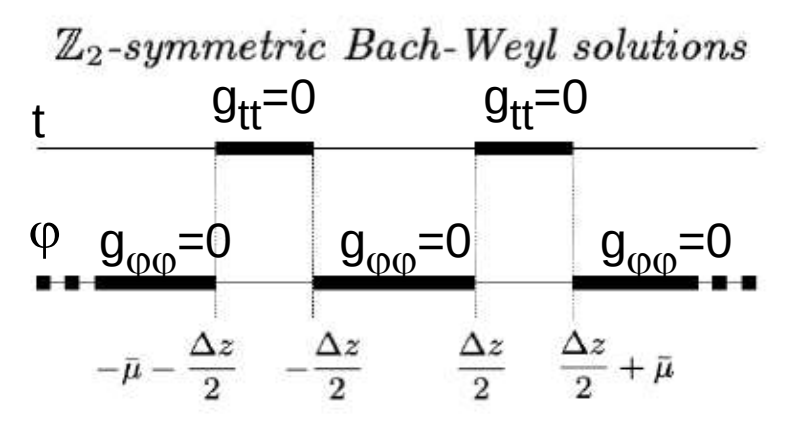
crucial (technical) point:

the use of Weyl-type coordinates+rod structure:

(the z-axis is divided into 5-intervals (called rods of the solution)

$$ds^2 = f_1(\rho,z)(d\rho^2+dz^2) + f_2(\rho,z)d\varphi^2 - f_0(\rho,z)dt^2$$

$$R_{\mu\nu}=0$$



two identical BHs

vacuum

$$f_0(\rho,z) = \frac{(r_1+r_2-\bar{\mu})}{(r_1+r_2+\bar{\mu})}\frac{(r_3+r_4-\bar{\mu})}{(r_3+r_4+\bar{\mu})}$$

$$f_2(\rho,z) = \frac{\rho^2}{f_0(\rho,z)}$$

$$r_1=\sqrt{\rho^2+\left(z-\frac{\Delta z}{2}-\bar{\mu}\right)^2},\quad r_2=\sqrt{\rho^2+\left(z-\frac{\Delta z}{2}\right)^2}$$

$$r_3=\sqrt{\rho^2+\left(z+\frac{\Delta z}{2}\right)^2},\quad r_4=\sqrt{\rho^2+\left(z+\frac{\Delta z}{2}+\bar{\mu}\right)^2}$$

$$f_1(\rho,z)=\left(\frac{\Delta z}{\Delta z+\bar{\mu}}\right)^2\left(\frac{(r_1+r_2)^2-\bar{\mu}^2}{4r_1r_2}\right)\left(\frac{(r_3+r_4)^2-\bar{\mu}^2}{4r_3r_4}\right)\left(\frac{(\Delta z+\bar{\mu})r_1+(\Delta z+2\bar{\mu})r_2-\bar{\mu}r_4}{\Delta z\ r_1+(\Delta z+\bar{\mu})r_2-\bar{\mu}r_3}\right)^2$$

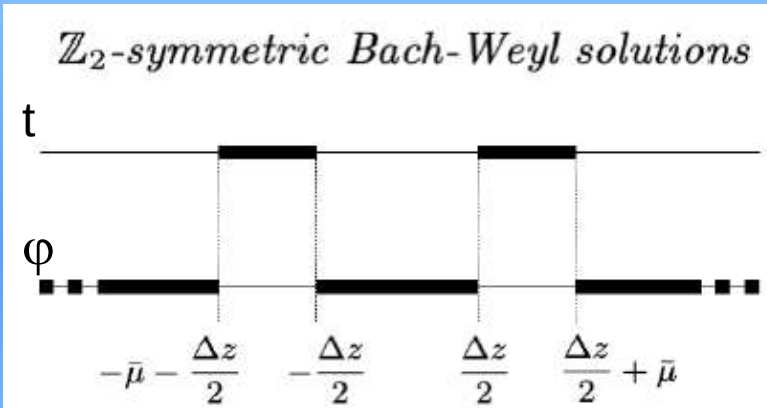
crucial (technical) point:

the use of Weyl-type coordinates+rod structure:

(the z-axis is divided into 5-intervals (called rods of the solution)

$$ds^2 = f_1(\rho,z)(d\rho^2+dz^2) + f_2(\rho,z)d\varphi^2 - f_0(\rho,z)dt^2$$

$$R_{\mu\nu} = 0$$



two identical BHs

$$M = \bar{\mu} > 0$$

$$A_H = 4\pi\bar{\mu}^2\frac{\Delta z+2\bar{\mu}}{\Delta z+\bar{\mu}}$$

$$\frac{\delta}{2\pi} = -\frac{\bar{\mu}^2}{(\Delta z+2\bar{\mu})\Delta z} < 0$$

$$T_H = \frac{1}{4\pi\bar{\mu}}\frac{\Delta z+\bar{\mu}}{\Delta z+2\bar{\mu}}$$

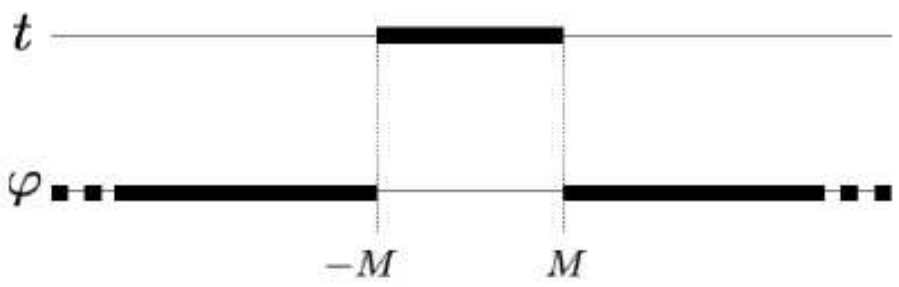
what about scalar hair?

crucial (technical) point:
the use of Weyl-type coordinates + rod structure:
(the z-axis is divided into 3-intervals (called rods of the solution)

$$ds^2 = f_1(\rho, z)(d\rho^2 + dz^2) + f_2(\rho, z)d\varphi^2 - f_0(\rho, z)dt^2$$

just a test of the method

Schwarzschild black hole with scalar hair



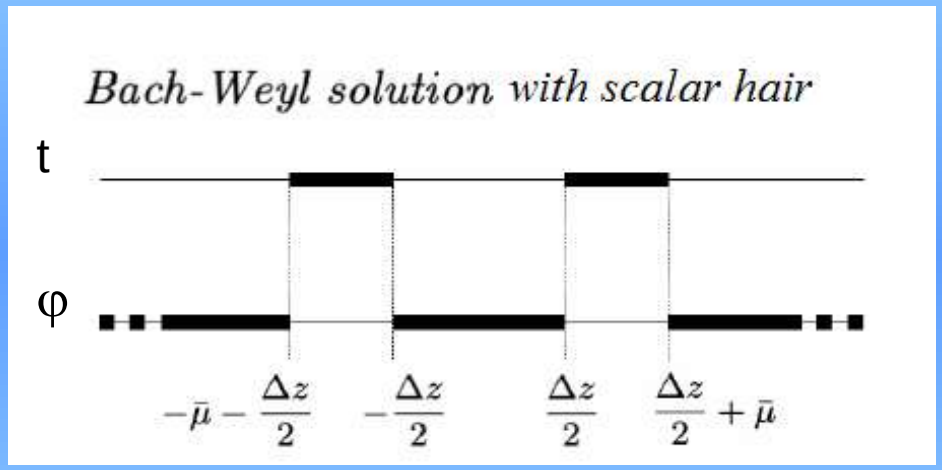
- single, non-vacuum Black Holes can be expressed in these coordinates
- we **recover** Schwarzschild Black Hole with scalar hair
- technically much more complicated

$\Phi V' < 0$

(solve a boundary value problem (set of four PDEs))

crucial (technical) point:
the use of Weyl-type coordinates+rod structure:
(the z-axis is divided into 5-intervals (called rods of the solution)

$$ds^2 = f_1(\rho,z)(d\rho^2+dz^2) + f_2(\rho,z)d\varphi^2 - f_0(\rho,z)dt^2$$



two identical BHs

$$\Phi V' < 0$$

we propose a general framework

- preserve the rod structure of the vacuum Back-Weyl solution
- work with *'deformed'* functions F_i

$$f_i = f_i^{(0)} e^{2F_i}$$

- solve a boundary value problem for F_i (set of four PDEs)

we propose a general framework

concrete model

$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left[\frac{R}{4G} - \partial_\alpha\Phi\partial^\alpha\Phi - U(\Phi)\right]$$

our choice

$$U = \mu^2\Phi^2 - \lambda\Phi^4 + \nu\Phi^6$$

real scalar field (odd parity)

$$\Phi = \phi(\rho, z)$$

same mechanism as for a single BH

$$\Phi dU/d\Phi < 0$$

generic expansion close to the z-axis:

$$f_i(\rho, z) = f_{i0}(z) + \rho^2 f_{i2}(z) + \dots$$

$$\phi(\rho, z) = \phi_0(z) + \rho^2\phi_2(z) + \dots$$

conical deficit/excess:

$$\delta = 2\pi\left(1 - \lim_{\rho\to 0}\sqrt{\frac{f_2(\rho, z)}{\rho^2 f_1(\rho, z)}}\right)$$

mass:

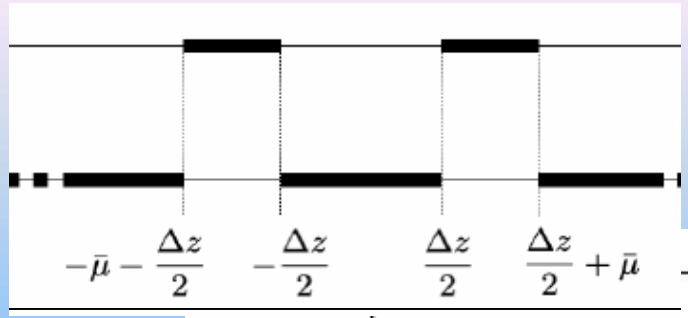
$$-g_{tt} = f_0 \sim 1 - \frac{2GM}{\sqrt{\rho^2 + z^2}} + \dots$$

horizon area:

$$A_H = 2\pi\int_{z_{H_1}}^{z_{H_2}} dz\sqrt{f_1(0, z)f_2(0, z)}$$

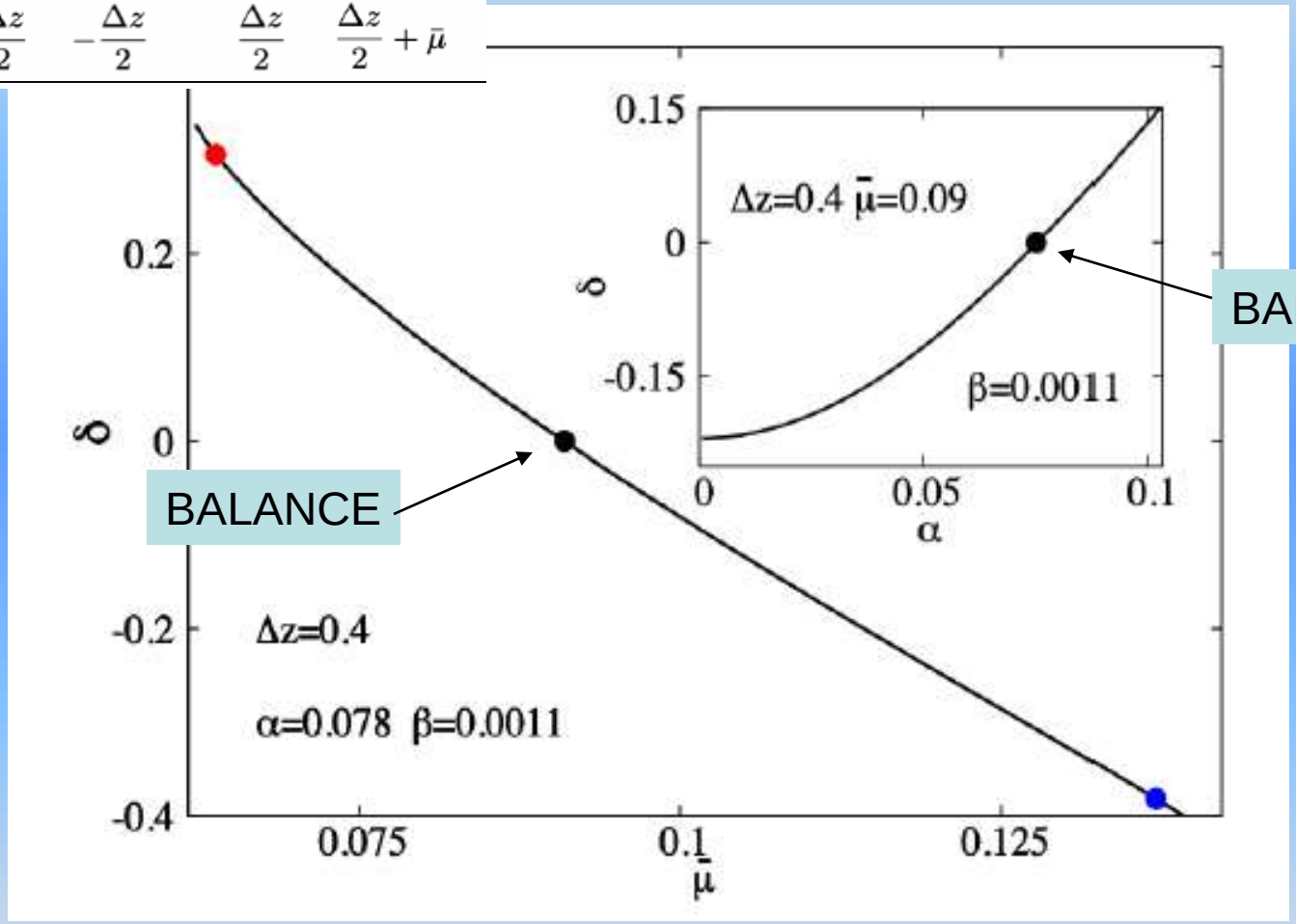
Hawking temperature:

$$T_H = \frac{1}{2\pi}\lim_{\rho\to 0}\sqrt{\frac{f_0(\rho, z)}{\rho^2 f_1(\rho, z)}}$$

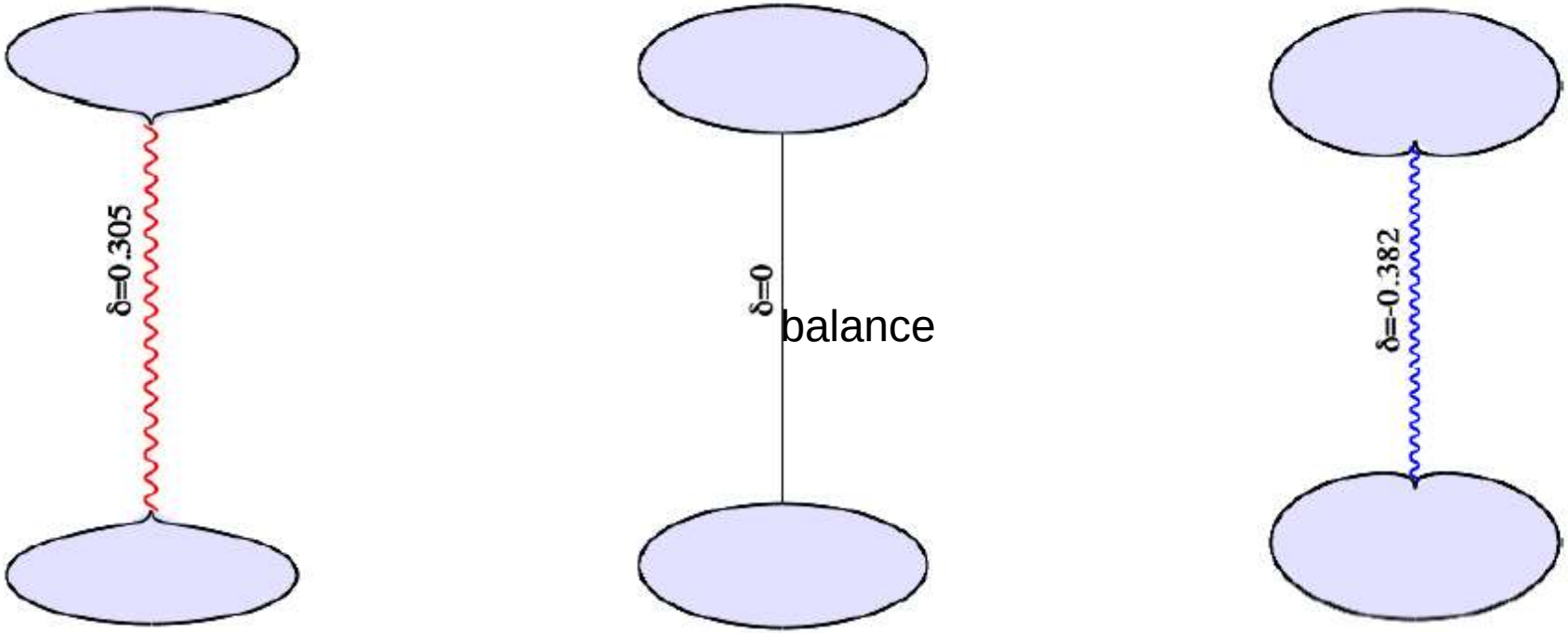


$$\alpha^2 = \frac{G\mu^2}{\lambda}$$

$$\beta^2 = \frac{\nu\mu^2}{\lambda}$$

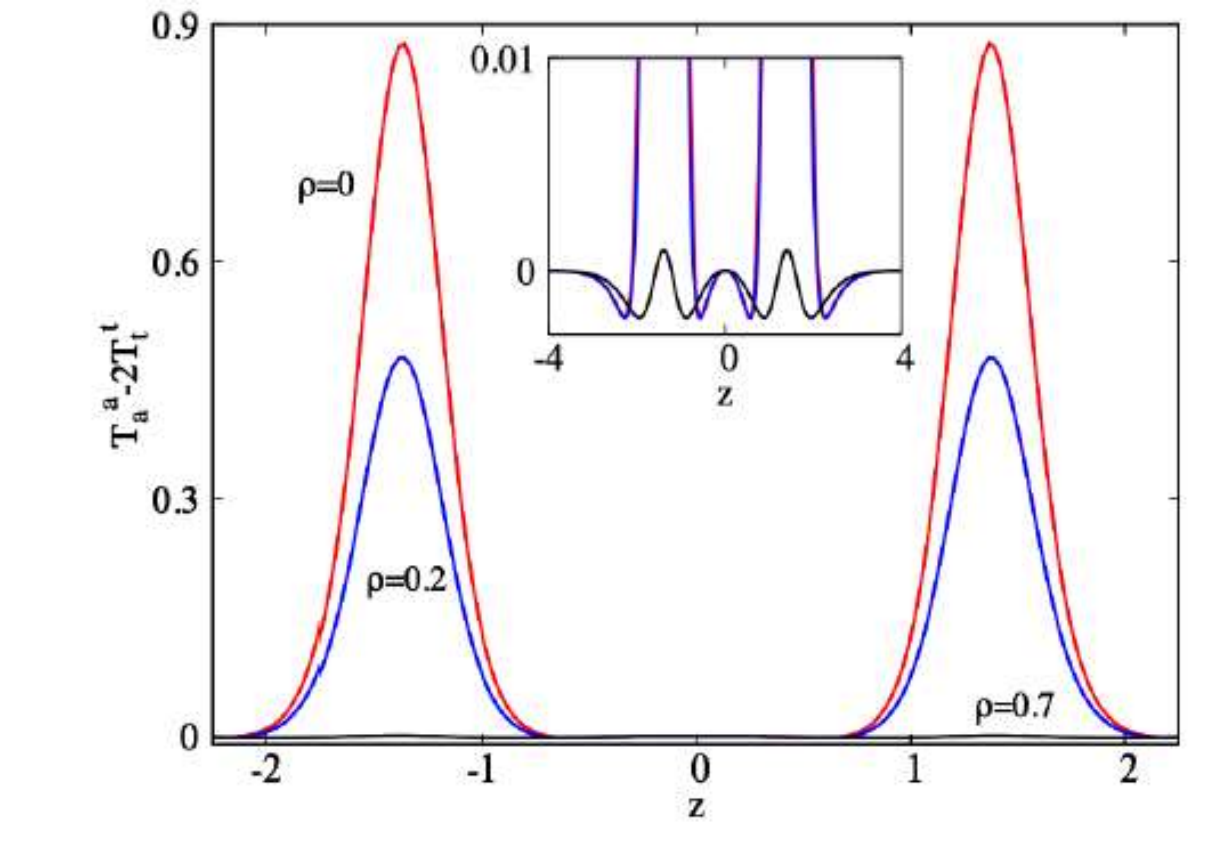


the conical excess/deficit δ s shown as a function of the horizon size (input parameter) and .the strength of gravity α (inset)..

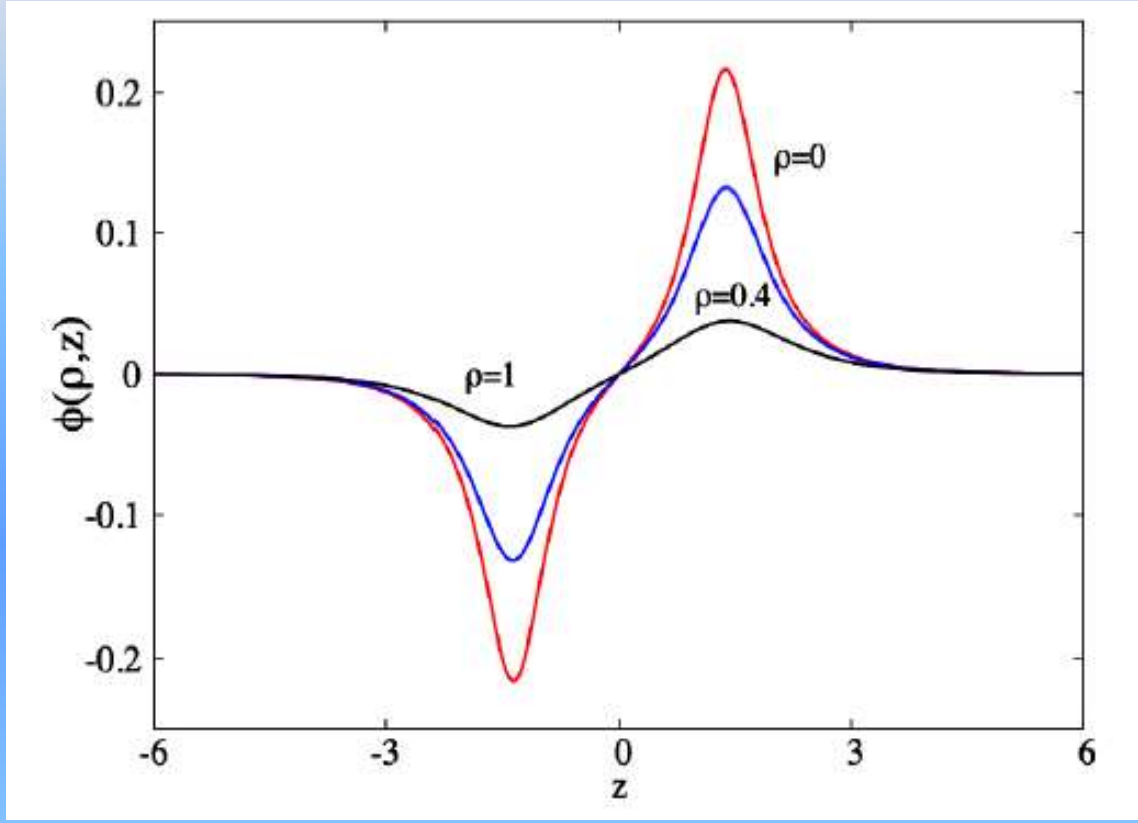


the *effective horizon shape* is shown for different 2BH solutions with scalar hair

(we use the approach in Costa and Perry, arXiv:hep-th/0008106
- not the usual embedding diagram*)*



the Komar mass-energy density Is shown for a typical balanced 2BHs system with scalar hair



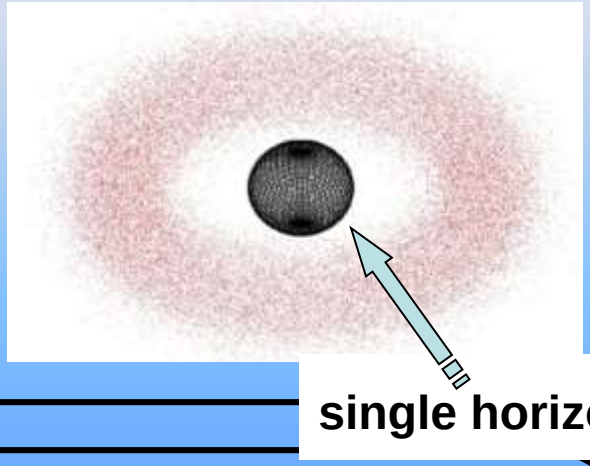
the scalar field profile is for a typical balanced 2BHs system with scalar hair

the problem we address:

- the ***no hair conjecture*** can be violated

- Schwarschild/Kerr Black Hole + scalar hair:

many interesting solutions



Q1: what about multi-Black Holes?

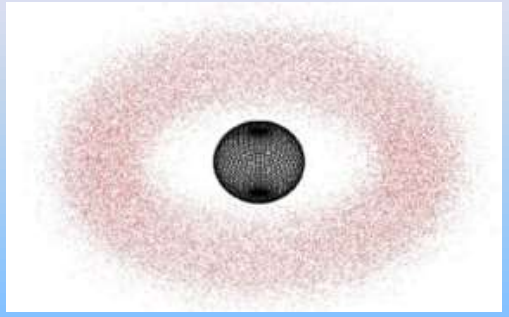
- **electro-vacuum:** N Black Holes *(exact solutions)*
- simplest case N=2: the Bach-Weyl vacuum solution (2Schwarzschild)
- generic presence of *conical singularities*

the scalar hair can balance the 2BH system

the problem we address:

- the ***no hair conjecture*** can be violated
- Schwarschild/Kerr Black Hole + scalar hair:

many interesting solutions



Q1: what about multi-Black Holes?

- electro-vacuum: N-Black Holes *(exact solutions)*
- simplest case N=2: the Bach-Weyl vacuum solution (2Schwarzschild)
- generic presence of *conical singularities*

the scalar hair can balance the system

Q2: what about higher dimensions?

D=5: the static Emparan-Reall vacuum Black Ring

- it possesses conical singularities

static, regular Black Rings with scalar hair

4**d** *vs.* higher **d** Black Holes

(asymptotically Minkowski solutions only!)

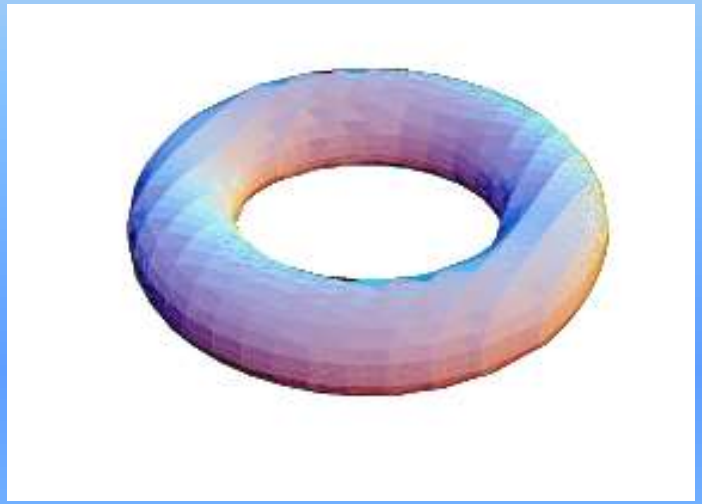
BHs in four dimensions

- the horizon: a sphere S^2
- Kerr-like metrics

BHs in d>4 dimensions?

d>4: the novel feature – non-uniqueness of black objects

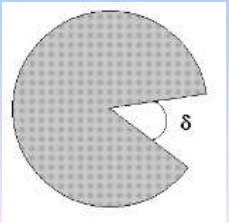
- much richer landscape
- extended black objects: (*black strings/branes*)



BLACK RINGS

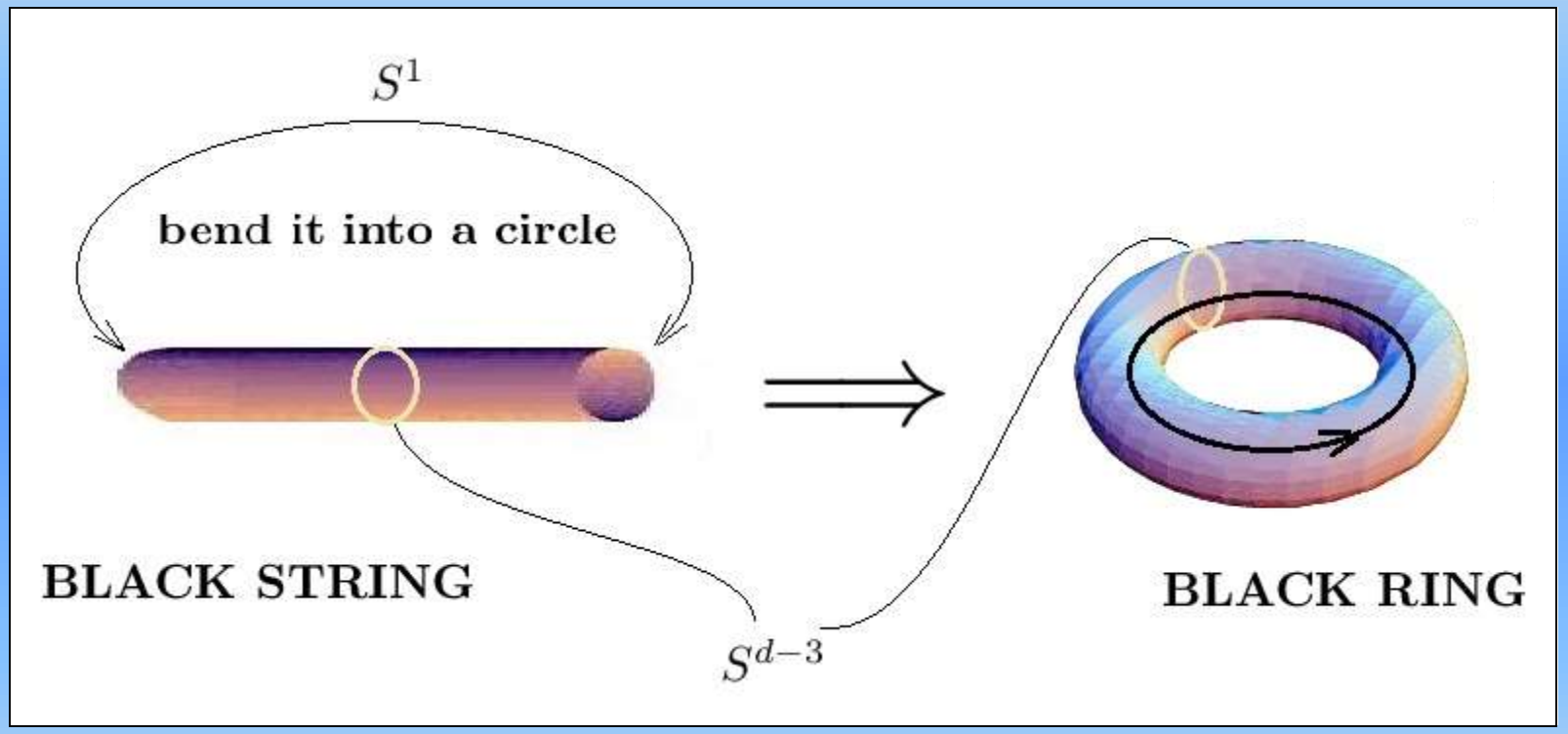
new types of solutions

the presence of conical singularities



heuristic construction of a black ring

Schwarzschild black hole in (d-1)-dimensions: black string in d-dimensions



five dimensions

• there's an explicit realisation, and fairly simple *(Emparan-Reall 2002)*

d>5

• approximate solution based on this heuristic construction (large ring radius)

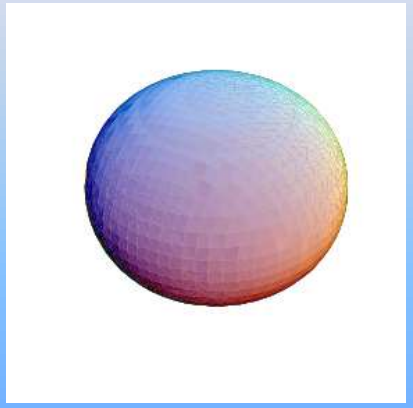
• numerical (nonperturbative) solutions

blackfolds

not really new in physics…

Field theory solutions in flat space

(stationary; four dimensions, no quantization)



• solitons (*3+1 dimensions; finite energy, localized, regular, particle-like configurations*)

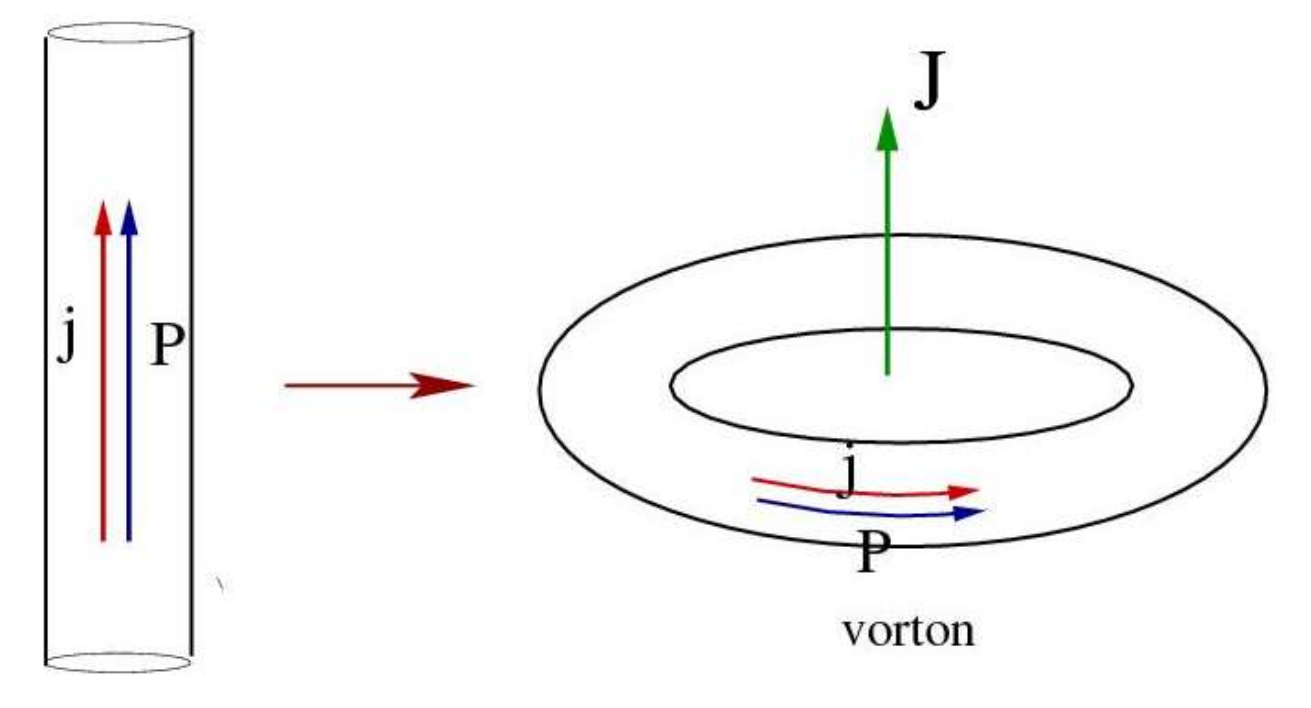
• vortex/string solutions (*2+1+1 dimensions; infinite extend; finite energy per unit length*)

examples:

vortices in Abelian-Higgs theory
vortices and Q-balls in Klein-Gordon theory
sphalerons in standard model
monopoles in Georgi–Glashow model

Vortex+soli***ton*** = ***vorton***

Vortons in field theory: *heuristic construction*



- ***applications:*** *astrophysics, condensed matter, nuclear physics (Skyrme model)*
- ***internal structure of particles:*** *- knots (old idea – lord Kelvin)*

- ***general formalism (large radius): before black rings*** **– B. Carter (1990's)**
- ***numerical solutions:*** **J. Garaud, E. Radu, M. Volkov (2013)**

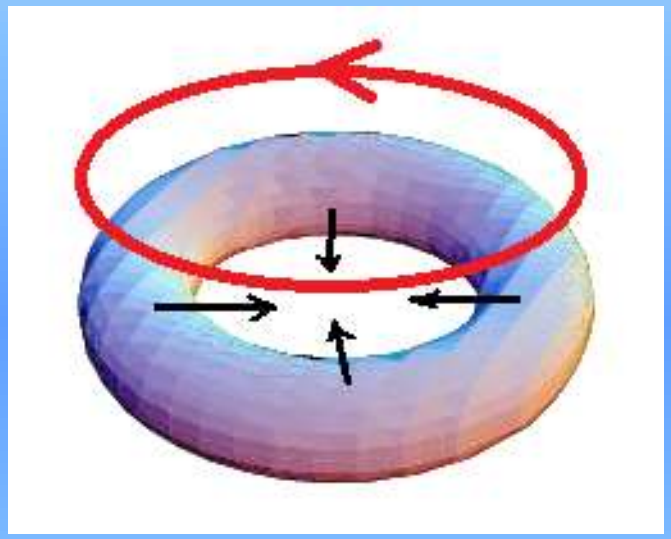
Black Rings without conical singularities?

how to balance a configuration with non-standard horizon topology ?

simplest solution: add rotation!

$$R_{\mu\nu} = 0$$

Emparan-Reall
exact solution (2002)



- *active area of research*
- *many interesting results*

Static Black Rings in more general models?

- ***EMd theory:*** *still conical singularities* (Kunduri and Lucietti, 2005)
- ***EGB theory:*** *the Gauss-Bonnet term reduces the conical excess, but not enough* (Kleihaus, Kunz, Radu, 2012)

here: ***Balanced Black Rings in Einstein-gauged scalar field model***

the Einstein-gauged scalar field model

$$S = \int d^D x \sqrt{-g} \left(\frac{R}{16\pi G} - \partial_\alpha \Psi^* \partial^\alpha \Psi - U(|\Psi|) \right)$$

local U(1) invariance:

$$\Psi \to \Psi e^{-i\chi(x^\alpha)}$$

the Einstein-Maxwell-scalar model

$$S = \int d^D x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - D_\alpha \Psi^* D^\alpha \Psi - U(|\Psi|) \right]$$

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

$$D_\alpha \Psi \equiv \partial_\alpha \Psi + iq A_\alpha \Psi$$

gauge coupling constant

- **more realistic**

(although not a Higgs field)

a different mechanism

$\Psi dU/d|\Psi| > 0$

the Einstein-scalar model

$$S = \int d^D x \sqrt{-g} \left(\frac{R}{16\pi G} - \partial_\alpha \Psi^* \partial^\alpha \Psi - U(|\Psi|) \right)$$

spherically symmetric sector:

Boson Stars

no black holes

(Pena -Sudarsly theorem)

---

the Einstein-Maxwell-scalar model

$$S = \int d^D x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - D_\alpha \Psi^* D^\alpha \Psi - U(|\Psi|) \right]$$

spherically symmetric sector:

gauged Boson Stars

no black holes

(Mayo-Bekenstein theorem)

the Einstein-Maxwell-scalar model

a different mechanism

$$\Psi dU/d|\Psi| > 0$$

$$S = \int d^D x \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - D_\alpha \Psi^* D^\alpha \Psi - U(|\Psi|)\right]$$

spherically symmetric sector:

gauged boson stars

no black holes

(Mayo-Bekenstein theorem)

however, loophole!

static, spherically symmetric black holes

D=4 result 2020

$$\Psi \sim e^{-iwt}$$

'resonance' condition

$$w = q\Phi_H$$

$$U(\psi) = \mu^2\psi^2 - \lambda\psi^4 + \nu\psi^6$$

- **nonlinearities are crucial**
- **one solves the full EMs problem** *(electric charge)*
- **the solutions are not connected with EM black holes**

D=4

J.-P. Hong, M. Suzuki and M. Yamada, *Phys. Lett. B* **803** (2020) 135324
C.Herdeiro and E. Radu, *Eur. Phys. J. C* **80** (2020) 390

Schwarzschild Black Holes with charged scalar hair

electric charge **Q**

D=5

:

Y. Brihaye, C. Herdeiro and E. Radu, *JHEP* 10 (2022) 153

Schwarzschild-Tangherlini Black Holes, Black Strings and *Black Rings* with charged scalar hair

static Black Rings without conical singularities

Crucial ingredient: **A new coordinate system**

E_D:

$$ds^2 = d\rho^2 + \rho^2(d\Theta^2 + \cos^2\Theta d\Omega_{D-3}^2 + \sin^2\Theta d\psi^2)$$

$$\rho = r\sqrt{U}, \quad \tan\Theta = \left(\frac{r^2+\rho^2+R^2}{r^2+\rho^2-R^2}\right)\tan\theta,$$

R-input parameter

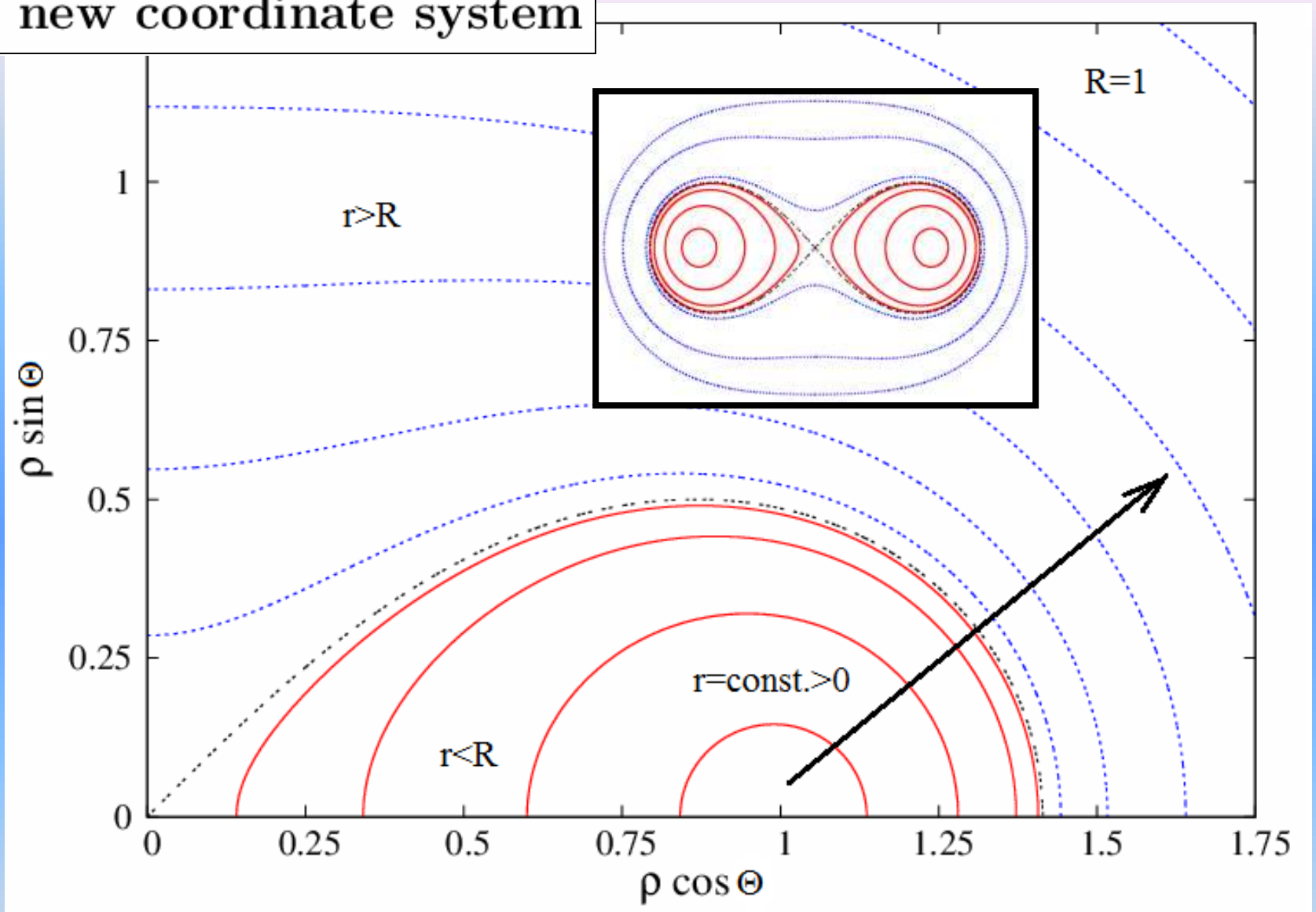
$$ds_D^2 = V_1(dr^2 + r^2 d\theta^2) + V_2 d\Omega_{D-3}^2 + V_3 d\psi^2,$$

$$V_1 = \frac{1}{U}, \quad V_2 = r^2\left(\cos^2\theta - \tfrac{1}{2}(1 + \tfrac{R^2}{r^2} - U)\right), V_3 = r^2\left(\sin^2\theta - \tfrac{1}{2}(1 - \tfrac{R^2}{r^2} - U)\right)$$

$$\text{with } U = \sqrt{1 + \frac{R^4}{r^4} - \frac{2R^2}{r^2}\cos 2\theta}.$$

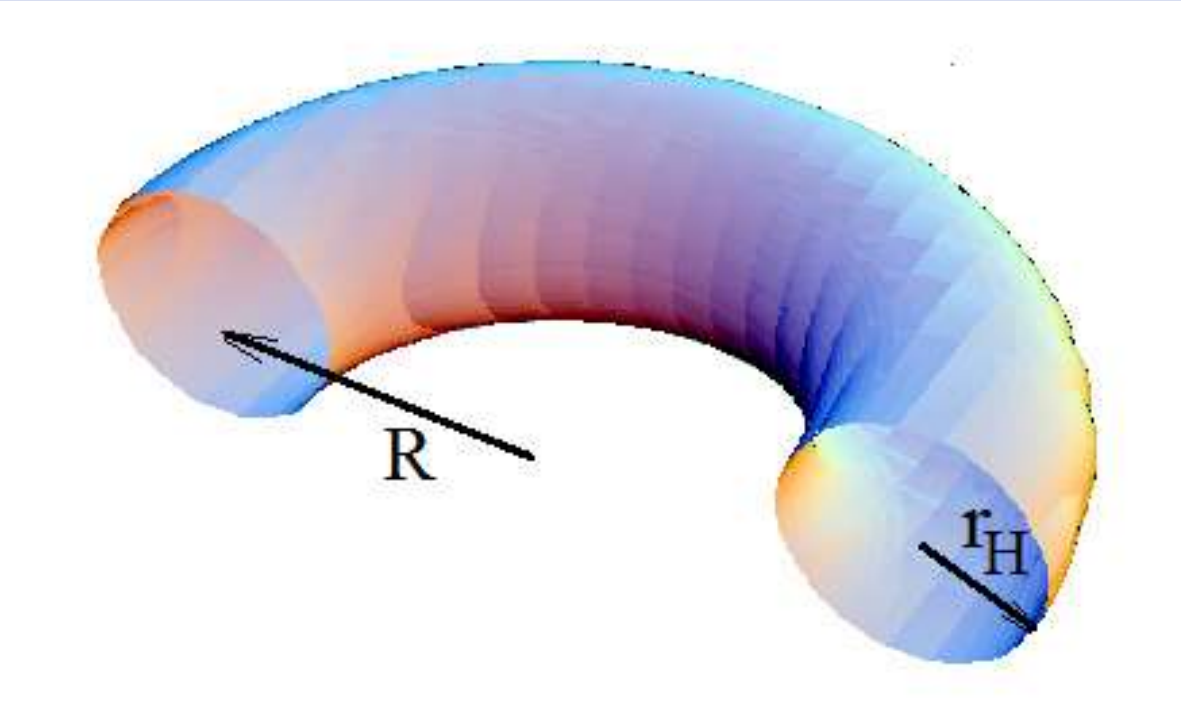
convenient for numerics: rectangular shape $0 \le r < \infty,\ 0 \le \theta \le \pi/2$

A new coordinate system



for $0 < r < R$, a surface of constant r has ring-like topology $S^2 \times S^1$

static *Black Rings* with charged scalar hair



Emparan-Reall static Black Ring

$$f_i = f_i^{(0)} e^{2F_i}$$

metric:

$$ds^2 = -f_0(r,\theta)dt^2 + f_1(r,\theta)(dr^2 + r^2 d\theta^2) + f_2(r,\theta)d\varphi_1^2 + f_3(r,\theta)d\varphi_2^2$$

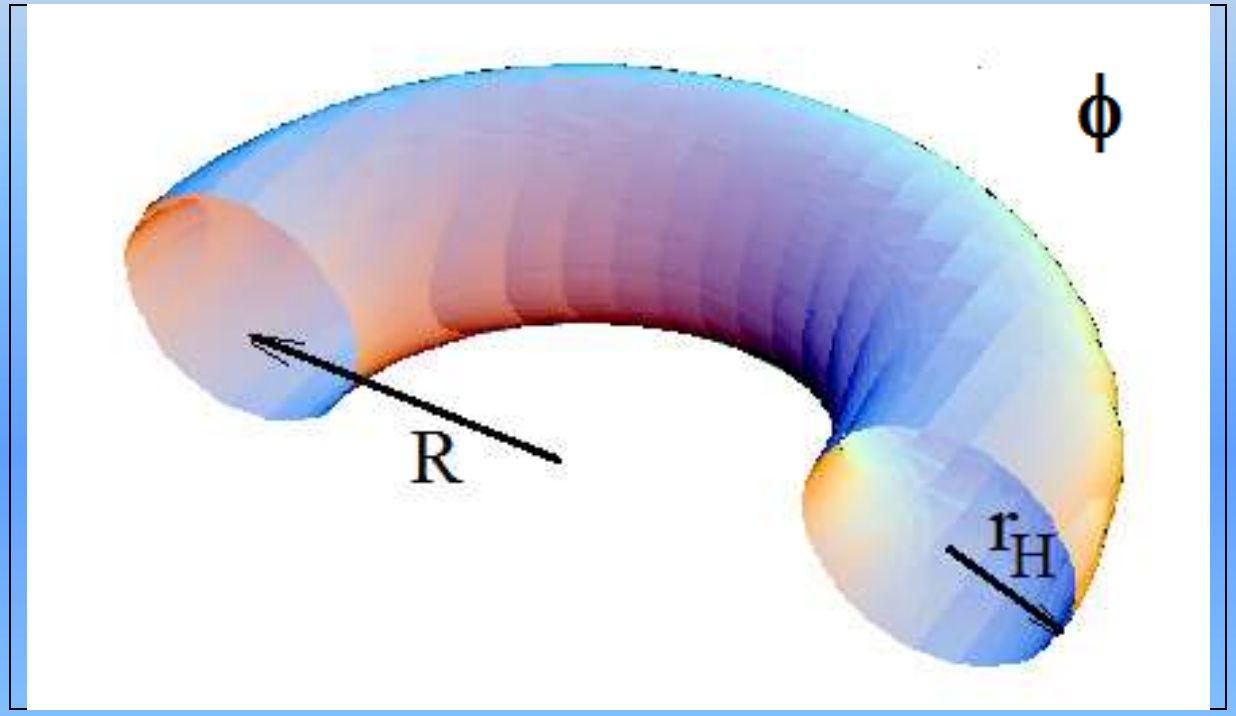
$$0 < r_H \leq r < \infty \qquad 0 \leq \theta \leq \pi/2 \qquad 0 \leq \varphi_{1,2} < 2\pi$$

scalar: $\Psi = \psi(r,\theta)e^{-iwt}$ ***U(1) field***: $A = V(r,\theta)dt$

static *Black Rings* with charged scalar hair

in numerics: **three input parameters:** $R,\ r_H,\ \phi$

+q, scalar potential



generic solutions: conical sing.

very important…

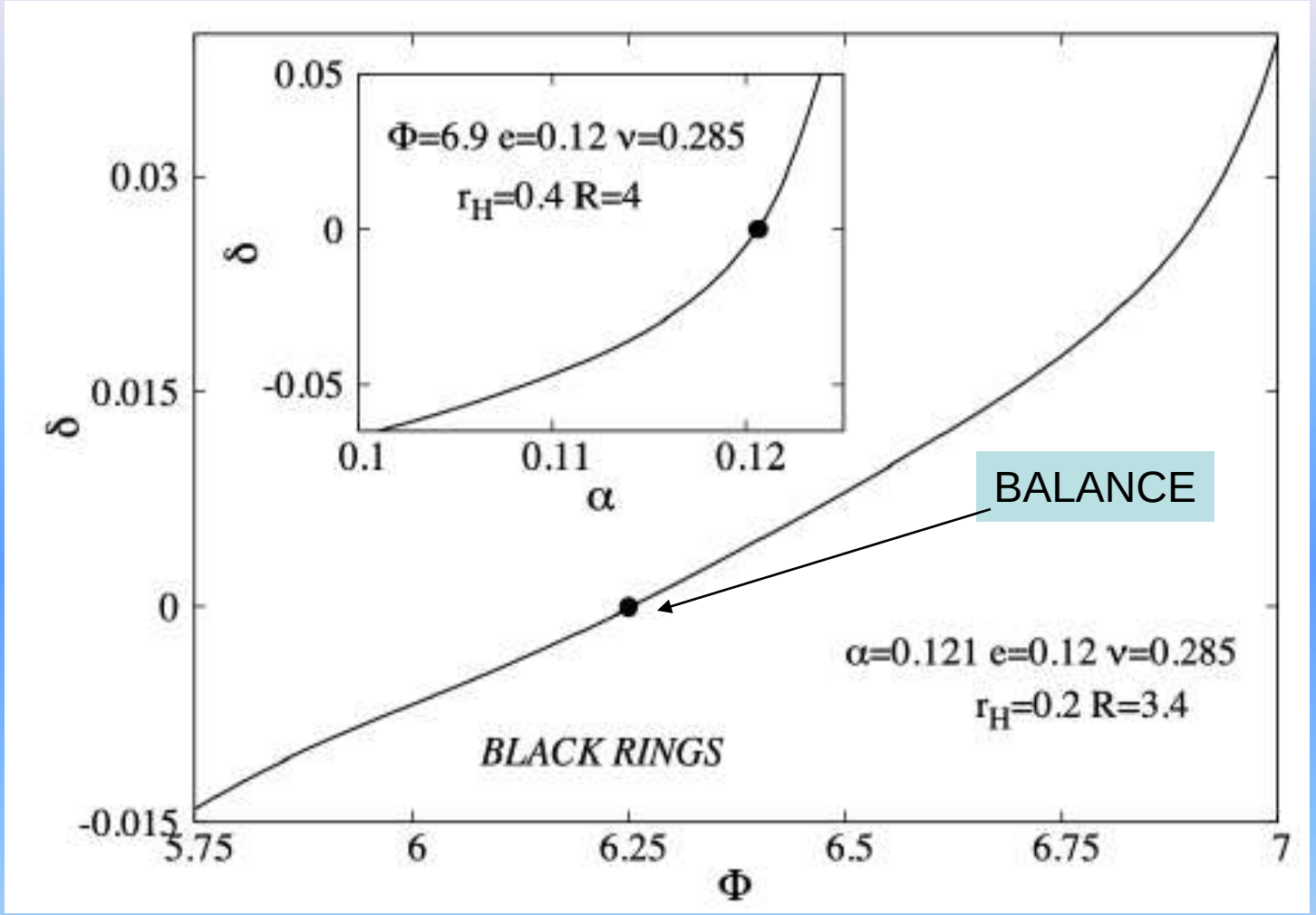
$T_H,\ A_H,\ M,\ Q$ **-computed from numerical output**

in practice:

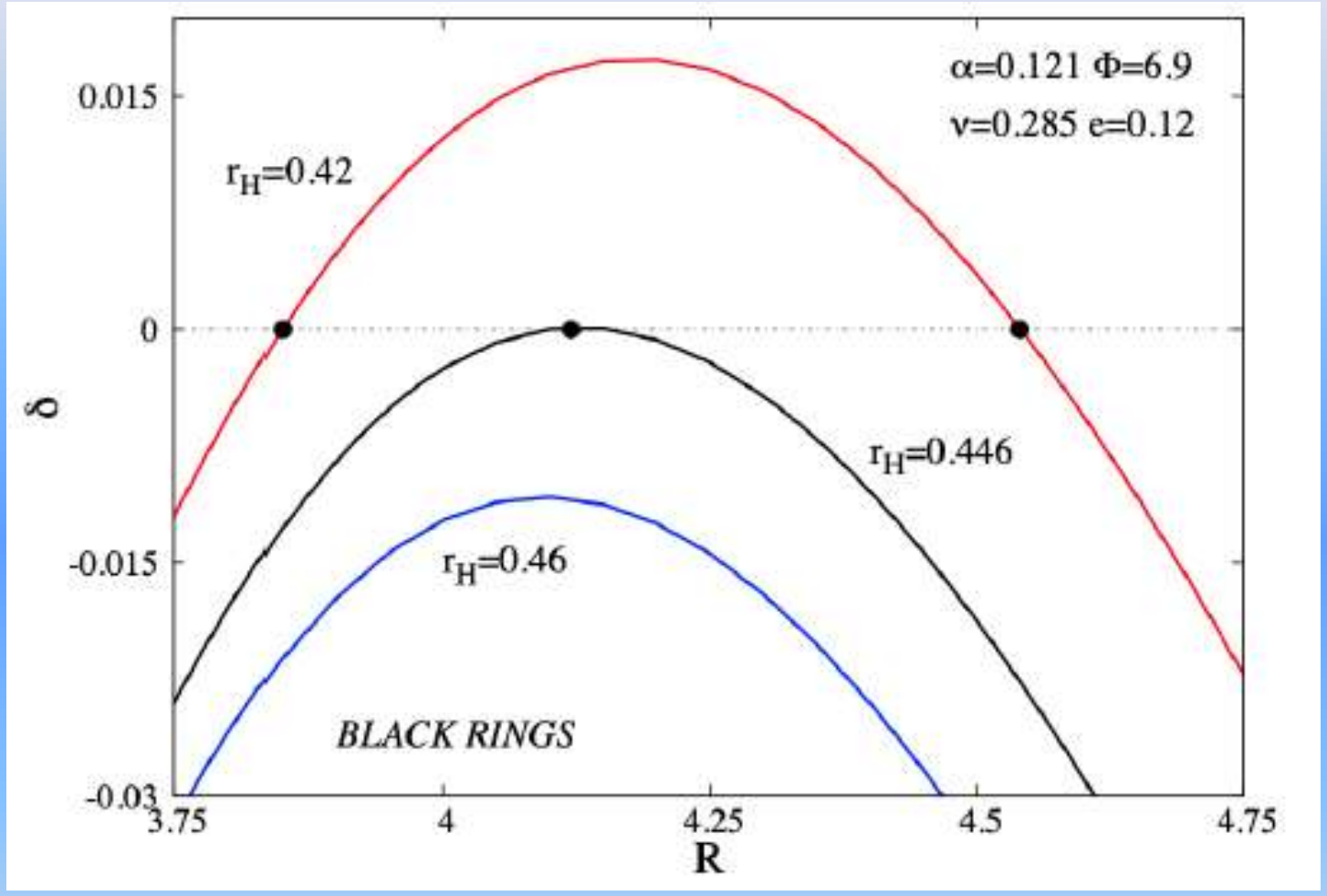
- **fix the model**
- **vary $R>r_H$ or Φ**
- **regular solutions exist for critical values of Φ only**

$$\alpha^2 = \frac{4\pi G\mu^2}{\lambda}$$

$$\beta^2 = \frac{\nu\mu^2}{\lambda}$$



the conical excess/deficit of a static BR is shown as a function of the electrostatic potential Φ and of the input parameter α which measures the strength of gravity (the inset).



the conical excess/deficit of a static BR is shown as a function of the ring radius R

to summarize::

- the (vacuum) two-Black Hole system/the static Black Ring posses conical singularities

- however, the scalar hair can provide the extra-force to balance the system

more than an exercise?

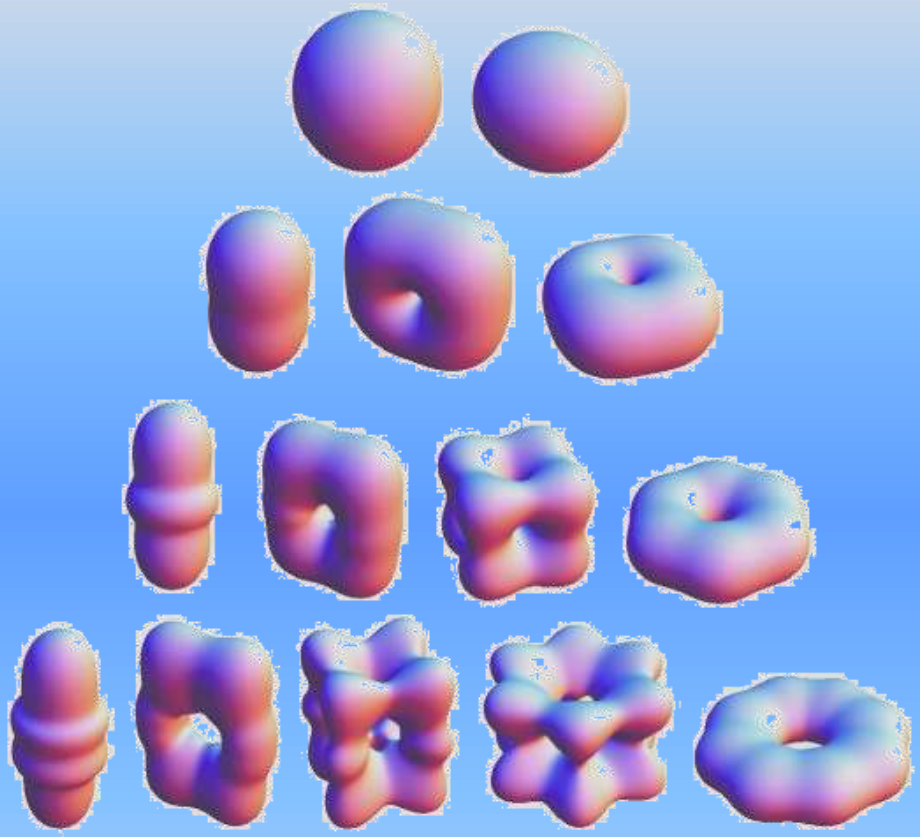
many open problems:

- including rotation
- stability?
- a more physical framework (**same methods**)

(scalar field – *just the simplest case)*

main message:

one cannot safely extrapolate the properties of the (electro-)vacuum Black Holes to any model



Vă mulţumesc pentru atenţie!