Minimal symmetry breaking of the cosmological principle and the Lemaître-Hubble diagram

André Tilquin, Galliano Valent

- Surface of the Earth $\leftrightarrow$ universe
- Minimal symmetry breaking of the cosmological prinicple
  with comoving dust $\Rightarrow$ axial Bianchi IX universes
- Lemaître-Hubble diagram of axial Bianchi IX universes
- Future experimental tests of axial Bianchi IX universes

To the memory of Vaughan Jones

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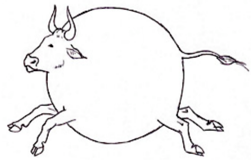

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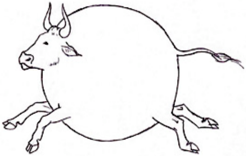
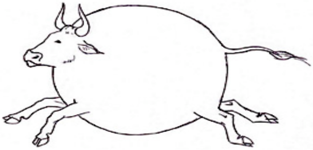


'geometric' sym. break. (less evident): (equatorial – polar) radius = 21.3 km, $21.3\,\mathrm{km} \cdot 2\pi/(40\,000\,\mathrm{km}) \approx 3.3 \cdot 10^{-3}$.



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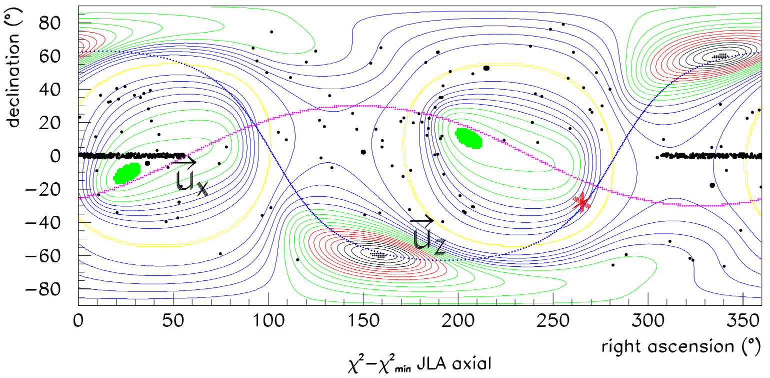

Figure: Black points represent 740 supernova positions. Note the accumulation of supernovae in the equatorial plane of the Earth and the absence of supernovae in the galactic plane (blue line). The red star is the direction towards our galactic center. Confidence level contours of privileged directions in arbitrary color codes for axial Bianchi I universes; best fit: $\vec{u}_z$.

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Again in 2014 Darling used tri-axial Bianchi I universes,

$$\mathrm{d}\tau^2 = \mathrm{d}t^2 - a(t)^2\,\mathrm{d}x^2 - b(t)^2\,\mathrm{d}y^2 - c(t)^2\,\mathrm{d}z^2,$$

Lemaître [1933], to fit the drift of 429 extra-galactic radio sources.

surface of the Earth	universe	sym. break.
sphericity*	cosmological principle	
static and isolated	Lemaître-Hubble flow	
mountains	over-densities	geographic
continental drift, variable duration of the day	peculiar velocities	geographic
constant rotation, oblate ellipticity	Bianchi I	geometric
precession period = 26 ky	position drift	geometric

*Flat-Earthers believe that sphericity is a conspiracy.
They must love flat universes too.

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Exciting years ahead, inviting us to propose already now finer mathematical models to be tested.

A few mathematical facts on maximal symmetry and its minimal breakings

Theorem: The isometry group of a d-dimensional space or spacetime (with $d \geq 2$) is a Lie group of dimension $n \leq d\,(d+1)/2$.

Examples in $d = 2$:
The sphere has the $n = d\,(d+1)/2 = 3$ dimensional isometry group $O(3)$, it is “maximally symmetric”;
oblate and prolate axial ellipsoids (pumpkin and rugby ball) have the $n = 1$ dimensional isometry group $O(2)$;
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In $d = 2$ dimensions the cylinder is a counter example, because it has two isometries.

If we say: axial ellipsoids realize **minimal symmetry breakings** of the sphere, we mean:

(1) These ellipsoids can be infinitesimally close to the sphere.

(2) They have the highest possible number of symmetries:

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Adding time as an orthogonal $\mathbb{R}$ to these 3-spaces of simultaneity, one obtains the 'Robertson-Walker' universes.

Definition: A **minimal symmetry breaking** of the cosmological principle is

(1) a smooth family of **deformations** of a maximally symmetric 3-space,

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$$\mathrm{d}\tau^2 = \mathrm{d}t^2 - a^2\,[\mathrm{d}x^2 + \mathrm{d}y^2] - c^2\,\mathrm{d}z^2,$$

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Counter-example: tri-axial Bianchi I universes, 3 scalefactors:

$$\mathrm{d}\tau^2 = \mathrm{d}t^2 - a^2\,\mathrm{d}x^2 - b^2\,\mathrm{d}y^2 - c^2\,\mathrm{d}z^2,$$

3 translations + no rotation = 3 symmetries.

Three symmetries: Bianchi universes

In 1898 Luigi Bianchi classifies all 3-dimensional, real Lie algebras. His list starts with seven 3-dimensional Lie algebras: Bianchi I, II, III, IV, V, VIII and IX.

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Bianchi also shows that all of these Lie algebras can be represented as **infinitesimal** isometries ('Killing vectors') on 3-spaces. Adding time as an orthogonal $\mathbb{R}$, one obtains the Bianchi universes.

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He finds that all of these 4-dimensional Lie algebras contain 3-dimensional Lie sub-algebras. (Today we know that any real or complex 4-dimensional Lie algebra has a 3-dimensional ideal.) Therefore these 3-spaces give rise to universes, that are special types of Bianchi universes. Let us call them **'axial'** Bianchi universes.

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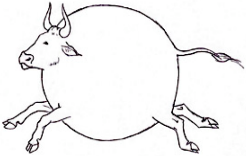
We remain with the axial Bianchi I, V and IX universes. They are smooth deformations of the Robertson-Walker universes with zero, negative and positive curvatures.

In pictures



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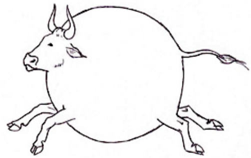

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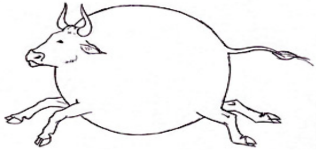

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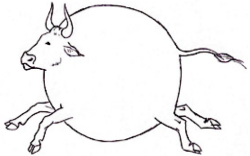

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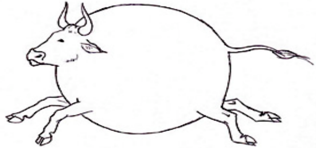

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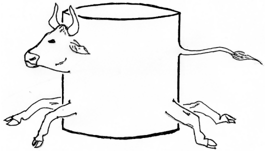

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In 2021 Akarsu, Di Valentino, Kumar, Ozyigit & Sharma considered tri-axial Bianchi V universes with $c^2 = ab$ and comoving dust. Galliano's bonus: exact solution of Einstein's equations. We also prove that the isometry group is 3-dimensional (except for $a = b$, the Friedman solution).

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• Seven parameters ($\cdot_0$ for evaluation *today* $= t_0$):
three for the underlying spherical Friedman universe

$$H_{F0},\, \Omega_{\Lambda 0},\, \Omega_{m0},$$

two for the direction of the axial symmetry axis

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and two for 'ellipticity' η_0 and 'Hubble stretch' η'_0, $(' := \mathrm{d}/\mathrm{d}t)$.

- Axial Bianchi I is the five-parameter sub-model with vanishing curvature $\Omega_{\Lambda 0} + \Omega_{m0} = 1$ and vanishing ellipticity $\eta_0 = 0$.

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$$z+1 \quad \sim \quad \frac{a_{F0}}{a_{Fe}}\left[1-\frac{1-3\cos^2\theta}{2}\,(\eta_0-\eta_e)\right], \qquad (0)$$

$$\ell \quad \sim \quad \ell_F\left[1+\frac{1-3\cos^2\theta}{2}\left(\eta_0-5\eta_e+4\,\frac{\chi}{\tan\chi}\,\bar{\eta}\right)\right], \qquad (1)$$

with $\cdot_e$ standing for evaluation at t_e and with

$$\ell_F=\frac{L}{4\pi\,a_{F0}^2\,\sin^2\chi}\left(\frac{a_{Fe}}{a_{F0}}\right)^2, \quad \chi:=\int_{t_e}^{t_0}\frac{1}{a_F}, \quad \bar{\eta}:=\frac{1}{\chi}\int_{t_e}^{t_0}\frac{\eta}{a_F}\,.$$

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In the limit of zero curvature, $\sin\chi\to\chi$ and $\chi/\tan\chi\to 1$ hold, and we recover our 2014 results for axial Bianchi I.

The dynamics is given by Einstein's equations:

- $a_F(t)$ satisfies Friedman's equations with final conditions:
  $a_F(t_0) = a_{F0}$ and $a'_F(t_0) = a_{F0}\, H_{F0}$.
- $\eta(t)$ satisfies

$$\eta'' + 3\, H_F\, \eta' + 8\, \frac{\eta}{a_F^2} \quad \sim \quad 0, \tag{2}$$

which admits a unique solution with final conditons:
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which admits a unique solution with final conditons:
ellipticity η_0 and Hubble stretch η'_0.

In the limit of zero curvature, the last term of equation (2) vanishes, and we recover our 2014 result for axial Bianchi I.

The dynamics is given by Einstein's equations:

- $a_F(t)$ satisfies Friedman's equations with final conditions:
  $a_F(t_0) = a_{F0}$ and $a'_F(t_0) = a_{F0}\, H_{F0}$.
- $\eta(t)$ satisfies

$$\eta'' + 3\, H_F\, \eta' + 8\, \frac{\eta}{a_F^2} \quad \sim \quad 0, \tag{2}$$

which admits a unique solution with final conditons:
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In the limit of zero curvature, the last term of equation (2) vanishes, and we recover our 2014 result for axial Bianchi I.

Finally the emission time t_e is eliminated in favour of the redshift by inverting $z(t_e)$, equation (0). Then the apparent luminosity is computed and compared to the observed one and the seven initial parameters are varied in order to optimize the fit for all observed supernovae.

Conclusions

In 2014 we confronted the 5-parameter axial Bianchi I with the 740 supernovae up to redshift 1.3 and found only a 1-σ signal.

Therefore we think that a fit of the 7-parameter axial Bianchi IX model to the Lemaître-Hubble diagram of type 1a supernovae becomes reasonable only once we can include the data expected from the Vera Rubin Observatory, the James Webb Space Telescope, the Chinese Space Station Telescope, ...

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In the same year 2014 Cea fitted the axial Bianchi I model to the WMAP and Planck data at redshift 1090. He also finds a 1-σ signal. Although his Hubble stretch has opposite sign and is smaller than ours by eight orders of magnitude, our results are compatible with his.

Again in 2014 Darling used the tri-axial Bianchi I model (7 parameters) to fit the drift of 429 extra-galactic radio sources measured by Titov & Lambert in 2013 using Very Long Baseline Interferometry. His main Hubble stretch has the same sign as ours but is ten time larger and the results are again compatible statistically.

Again in 2014 Darling used the tri-axial Bianchi I model (7 parameters) to fit the drift of 429 extra-galactic radio sources measured by Titov & Lambert in 2013 using Very Long Baseline Interferometry. His main Hubble stretch has the same sign as ours but is ten time larger and the results are again compatible statistically.

• Waiting and preparing for the promised promising data of type 1a supernovae, a combined analysis of axial Bianchi IX universes with type 1a supernovae, Cosmic Microwave Background, drift of radio sources and Baryonic Acoustic Oscillations (and maybe weak lensing or black-hole mergers) is called for now.

Traudi of Oberwolfach

To illustrate the cosmological principle here is a story set in the Black Forest. Next to the Oberwolfach Research Institute for Mathematics there is a farm, home to Traudi, her farmer's preferred cow. Traudi is ill and the veterinarian helpless. Sparing no effort, the farmer calls on a physician, more expensive, but as helpless as his colleague. The farmer has a nephew with a PhD in biology and asks him to see Traudi, again without success. Finally, when he sees a theoretical physicist on his way to a conference, the farmer, driven to despair, asks him for help. The physicist sits down next to Traudi, pulls out his note pad and starts calculating. During hours the farmer watches the physicist's intense concentration from a respectful distance and feels a timid ripple of optimism. He pulls closer, caresses Traudi between the horns and asks: 'Is there any hope?' 'Indeed there is', replies the physicist with unconcealed pride, 'I just solved the case of the spherical cow.'