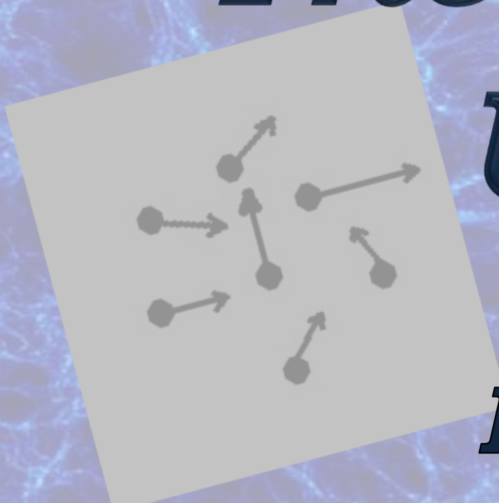
The kinetic gas Universe



$$\mathfrak{G} = -\kappa^2\varphi$$

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1 Main idea of the talk

✓ We propose a **Finsler-geometric** model for the description of a **gravitating multiparticle system**, viewed as a **kinetic gas.**

✓ This is a **more accurate description** of gravity generated by many-particle sources, **taking the full velocity distribution of the sources into account** $\rightarrow$ it could account for (some of the) DE phenomenology.

Main references:

1. M. Hohmann, C. Pfeifer, N. Voicu, *Kinetic gases as direct gravity sources*, Phys. Rev. D 101, 024062 (2020).
2. M. Hohmann, C. Pfeifer, N. Voicu, *Mathematical foundations for field theories on Finsler spacetimes*, Journal of Mathematical Physics 63, 032503 (2022).

2 Finsler geometry in physics

✓ QG phenomenology - modified dispersion relations;

✓ Standard Model extensions - study of Lorentz invariance violation;

✓ minimal extension of Ehlers-Pirani-Schild axioms;

✓ more accurate description of gravity, takings into account *multiple sources, with different velocities* - **kinetic gas** (Hohmann, Pfeifer, NV, 2020);

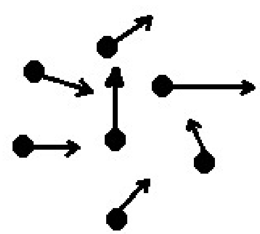
✓ wave propagation in media.

★ **Main feature of (pseudo-)Finsler geometry:**
most general geometry with a geometric clock (= a well-defined notion of arc length)

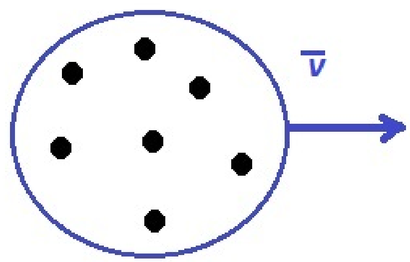
3 Kinetic gases in general relativity

$\boxed{\textbf{Kinetic gas}}$ = a large number N of interacting point particles, described by a smooth **1-particle distribution function:**

$$\varphi = \varphi\,(x, \dot{x})$$



kinetic gas
(individual velocities)



fluid
(averaged velocity)

Particles move on piecewise smooth *normalized* **geodesics** γ.

✓ **Number** N_σ **of particle trajectories** $(\gamma, \dot{\gamma})$ crossing a hypersurface $\sigma \subset \mathcal{O}$:

$$N_\sigma = \int_\sigma \varphi d\Omega, \tag{1}$$

✓ $\mathcal{O} = \{(x, \dot{x}) \in TM \mid g_x(\dot{x}, \dot{x}) = 1,\ \dot{x}\text{- future pointing}\}$ - **observer space.**

★ **Einstein-Vlasov equations** (see Sarbach-Zannias, [4]):

$$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$
$$T^{\mu\nu}(x) = \int_{\mathcal{O}_x} m\varphi \dot{x}^\mu \dot{x}^\nu d\mathbf{\Sigma}.$$

✓ For **collisionless gases** $\Rightarrow$ **Liouville equation:**

$$\nabla_{\mathbf{r}} \varphi = 0.$$

4 The Finslerian approach: motivation

M - 4-dim. connected, orientable manifold, $(x^\mu, \dot{x}^\mu)$ - local coords on TM.

1. Couple gravity directly to φ (no averaging over velocities):
Coupling - on **observer space** **(**$\subset$ **tangent bundle)** of spacetime
(similarly: on *momentum space* T^*M)

$$TM \equiv \{(x, \dot{x}) \,|\, x \in M, \dot{x} \in T_x M\} \text{ "position\&velocity space"}$$

2. Natural framework - Finsler spacetime geometry:
✓ $\varphi = \varphi(x, \dot{x})$ - scalar function on some subset of TM
✓ Finsler function $L = L(x, \dot{x}) \Rightarrow$ field equation $=$ scalar eq. on TM (**same space!**)

5 Finsler spacetimes

✓ An open subset $\mathcal{Q} \subset TM \backslash \{0\}$ is a **conic subbundle** if:

- for $\forall x \in M$, $\mathcal{Q}_x := \mathcal{Q} \cap T_x M$ is non-empty;

- *conic property:* $(x, \dot{x}) \in \mathcal{Q} \Rightarrow (x, \alpha\dot{x}) \in \mathcal{Q}, \quad \forall \alpha > 0$.

★ $\boxed{\textbf{Finsler spacetime}}$ $= (M, L)$, where: $L : \mathcal{A} \to \mathbb{R}$ - smooth on a *conic subbundle* $\mathcal{A} \subset TM$ and:

(i) $L(x, \alpha\dot{x}) = \alpha^2 L(x, \dot{x})$, $\forall \alpha > 0$;

(ii) $g_{\mu\nu}(x, \dot{x}) = \dfrac{1}{2}\dfrac{\partial^2 L}{\partial \dot{x}^\mu \partial \dot{x}^\nu}$ is **nondegenerate** on $\mathcal{A}$;

(iii) $\exists$ a conic subbundle $\mathcal{T} \subset \mathcal{A}$, with connected fibers $\mathcal{T}_x$, on which:

✓ $L > 0$, g has Lorentzian signature $(+, -, -, -)$

✓ L can be continuously extended as 0 to $\partial\mathcal{T}$.

✓ **Interpretation:**

$$ds^2 = L(x, dx) = g_{\mu\nu}(x, \dot{x})\, dx^\mu dx^\nu.$$

✓ Length of a curve $\gamma : [a, b] \to M$:

$$\ell(\gamma) = \int_\gamma \sqrt{|L(\gamma(t), \dot{\gamma}(t))|}dt$$

- independent on the choice of the parameter t.

Finsler geometry = most general geometry with a geometric clock:

$$\mathbf{ds} = \mathbf{c}\mathbf{d}\boldsymbol{\tau}.$$

★ Examples of Finsler spacetime functions L :

$\surd$ **Lorentzian** (quadratic in $\dot{x}$)**:**

$$L(x, \dot{x}) = a_{\mu\nu}(x)\dot{x}^{\mu}\dot{x}^{\nu}$$

$\surd$ **Bogoslovsky/Kropina** *(VSR,VGR - Cohen&Glashow):*

$$L(x, \dot{x}) = \epsilon|a_{\mu\nu}(x)\dot{x}^{\mu}\dot{x}^{\nu}|^{1-q}(b_{\rho}(x)\dot{x}^{\rho})^{2q},$$

where: $\epsilon = sign(a_{\mu\nu}(x)\dot{x}^{\mu}\dot{x}^{\nu})$.

$\surd$ **quartic metrics** *($\rightarrow$ birefringence - Pfeifer&Wohlfarth, Perlick etc.):*

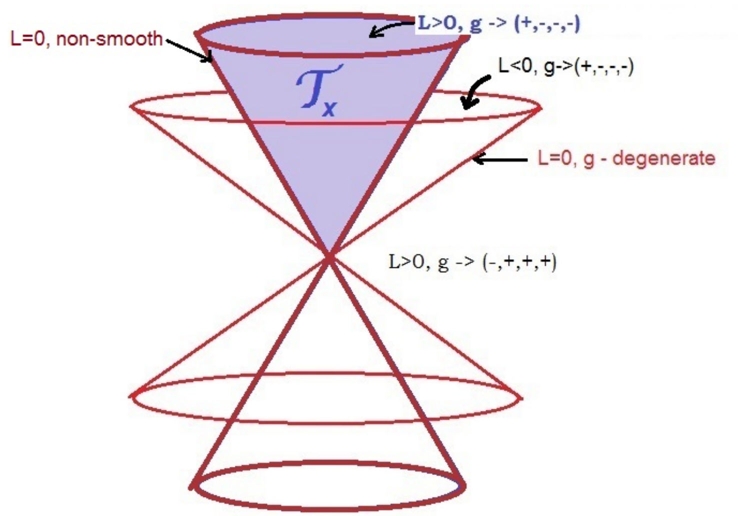
$$L(x, \dot{x}) = \epsilon\sqrt{|(a_{\mu\nu}\left(x\right)\dot{x}^{\mu}\dot{x}^{\nu})\left(h_{\rho\sigma}\left(x\right)\dot{x}^{\rho}\dot{x}^{\sigma}\right)|},$$

where $\epsilon = sign(...)$.

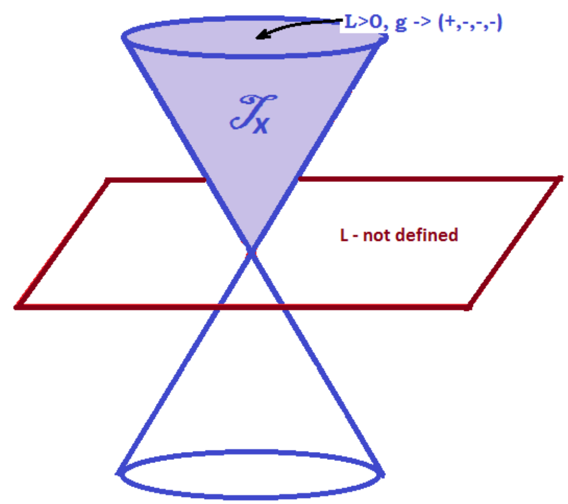
★ **Remarkable subsets of** $TM\backslash\{0\}$:

✓ $\mathcal{A}$ - set of **admissible vectors** (where L is defined, smooth, g - nondegenerate)**;**

✓ $\mathcal{T}$ - future directed **timelike** vectors;



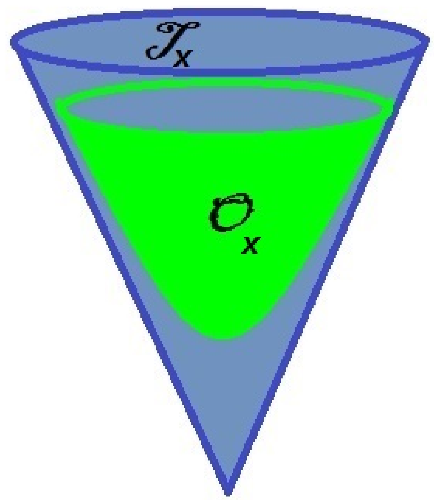
cone of a quartic metric



cone of a Kropina metric

★ **Observer space:**

$$\mathcal{O} = \{(x, \dot{x}) \in \mathcal{T} \mid L(x, \dot{x}) = 1\}\,.$$



✓ $\mathcal{O}$ has a natural **contact structure** - the *Hilbert form***:**

$$\omega = \dot{\partial}_i \left(\sqrt{L}\right) dx^i.$$

★ Geometry of $\mathcal{O}$ - naturally determined by ω :

- Length of a timelike curve γ on M : $\ell(\gamma) = \int\limits_{(\gamma,\dot\gamma)} \omega$.

- Canonical *volume form* on $\mathcal{O}$: $d\mathbf{\Sigma} = \omega \wedge d\omega \wedge d\omega \wedge d\omega$.

- *Geodesic (Reeb) vector field* $\mathbf{r}$: $\omega(\mathbf{r}) = 1$, $d\omega(\mathbf{r}, \cdot) = 0 \Rightarrow$

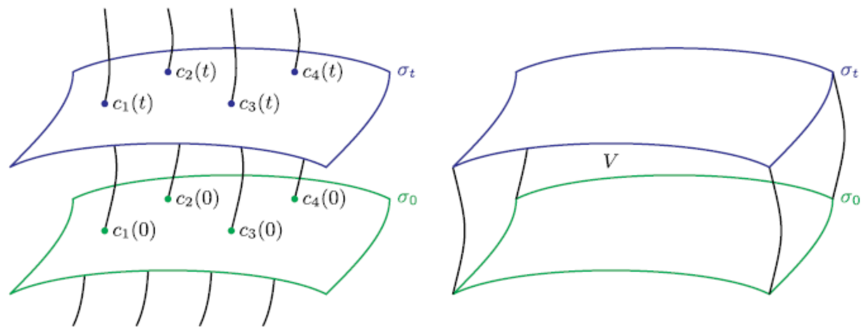
$$\mathbf{r} = \frac{1}{\sqrt{L}}(\dot x^\mu \frac{\partial}{\partial x^\mu} - 2G^\mu \frac{\partial}{\partial \dot x^\mu}).$$

$\Rightarrow$ Flow of $\mathbf{r}$ - lifts $(\gamma, \dot\gamma)$ of **normalized geodesics of** (M, L) :

$$\ddot\gamma^\mu(s) + 2G^\mu(\gamma(s), \dot\gamma(s)) = 0.$$

Remark: $G^\mu(x, \dot x) = \Gamma^\mu{}_{\rho\sigma}(x, \dot x)\dot x^\rho \dot x^\sigma$.

6 Kinetic gas action



Kinetic gas: volume spanned by particle worldlines

✓ $\sigma \subset \mathcal{O}$ hypersurface transversal to $\mathbf{r} \Rightarrow$ number of particles crossing σ :

$$N(\sigma) = \int_{\sigma} \varphi \Omega,$$

where $\Omega = d\omega \wedge d\omega \wedge d\omega$ - canonical volume form on σ.

Action for the kinetic gas:

$$
\begin{aligned}
S_{gas} &= Nmt = mN\int_{0}^{t} ds \\
&= m\int_{0}^{t}\left(\int_{\sigma_s}\varphi\Omega\right) ds = \ldots = m\int_{V}\varphi d\boldsymbol{\Sigma}
\end{aligned}
$$

That is:

$$
S_{gas} = m\int_{V}\varphi d\boldsymbol{\Sigma}
$$

Energy-momentum distribution tensor and its conservation law

★ Coordinate invariance of S_{gas} $\Rightarrow$ **averaged conservation law** (on M):

$$\int_{\mathcal{O}_x} D_\mu \Theta^\mu{}_\nu \, d\boldsymbol{\Sigma}_x = 0, \qquad (*)$$

where Θ is the **energy-momentum distribution tensor:**

$$\Theta^\mu{}_\nu(x, \dot{x}) = m\varphi \frac{\dot{x}^\mu \dot{x}_\nu}{L}$$

Liouville equation for collisionless gases = pointwise conservation law:

$$\nabla_{\mathbf{r}} \varphi = 0 \qquad \Leftrightarrow \qquad D_\mu \Theta^\mu{}_\nu = 0$$

★ **Energy-momentum tensor density** on M - by averaging over $\dot{x}$:

$$\mathcal{T}^{\mu}_{\ \nu}(x) = \int_{\mathcal{O}_x} \Theta^{\mu}{}_{\nu}(x, \dot{x})\ d\mathbf{\Sigma}_x$$

★ **In particular, if** L **- semi-Riemannian** $\Rightarrow$ we can "un-densitize" $\mathcal{T}$:

$$T^{\mu}_{\ \nu}(x) = \frac{1}{\sqrt{|\det g|}} \mathcal{T}^{\mu}_{\ \nu}(x)$$

The averaged conservation law $(*)$ is then equivalent to:

$$T^{\mu}_{\ \nu;\mu} = 0.$$

Total action and field equation

★ **Total action:**

$$S := \frac{1}{2\kappa^2}\int\limits_V \frac{R}{L}\, d\boldsymbol{\Sigma} \; + \; m\int\limits_V \varphi \, d\boldsymbol{\Sigma}$$

($R = R(x, \dot{x})$ - Finslerian Ricci scalar)

★ **Field equation** $\longrightarrow$ scalar equation on TM :

$$\mathfrak{G} = -\kappa^2 \varphi,$$

where:

$$\mathfrak{G} := 3L^{-2}R - \frac{1}{2}g^{\mu\nu}\partial_{\dot{x}^\mu}\partial_{\dot{x}^\nu}R + g^{\mu\nu}[\partial_{\dot{x}^\nu}(D_{\mathbf{r}}P_\mu) + D_\nu P_\mu - P_\mu P_\nu]$$

$$P_\mu = D_{\mathbf{r}}(\partial_{\dot{x}^\mu} \ln|\det g|).$$

Averaged Finslerian field equations vs. Einstein-Vlasov equations:

$$\underbrace{\int\limits_{\mathcal{O}_x} \mathfrak{G} \frac{\dot{x}^\mu \dot{x}_\nu}{L^{-1}} d\Sigma_x}_{\mathfrak{G}^\mu{}_\nu} = -\kappa^2 \mathcal{T}^\mu_\nu \qquad \overset{a}{R}{}^\mu{}_\nu - \overset{a}{R}\delta^\mu{}_\nu = \frac{8\pi G}{c^4} T^\mu_\nu$$

averaged Finslerian — Einstein-Vlasov

$\boxed{\textbf{This talk: Gravitational field of a kinetic gas - no averaging (!)}}$

$$\mathfrak{G} = -\kappa^2 \varphi$$

$\boxed{\textbf{takes velocity distribution of gas particles into account.}}$

Next step: Find a meaningful $a_{\mu\nu}(x)$ such that:

$$\mathfrak{G}^\mu{}_\nu = \overset{a}{R}{}^\mu{}_\nu - \overset{a}{R}\delta^\mu{}_\nu + \Lambda\delta^\mu{}_\nu + \ldots$$

7 Cosmologically symmetric Finsler spacetimes

✓ Generators of cosmological symmetry - same as in GR, [4].

Theorem (Cosmologically symmetric Finsler spacetime functions), [6]:
If (M, L) is a cosmological Finsler spacetime, then:

$$L = L(t, \dot{t}, w),$$

where

$$w^2 = \frac{\dot{r}^2}{1 - \kappa r^2} + r^2 \left(\dot{\theta}^2 + \dot{\varphi}^2 \sin^2 \theta \right), \qquad \kappa \in \{0, \pm 1\} \, .$$

Examples of cosmologically symmetric Finsler functions:

$\surd$ FLRW metric (pseudo-Riemannian):

$$L = \dot{t}^2 - a^2(t)w^2;$$

$\surd$ Bogoslovsky-Kropina type:

$$L = \epsilon \left| \dot{t}^2 - a^2(t)w^2 \right|^{1-q} \dot{t}^{2q},$$

with: $\epsilon = sign(\dot{t}^2 - w^2)$.

$\surd$ More generally:

$$L = \dot{t}^2 \Phi \left(t, \frac{w}{\dot{t}} \right).$$

★**Next steps:**

1. Find a reasonable cosmologically symmetric $\varphi = \varphi(t, \dot{t}, w)$ and integrate the resulting field equation.

Candidate:

$$\varphi \sim e^{-\alpha \mathbf{v}^2}.$$

2. Spherically symmetric solutions of field equation (with appropriate φ).

...work in progress...

References:

[1] M. Hohmann, C. Pfeifer, N. Voicu, *Relativistic kinetic gases as direct sources of gravity*, Phys. Rev. D 101, 024062 (2020).

[2] M. Hohmann, C. Pfeifer, N. Voicu, *Mathematical foundations for field theories on Finsler spacetimes*, Journal of Mathematical Physics 63, 032503 (2022).

[3] M. Hohmann, C. Pfeifer, N. Voicu, The kinetic gas universe, *The kinetic gas universe,* Eur. Phys. J. C 80, 809 (2020).

[4] M. Hohmann, C. Pfeifer, N. Voicu, *Cosmological Finsler spacetimes*, Universe 6 (5), 65 (2020).

[5] O. Sarbach and T. Zannias, *The geometry of the tangent bundle and the relativistic kinetic theory of gases*, Classical and Quantum Gravity 31, 085013 (2014).

[6] M. Hohmann, C. Pfeifer, *Geodesics and the magnitude-redshift relation on cosmologically symmetric Finsler spacetimes,* Phys.Rev.D 95 (10), 104021 (2017).

Thank you !