The boundary problem in (super)gravity from a geometric perspective

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Boundary problem in (SU)GRA

Gravity and supergravity Lagrangians in the presence of a boundary studied from the early seventies on...

- York-Gibbons-Hawking (1972, 1977): Need of adding a boundary term to the gravity action such as to implement Dirichlet boundary conditions for the metric field in early attempts to study the quantization of gravity with a path integral approach

- Horava-Witten (1996): Addition of boundary terms considered to cancel gauge and gravitational anomalies in the Horava-Witten model in $11D$

- AdS/CFT (1997): Bulk fields (metric) diverge at $\partial\mathcal{M} \to$ Cured by inclusion of counterterms at the boundary (Holographic renormalization)

General lesson: For $\partial\mathcal{M} \neq 0$, the bulk theory needs to be supplemented by boundary terms

Overview

- ☞ Gravity case in the geometric (Cartan) approach
- ☞ (Super)group-manifold approach to (super)gravity [key aspects]
- ☞ Geometric construction of pure $D = 4$ SUGRA with negative cosmological constant in four dimensions in the presence of a non-trivial spacetime boundary [$\mathcal{N} = 1$]

  L. Andrianopoli, R. D'Auria, 1405.2010
- ☞ Case of vanishing cosmological constant ("flat" SUGRA, no explicit internal scale in the Lagrangian)

  P. Concha, L. R., E. Rodríguez, 1809.07871
- ☞ Application to specific problems in cases where the boundary is located asymptotically + Open directions

Geometric (Cartan) approach at the gravity level

Aros, Contreras, Olea, Troncoso, Zanelli (1999); Olea (2005): Diffeomorphism invariance of the bulk Einstein Lagrangian + cosmological constant Λ is broken in the presence of a boundary

$\Rightarrow$ Restored by adding a topological term (Euler-Gauss-Bonnet):

$$\mathcal{L}_{\text{EGB}} = \mathcal{R}^{ab} \wedge \mathcal{R}^{cd} \epsilon_{abcd} = \mathrm{d}\left(\omega^{ab} \wedge \mathcal{R}^{cd} + \omega^{a}{}_{\ell} \wedge \omega^{\ell b} \wedge \omega^{cd}\right) \epsilon_{abcd}$$

$\Rightarrow$ Background-independent definition of Noether charges, without the need of explicitly imposing Dirichlet boundary conditions on the fields

The expansion of $\mathcal{L}_{\text{EGB}}$ in the radial coordinate $\perp$ to the boundary

- Regularizes action and the related (background-independent) conserved charges
- Reproduces holographic renormalization counterterms

Differential forms, Lie (super)algebras and Maurer-Cartan equations

Lie superalgebra

$$[T_A, T_B\} = C_{AB}{}^{C} T_C$$

T_A: Generators in the adjoint representation of the Lie group

Dual formulation

$$\longrightarrow$$

$$\sigma^A(T_B) = \delta^A_B$$

σ^A: Differential 1-forms

Maurer-Cartan equations

$$R^A \equiv d\sigma^A + \frac{1}{2} C_{BC}{}^{A} \sigma^B \wedge \sigma^C = 0$$

$d^2 = 0 \leftrightarrow$ Jacobi identities

- R^A are the supercurvatures (super field strengths), building blocks of sugra in the geometric framework
- The Maurer-Cartan equations $R^A = 0$ define the vacuum of a SUGRA theory
- Geometric formulation in superspace, spanned by the supervielbein $\{V^a, \Psi\}$ (dual to P_a, Q)

GEOMETRIC APPROACH TO SUGRA IN SUPERSPACE

- Geometric (rheonomic) approach to SUGRA in superspace
- Superfields $\mu^A(x, \theta)$, supercurvatures R^A; θ spinorial anticommuting coordinates $\rightarrow$ Restriction to spacetime: $\theta = \mathrm{d}\theta = 0$
- Superspace basis: $\{V^a, \psi\}$ (supervielbein)

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- In principle: Extra dynamical info. in superspace $\Rightarrow$ Constraints to have same dynamical info. we have on spacetime ("rheonomic constraints" on the parametrization of the supercurvatures)

  $\Rightarrow$ Bianchi identities become relations among the superfields and their curvatures (satisfied on-shell)

  Realized by requiring the supercurvatures (defined off-shell) to be identified on-shell as particular 2-forms in superspace: Parametrization on a basis of 2-forms in superspace, det. by requiring the "Bianchi relations" to be satisfied

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$$R^A = R^A_{ab} V^a \wedge V^b + R^A_{a\alpha} V^a \wedge \psi^\alpha + R^A_{\alpha\beta} \psi^\alpha \wedge \psi^\beta$$

  R^A_{ab} inner components, $R^A_{a\alpha}$ and $R^A_{\alpha\beta}$ outer components

  Bianchi $\rightarrow$ Outer as linear tensor comb. of inner (constraints, phys. equiv. to on-shell ones) $\Rightarrow$ No extra d.o.f.
- SUSY tr. of the fields on spacetime corresponds to diffeo. in the fermionic (θ) directions of superspace $\rightarrow$ Lie derivatives in those directions

SUPERGRAVITY CASE AND THE GEOMETRIC SUPERSPACE APPROACH

Boundary problem considered from several authors, different approaches

Point of contact: To restore all the invariances of a SU(GRA) Lagrangian with Λ, add topological contributions

A systematic way to face the boundary problem in SUGRA:

Geometric approach to SUGRA in superspace

- The theory is given in terms of superfields 1-forms $\mu^{\mathcal{A}}$ defined on superspace $\mathcal{M}_{4|4\mathcal{N}}$ (4 spacetime dims.)
- The Lagrangian, $\mathcal{L}[\mu^{\mathcal{A}}]$, is a bosonic 4-form in superspace and the action is obtained by integrating $\mathcal{L}$ on a generic bosonic hypersurface $\mathcal{M}_4(x,\theta) \subset \mathcal{M}_{4|4\mathcal{N}}$ immersed in superspace

$$\mathcal{S} = \int_{\mathcal{M}_4} \mathcal{L}[\mu^{\mathcal{A}}]$$

- SUSY transformations in spacetime are diffeomorphisms in the fermionic (θ) directions of superspace:

$$\text{SUSY:} \quad \mathcal{M}_4(x,\theta) \to \mathcal{M}_4(x,\delta\theta)$$

$\Rightarrow$ Can be described in terms of Lie derivatives ℓ_ϵ with fermionic parameter $\epsilon(x,\theta)$ (SUSY parameter)

$$\ell_\epsilon = \imath_\epsilon \mathrm{d} + \mathrm{d}\imath_\epsilon \,, \qquad \imath_\epsilon : \text{ contraction operator} \quad \imath_\epsilon\left(V^a\right) = 0 \,,\ \imath_\epsilon\left(\psi\right) = \epsilon$$

SUGRA theory $\to$ Invariance of the action under SUSY transformations: $\delta_\epsilon \mathcal{S} \equiv \int_{\mathcal{M}_4} \delta_\epsilon \mathcal{L} = 0$

- Condition for the superspace Lagrangian to be invariant under local SUSY:

$$\delta_\epsilon \mathcal{L} = \ell_\epsilon \mathcal{L} = \imath_\epsilon(\mathrm{d}\mathcal{L}) + \mathrm{d}(\imath_\epsilon \mathcal{L}) = 0$$

$\Rightarrow$ Necessary condition for a SUSY-invariant SUGRA Lagrangian:

$$\imath_\epsilon(\mathrm{d}\mathcal{L}) = 0$$

Corresponding to requiring SUSY invariance in the bulk of superspace
$\to$ Assumed true from now on, the Lagrangian satisfying it: Bulk-supergravity Lagrangians, $\mathcal{L}_{\text{bulk}}$

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- SUSY invariance of the action then requires the weaker condition on the bulk Lagrangian

$$\delta_\epsilon \mathcal{S} = \int_{\mathcal{M}_4} \mathrm{d}(\imath_\epsilon \mathcal{L}_{\text{bulk}}) = \int_{\partial\mathcal{M}_4} \imath_\epsilon \mathcal{L}_{\text{bulk}} = 0 \quad \Rightarrow \quad \imath_\epsilon \mathcal{L}_{\text{bulk}}|_{\partial\mathcal{M}_4} = \mathrm{d}\phi$$

In general not satisfied by $\mathcal{L}_{\text{bulk}}$ in the presence of non-trivial boundary conditions on $\partial\mathcal{M}_4 \neq 0$
$\Rightarrow$ SUSY invariance requires to add boundary terms $\rightarrow$ Consider the full Lagrangian

$$\mathcal{L}_{\text{full}} = \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\text{bdy}}\,, \quad \mathcal{L}_{\text{bdy}} = \mathrm{d}\mathcal{B}_{(3)} \quad \Rightarrow \quad \imath_\epsilon(\mathrm{d}\mathcal{L}_{\text{full}}) = 0 \quad \text{and} \quad \imath_\epsilon \mathcal{L}_{\text{full}}|_{\partial\mathcal{M}_4} = 0$$

Pure $\mathcal{N}=1$, $D=4$ SUGRA with negative cosmological constant

- Fields: V^a ($a=0,1,2,3$), spin connection ω^{ab}, gravitino ψ^α (Majorana spinor, $\alpha=1,2,3,4$)
- Lorentz-covariant supercurvatures:

$$\mathcal{R}^{ab} \equiv \mathrm{d}\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}$$

$$R^a \equiv \mathcal{D}V^a - \frac{\mathrm{i}}{2}\bar{\psi}\gamma^a \wedge \psi = \mathrm{d}V^a + \omega^a{}_b \wedge V^b - \frac{\mathrm{i}}{2}\bar{\psi}\gamma^a \wedge \psi$$

$$\rho \equiv \mathcal{D}\psi = \mathrm{d}\psi + \frac{1}{4}\omega^{ab}\gamma_{ab} \wedge \psi$$

- Bulk Lagrangian of pure $\mathcal{N}=1$, $D=4$ SUGRA in superspace, whose e.o.m. admit an AdS_4 vacuum solution with cosmological constant $\Lambda = -3/\ell^2$:

$$\mathcal{L}^{\mathcal{N}=1}_{\text{bulk}} = \frac{1}{4}\mathcal{R}^{ab}V^cV^d\epsilon_{abcd} - \bar{\psi}\gamma_5\gamma_a\rho V^a - \frac{\mathrm{i}}{2\ell}\bar{\psi}\gamma_5\gamma_{ab}\psi V^a V^b - \frac{1}{8\ell^2}V^aV^bV^cV^d\epsilon_{abcd}$$

- Invariant (in the bulk) under SUSY:

$$\imath_\epsilon(\mathrm{d}\mathcal{L}^{\mathcal{N}=1}_{\text{bulk}}) = 0$$

When the background spacetime has a non-trivial boundary:

$$\imath_\epsilon \mathcal{L}^{\mathcal{N}=1}_{\text{bulk}}|_{\partial\mathcal{M}_4} \neq \mathrm{d}\varphi \quad \Rightarrow \quad \delta_\epsilon \mathcal{S}_{\text{bulk}} \neq 0$$

- To restore SUSY invariance: Add boundary terms $\mathcal{L}^{\mathcal{N}=1}_{\text{bdy}} = d\mathcal{B}_{(3)}$ to the superspace Lagrangian which do not alter $\mathrm{d}\mathcal{L}^{\mathcal{N}=1}_{\text{bulk}}$ so that still $\imath_\epsilon(\mathrm{d}\mathcal{L}^{\mathcal{N}=1}_{\text{full}}) = 0$
- Possible boundary terms:

$$\mathrm{d}\left(\omega^{ab}\wedge\mathcal{R}^{cd} + \omega^{a}{}_{\ell}\wedge\omega^{\ell b}\wedge\omega^{cd}\right)\epsilon_{abcd} = \mathcal{R}^{ab}\wedge\mathcal{R}^{cd}\epsilon_{abcd}$$

$$\mathrm{d}\left(\bar{\psi}\wedge\gamma_5\rho\right) = \bar{\rho}\wedge\gamma_5\rho - \frac{1}{4}\mathcal{R}^{ab}\wedge\bar{\psi}\gamma_5\gamma_{ab}\wedge\psi$$

- Therefore consider the boundary Lagrangian

$$\mathcal{L}^{\mathcal{N}=1}_{\text{bdy}} = \alpha\mathcal{R}^{ab}\mathcal{R}^{cd}\epsilon_{abcd} - \mathrm{i}\beta\left(\bar{\rho}\gamma_5\rho - \frac{1}{4}\mathcal{R}^{ab}\bar{\psi}\gamma_5\gamma_{ab}\psi\right)$$

- Modify the Lagrangian $\rightarrow$ Full Lagrangian:

$$\mathcal{L}^{\mathcal{N}=1}_{\text{bulk}} \quad \rightarrow \quad \mathcal{L}^{\mathcal{N}=1}_{\text{full}} \equiv \mathcal{L}^{\mathcal{N}=1}_{\text{bulk}} + \mathcal{L}^{\mathcal{N}=1}_{\text{bdy}}$$

- Consider the boundary contributions in the field eqs. from $\mathcal{L}_{\text{full}}^{\mathcal{N}=1}$
  $\Rightarrow$ Constraints on the supercurvatures to hold on the boundary:

$$\begin{cases} \dfrac{\delta \mathcal{L}_{\text{full}}^{\mathcal{N}=1}}{\delta \omega^{ab}} = 0 \quad \Rightarrow \quad \mathcal{R}^{ab}|_{\partial \mathcal{M}_4} = -\dfrac{1}{8\alpha}\left(V^a V^b + \dfrac{1}{2}\beta \bar{\psi}\gamma^{ab}\psi \right)_{\partial \mathcal{M}_4} \\ \dfrac{\delta \mathcal{L}_{\text{full}}^{\mathcal{N}=1}}{\delta \psi} = 0 \quad \Rightarrow \quad \rho|_{\partial \mathcal{M}_4} = \dfrac{\mathrm{i}}{2\beta}\left(\gamma_a \psi V^a\right)_{\partial \mathcal{M}_4} \end{cases}$$

$\mathcal{R}^{ab}$, ρ on $\partial\mathcal{M}_4$ dynamically fixed to const. values in the anholonomic basis of the bos. and ferm. vielbeins

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- Impose SUSY invariance $\to$ Using the above eqs. we find:

$$\imath_\epsilon (\mathcal{L}^{\mathcal{N}=1}_{\text{full}})|_{\partial \mathcal{M}} = 0 \quad \Leftrightarrow \quad \frac{\beta}{16\alpha} - \frac{1}{2\beta} = -\frac{1}{\ell}$$

$\to$ Can be solved in terms of the real parameter $k \neq -1$:

$$\alpha = -\frac{1}{8}\frac{\ell^2}{1-k^2}\,, \quad \beta = \frac{\ell}{1-k}$$

that is

$$\begin{cases} \mathcal{R}^{ab}|_{\partial \mathcal{M}_4} = \left[\dfrac{1-k^2}{\ell^2} V^a V^b + \dfrac{1+k}{2\ell}\bar{\psi}\gamma^{ab}\psi\right]_{\partial \mathcal{M}_4} \\ \rho|_{\partial \mathcal{M}_4} = \dfrac{\mathrm{i}(1-k)}{2\ell}\left[\gamma_a \psi V^a\right]_{\partial \mathcal{M}_4} \end{cases}$$

- Setting $k = 0$, which implies $\alpha = -\ell^2/8$ and $\beta = \ell$, $\mathcal{L}_{\text{full}}^{\mathcal{N}=1}$ takes the form

$$\mathcal{L}_{\text{full}}^{\mathcal{N}=1} = -\frac{\ell^2}{8} \mathbf{R}^{ab} \wedge \mathbf{R}^{cd} \epsilon_{abcd} - \mathrm{i}\ell \bar{\boldsymbol{\rho}} \gamma_5 \wedge \boldsymbol{\rho}$$

in terms of the $\mathrm{OSp}(1|4)$-covariant supercurvatures

$$\mathbf{R}^{ab} \equiv \mathcal{R}^{ab} - \frac{1}{\ell^2} V^a V^b - \frac{1}{2\ell} \bar{\psi} \gamma^{ab} \psi$$
$$\boldsymbol{\rho} \equiv \rho - \frac{\mathrm{i}}{2\ell} \gamma_a \psi V^a$$
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- The constraints coming from the boundary contributions to the field eqs. take the simple form (for $k = 0$)

$$\mathbf{R}^{ab}|_{\partial\mathcal{M}_4} = 0\,, \quad \boldsymbol{\rho}|_{\partial\mathcal{M}_4} = 0\,, \quad \mathbf{R}^a|_{\partial\mathcal{M}_4} = 0$$

$\Rightarrow$ The $\mathrm{OSp}(1|4)$ supercurvatures vanish at the boundary $\rightarrow$ Boundary enjoys global inv. under $\mathrm{OSp}(1|4)$

- SUSY extension of Olea's results where the invariance of the gravity Lagrangian under spacetime diffeomorphisms was required: Boundary Lagrangian is the $\mathcal{N} = 1$ SUSY extension of the EGB term
- $\mathcal{N} = 1$ SUGRA also allows $k \neq 0$, peculiar freedom of the minimal theory

SUSY invariance of flat $\mathcal{N}=1$, $D=4$ SUGRA with boundary

How does the $\Lambda \to 0$ (that is $\ell \to \infty$) limit work?

- As we can see, direct flat limit of the MacDowell-Mansouri Lagrangian does not appear to be well-defined:

$$\mathcal{L}_{\text{full}}^{\mathcal{N}=1} = -\frac{\ell^2}{8}\mathbf{R}^{ab} \wedge \mathbf{R}^{cd}\epsilon_{abcd} - \mathrm{i}\ell\bar{\boldsymbol{\rho}}\gamma_5 \wedge \boldsymbol{\rho}$$

- Case where the boundary is placed asymptotically at infinity: BMS group emerges as asymptotic symmetry
- $\exists$ a geometric $\mathcal{L}_{\text{bdy}}$ exhibiting super-BMS symmetry?
  $\to$ Consider boundary at asymptotic infinity to allow the BMS symmetry to possibly emerge
- But to implement the geometric approach scheme the boundary is not required to be specified

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- But to implement the geometric approach scheme the boundary is not required to be specified
- Focus here: Restore the SUSY invariance when $\partial\mathcal{M} \neq 0$ by adding boundary terms

$$\mathcal{L}_{\text{bulk}}^{\text{flat}} = \frac{1}{4}\mathcal{R}^{ab}V^cV^d\epsilon_{abcd} - \bar{\psi}\gamma_5\gamma_a\rho V^a$$

The boundary terms that can be constructed using ω^{ab}, V^a, ψ scale as L^0 and L (while EH and RS scale as L^2)

Alternative approach proposed in 1809.07871

- Add new gauge fields with higher scale-weight: $A^{ab} = -A^{ba}$ (s.w. L^2) and χ (s.w. $\mathrm{L}^{3/2}$)
- They appear only in the boundary Lagrangian necessary to restore SUSY in the geometric approach (topological role)
- They act as auxiliary fields (off-shell matching of the bosonic and fermionic d.o.f.) under the bulk perspective, implementing the Bianchi identities of Lorentz and supersymmetry respectively, associated with ω^{ab} and ψ

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Boundary contributions (not involving a scale parameter):

$$\mathrm{d}\left(A^{ab}\wedge\mathcal{R}^{cd}+\omega^{a}{}_{f}\wedge\omega^{fb}\wedge A^{cd}+2\omega^{a}{}_{f}\wedge A^{fb}\wedge\omega^{cd}+\omega^{ab}\wedge\mathcal{F}^{cd}\right)\epsilon_{abcd}=2\mathcal{R}^{ab}\wedge\mathcal{F}^{cd}\epsilon_{abcd}$$

$$\mathrm{d}\left(\bar{\psi}\gamma_5\wedge\sigma+\bar{\chi}\gamma_5\wedge\rho\right)=2\bar{\sigma}\gamma_5\wedge\rho-\frac{1}{2}\mathcal{R}^{ab}\wedge\bar{\chi}\gamma_5\gamma_{ab}\wedge\psi$$

where we have defined $\sigma\equiv\mathcal{D}\chi$ and $\mathcal{F}^{ab}\equiv\mathcal{D}A^{ab}$

Boundary Lagrangian:

$$\mathcal{L}^{\text{flat}}_{\text{bdy}}=\alpha'\left(2\mathcal{R}^{ab}\mathcal{F}^{cd}\epsilon_{abcd}\right)-\mathrm{i}\beta'\left(2\bar{\sigma}\gamma_5\rho-\frac{1}{2}\mathcal{R}^{ab}\bar{\chi}\gamma_5\gamma_{ab}\psi\right)$$

- α' and β' are constant dimensionless parameters amounting to the normalization of the auxiliary fields
- $\mathcal{L}^{\text{flat}}_{\text{bdy}}$ has scale-weight L^2 as the bulk Lagrangian

Full Lagrangian:

$$\begin{aligned}\mathcal{L}^{\text{flat}}_{\text{full}} &= \mathcal{L}^{\text{flat}}_{\text{bulk}} + \mathcal{L}^{\text{flat}}_{\text{bdy}} \\ &= \frac{1}{4}\mathcal{R}^{ab} V^c V^d \epsilon_{abcd} - \bar{\psi}\gamma_5\gamma_a\rho V^a + \alpha' \left(2\mathcal{R}^{ab}\mathcal{F}^{cd}\epsilon_{abcd}\right) - \mathrm{i}\beta' \left(2\bar{\sigma}\gamma_5\rho - \frac{1}{2}\mathcal{R}^{ab}\bar{\chi}\gamma_5\gamma_{ab}\psi\right)\end{aligned}$$

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SUSY invariance of $\mathcal{L}^{\text{flat}}_{\text{full}}$ requires to verify the condition $\imath_\epsilon\left(\mathcal{L}^{\text{flat}}_{\text{full}}\right)|_{\partial\mathcal{M}_4} = 0$

Boundary contributions to the field eqs. result in

$$\begin{cases} \mathcal{R}^{ab}|_{\partial\mathcal{M}_4} = 0 \\ \mathcal{F}^{ab}|_{\partial\mathcal{M}_4} = -\dfrac{1}{8\alpha'}\left(V^aV^b + \beta'\bar{\chi}\gamma^{ab}\psi\right)_{\partial\mathcal{M}_4} \\ \rho|_{\partial\mathcal{M}_4} = 0 \\ \sigma|_{\partial\mathcal{M}_4} = \dfrac{\mathrm{i}}{2\beta'}\left(\gamma_a\psi V^a\right)_{\partial\mathcal{M}_4} \end{cases}$$

$\Rightarrow$ Supercurvatures dynamically fixed on $\partial\mathcal{M}_4$ to constant values in an enlarged anholonomic basis, and

$$\imath_\epsilon\left(\mathcal{L}^{\text{flat}}_{\text{full}}\right)|_{\partial\mathcal{M}_4} = 0\,, \quad \alpha' \neq 0\,,\ \beta' \neq 0$$

For $\alpha' = -1/8$ and $\beta' = 1$ (normalization) the emerging algebraic structure is more transparent:

$$\mathcal{L}^{\text{flat}}_{\text{full}} = -\frac{1}{4}\mathcal{R}^{ab} \wedge \hat{\mathcal{F}}^{cd}\epsilon_{abcd} - 2\mathrm{i}\bar{\Xi}\gamma_5 \wedge \rho$$

$\Rightarrow$ "MacDowell-Mansouri-like" Lagrangian, where

$$\hat{\mathcal{F}}^{ab} \equiv \mathcal{F}^{ab} - V^a V^b - \bar{\chi}\gamma^{ab}\psi$$

$$\Xi \equiv \sigma - \frac{\mathrm{i}}{2}\gamma_a \psi V^a$$

The latter, along with

$$R^{ab} \equiv \mathcal{R}^{ab}$$

$$\Psi \equiv \rho$$

$$R^a \equiv \mathcal{D}V^a - \frac{\mathrm{i}}{2}\bar{\psi}\gamma^a\psi$$

reproduce the so-called (minimal) Maxwell-covariant supercurvatures

Interpret the boundary constraints

$$R^{ab}|_{\partial\mathcal{M}_4} = 0\,, \quad \hat{\mathcal{F}}^{ab}|_{\partial\mathcal{M}_4} = 0\,, \quad \Psi|_{\partial\mathcal{M}_4} = 0\,, \quad \Xi|_{\partial\mathcal{M}_4} = 0$$

as the condition that the super-Maxwell algebra emerges as global symmetry at the boundary
(Consistency of the bulk theory: $R^a = 0 \Rightarrow$ For continuity, we also require $R^a|_{\partial\mathcal{M}_4} = 0$)

Super-Maxwell algebra:

$$[J_{ab}, J_{cd}] \propto \eta_{bc}J_{ad} - \eta_{ac}J_{bd} - \eta_{bd}J_{ac} + \eta_{ad}J_{bc}$$
$$[J_{ab}, P_c] \propto \eta_{bc}P_a - \eta_{ac}P_b\,, \quad [P_a, P_b] \propto Z_{ab}$$
$$[J_{ab}, Z_{cd}] \propto \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac} + \eta_{ad}Z_{bc}$$
$$[J_{ab}, Q] \propto \gamma_{ab}Q\,, \quad [J_{ab}, \Sigma] \propto \gamma_{ab}\Sigma\,, \quad [P_a, Q] \propto \gamma_a\Sigma$$
$$\{Q, Q\} \propto C\gamma^a P_a\,, \quad \{Q, \Sigma\} \propto C\gamma^{ab}Z_{ab}$$

$\Rightarrow$ Full Lagrangian in terms of the Maxwell supercurvatures:

$$\mathcal{L}^{\text{flat}}_{\text{full}} = -\frac{1}{4}R^{ab}\wedge\hat{\mathcal{F}}^{cd}\epsilon_{abcd} - 2\mathrm{i}\bar{\Xi}\gamma_5\wedge\Psi$$

- A^{ab} and χ auxiliary fields under the bulk perspective, they implement through their field eqs. the Bianchi identities of Lorentz and SUSY (with $R^a = 0$, consistency requirement):

$$\text{e.o.m. } A^{ab} \quad \leftrightarrow \quad \mathcal{D}\mathcal{R}^{ab} = 0$$

$$\text{e.o.m. } \chi \quad \leftrightarrow \quad \mathcal{D}\rho - \frac{1}{4}\mathcal{R}^{ab}\gamma_{ab}\psi = 0$$

- E.o.m. of ω^{ab} and ψ:

$$\text{e.o.m. } \omega^{ab} \quad \leftrightarrow \quad \mathcal{D}\hat{\mathcal{F}}^{ab} - 2R^{[a}{}_{c}A^{c|b]} + \bar{\Xi}\gamma^{ab}\psi - \bar{\chi}\gamma^{ab}\Psi = 0$$

$$\text{e.o.m. } \psi \quad \leftrightarrow \quad \mathcal{D}\Xi - \frac{1}{4}R^{ab}\gamma_{ab}\chi + \frac{\mathrm{i}}{2}\gamma_a\Psi V^a = 0$$

The RS e.o.m. of the gravitino is hidden in the 2nd $\rightarrow$ It can be retrieved if we restrict the auxiliary field χ to be defined only on the boundary

- E.o.m. of V^a:

$$\frac{1}{2}V^b\mathcal{R}^{cd}\epsilon_{abcd} - \bar{\psi}\gamma_a\gamma_5\rho = 0$$

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$\mathcal{L}^{\text{flat}}_{\text{full}}$ cannot be directly obtained as a flat limit of $\mathcal{L}^{\mathcal{N}=1}_{\text{full}}$ $\rightarrow$ Nevertheless, $\mathcal{L}^{\text{flat}}_{\text{full}}$ as $\ell \rightarrow \infty$ limit of a theory originating from AdS_4 SUGRA (but with super AdS-Lorentz covariance), extra 1-form gauge fields not only in the boundary Lagrangian but also in the bulk one

Recovering flat SUGRA with boundary from super-AdS$_4$

Start from the AdS$_4$ SUGRA and perform the following redefinition:

- Introduce a torsionful spin connection: $\hat{\omega}^{ab} \equiv \omega^{ab} + \frac{1}{\ell^2} A^{ab}$ so that

$$\mathcal{R}^{ab} \to \hat{\mathcal{R}}^{ab} = \mathrm{d}\omega^{ab} + \omega^a{}_c \omega^{cb} + \frac{1}{\ell^2} \mathcal{D}_{(\omega)} A^{ab} + \frac{1}{\ell^4} A^a{}_c A^{cb} \equiv \mathcal{R}^{ab} + \frac{1}{\ell^2} \mathbb{F}^{ab}$$

$$R^a \to \hat{R}^a = \mathcal{D}_{(\omega)} V^a + \frac{1}{\ell^2} A^a{}_b V^b - \frac{\mathrm{i}}{2} \bar{\psi} \gamma^a \psi$$

where $\mathbb{F}^{ab} \equiv \mathcal{D}_{(\omega)} A^{ab} + \frac{1}{\ell^2} A^a{}_c A^{cb}$

- Redefine the gravitino 1-form with the introduction of the new spinor 1-form χ: $\psi \to \psi + \frac{1}{\ell} \chi$ so that

$$\hat{R}^a \to \mathfrak{R}^a \equiv \mathcal{D}_{(\omega)} V^a - \frac{\mathrm{i}}{2} \bar{\psi} \gamma^a \psi + \frac{1}{\ell^2} A^a{}_b V^b - \frac{\mathrm{i}}{\ell} \bar{\psi} \gamma^a \chi - \frac{\mathrm{i}}{2\ell^2} \bar{\chi} \gamma^a \chi$$

$$\rho \to \hat{\rho} = \mathcal{D}_{(\omega)} \psi + \frac{1}{\ell} \left(\mathcal{D}_{(\omega)} \chi + \frac{1}{4\ell} A^{ab} \gamma_{ab} \psi + \frac{1}{4\ell^2} A^{ab} \gamma_{ab} \chi \right) \equiv \rho + \frac{1}{\ell} \Phi$$

where $\Phi \equiv \mathcal{D}_{(\omega)} \chi + \frac{1}{4\ell} A^{ab} \gamma_{ab} \psi + \frac{1}{4\ell^2} A^{ab} \gamma_{ab} \chi$

⇒ Redefined super field strengths:

$$\mathcal{R}^{ab} \equiv \mathrm{d}\omega^{ab} + \omega^{a}{}_{c}\omega^{cb}$$
$$\mathfrak{R}^{a} \equiv \mathcal{D}V^{a} - \frac{\mathrm{i}}{2}\bar{\psi}\gamma^{a}\psi + \frac{1}{\ell^{2}}A^{a}{}_{b}V^{b} - \frac{\mathrm{i}}{\ell}\bar{\psi}\gamma^{a}\chi - \frac{\mathrm{i}}{2\ell^{2}}\bar{\chi}\gamma^{a}\chi$$
$$\rho \equiv \mathcal{D}\psi$$
$$\mathbb{F}^{ab} \equiv \mathcal{D}A^{ab} + \frac{1}{\ell^{2}}A^{a}{}_{c}A^{cb}$$
$$\Phi \equiv \mathcal{D}\chi + \frac{1}{4\ell}A^{ab}\gamma_{ab}\psi + \frac{1}{4\ell^{2}}A^{ab}\gamma_{ab}\chi$$

⇒ Bulk Lagrangian:

$$\begin{aligned}\mathcal{L}^{\ell}_{\text{bulk}} = {} & \frac{1}{4}\epsilon_{abcd}R^{ab}V^{c}V^{d} + \frac{1}{4\ell^{2}}\epsilon_{abcd}\mathbb{F}^{ab}V^{c}V^{d} - \bar{\psi}\gamma_{5}\gamma_{a}\rho V^{a} - \frac{1}{\ell}\bar{\psi}\gamma_{5}\gamma_{a}\Phi V^{a} \\ & - \frac{1}{\ell^{2}}\bar{\chi}\gamma_{5}\gamma_{a}\Phi V^{a} - \frac{1}{\ell}\bar{\chi}\gamma_{5}\gamma_{a}\rho V^{a} - \frac{1}{8\ell^{2}}\epsilon_{abcd}V^{a}V^{b}V^{c}V^{d} \\ & - \frac{\mathrm{i}}{2\ell}\bar{\psi}\gamma_{5}\gamma_{ab}\psi V^{a}V^{b} - \frac{\mathrm{i}}{\ell^{2}}\bar{\chi}\gamma_{5}\gamma_{ab}\psi V^{a}V^{b} - \frac{\mathrm{i}}{2\ell^{3}}\bar{\chi}\gamma_{5}\gamma_{ab}\chi V^{a}V^{b}\end{aligned}$$

In the presence of a non-trivial boundary, consider the full Lagrangian:

$$
\begin{aligned}
\mathcal{L}^{\ell}_{\text{full}} &= \mathcal{L}^{\ell}_{\text{bulk}} + \mathcal{L}^{\ell}_{\text{bdy}} \\
&= \frac{1}{4}\epsilon_{abcd}R^{ab}V^cV^d + \frac{1}{4\ell^2}\epsilon_{abcd}\mathbb{F}^{ab}V^cV^d - \bar{\psi}\gamma_5\gamma_a\rho V^a - \frac{1}{\ell}\bar{\psi}\gamma_5\gamma_a\Phi V^a \\
&- \frac{1}{\ell^2}\bar{\chi}\gamma_5\gamma_a\Phi V^a - \frac{1}{\ell}\bar{\chi}\gamma_5\gamma_a\rho V^a - \frac{1}{8\ell^2}\epsilon_{abcd}V^aV^bV^cV^d \\
&- \frac{\mathrm{i}}{2\ell}\bar{\psi}\gamma_5\gamma_{ab}\psi V^aV^b - \frac{\mathrm{i}}{\ell^2}\bar{\chi}\gamma_5\gamma_{ab}\psi V^aV^b - \frac{\mathrm{i}}{2\ell^3}\bar{\chi}\gamma_5\gamma_{ab}\chi V^aV^b \\
&+ \mu\epsilon_{abcd}\left(2R^{ab}\mathbb{F}^{cd} + \frac{1}{\ell^2}\mathbb{F}^{ab}\mathbb{F}^{cd}\right) \\
&- \mathrm{i}\nu\left(2\bar{\rho}\gamma_5\Phi + \bar{\Phi}\gamma_5\Phi - \frac{1}{2}R^{ab}\bar{\psi}\gamma_5\gamma_{ab}\chi - \frac{1}{4\ell}\mathbb{F}^{ab}\bar{\psi}\gamma_5\gamma_{ab}\psi\right. \\
&\left.-\frac{1}{2\ell^2}\mathbb{F}^{ab}\bar{\psi}\gamma_5\gamma_{ab}\chi - \frac{1}{4\ell}R^{ab}\bar{\chi}\gamma_5\gamma_{ab}\chi - \frac{1}{4\ell^3}\mathbb{F}^{ab}\bar{\chi}\gamma_5\gamma_{ab}\chi\right)
\end{aligned}
$$

Boundary contributions to the field eqs. $\Rightarrow$ Supercurvatures fixed to constant values on $\partial\mathcal{M}_4$ in an enlarged anholonomic basis:

$$\begin{cases} R^{ab}|_{\partial\mathcal{M}_4} = -\frac{\nu}{16\mu\ell}\left(\bar{\psi}\gamma^{ab}\psi\right)_{\partial\mathcal{M}_4} \\ \mathbf{F}^{ab}|_{\partial\mathcal{M}_4} = -\frac{1}{8\mu}\left(V^a V^b + \nu\bar{\chi}\gamma^{ab}\psi + \frac{\nu}{2\ell}\bar{\chi}\gamma^{ab}\chi\right)_{\partial\mathcal{M}_4} \\ \rho|_{\partial\mathcal{M}_4} = 0 \\ \Phi|_{\partial\mathcal{M}_4} = \frac{\mathrm{i}}{2\nu}\left(\gamma_a\psi V^a + \frac{1}{\ell}\gamma_a\chi V^a\right)_{\partial\mathcal{M}_4} \end{cases}$$

Condition $\imath_\epsilon\left(\mathcal{L}^\ell_{\text{full}}\right)|_{\partial\mathcal{M}_4} = 0$ for SUSY of the full Lagrangian realized when ($h \neq -1$)

$$\mu = -\frac{1}{8}\frac{1}{1-h^2}\,, \quad \nu = \frac{1}{1-h}$$

Setting $h = 0 \Rightarrow \mu = -1/8$ and $\nu = 1$; $\mathcal{L}^\ell_{\text{full}} \to$ MacDowell-Mansouri-like:

$$\mathcal{L}^\ell_{\text{full}} = -\frac{1}{4}\mathfrak{R}^{ab}\mathfrak{F}^{cd}\epsilon_{abcd} - \frac{1}{8\ell^2}\mathfrak{F}^{ab}\mathfrak{F}^{cd}\epsilon_{abcd} - 2\mathrm{i}\bar{\Omega}\gamma_5\rho - \frac{\mathrm{i}}{\ell}\bar{\Omega}\gamma_5\Omega$$

Super field strengths in $\mathcal{L}^{\ell}_{\text{full}}$:

$$
\begin{aligned}
\mathfrak{R}^{ab} &\equiv d\omega^{ab} + \omega^a{}_c\omega^{cb} - \frac{1}{2\ell}\bar{\psi}\gamma^{ab}\psi \\
\mathfrak{R}^{a} &\equiv \mathcal{D}V^a - \frac{i}{2}\bar{\psi}\gamma^a\psi + \frac{1}{\ell^2}A^a{}_bV^b - \frac{i}{\ell}\bar{\psi}\gamma^a\chi - \frac{i}{2\ell^2}\bar{\chi}\gamma^a\chi \\
\rho &\equiv \mathcal{D}\psi \\
\mathfrak{F}^{ab} &\equiv \mathcal{D}A^{ab} - V^aV^b - \bar{\chi}\gamma^{ab}\psi + \frac{1}{\ell^2}A^a{}_cA^{cb} - \frac{1}{2\ell}\bar{\chi}\gamma^{ab}\chi \\
\Omega &\equiv \mathcal{D}\chi - \frac{i}{2}\gamma_a\psi V^a - \frac{i}{2\ell}\gamma_a\chi V^a + \frac{1}{4\ell}A^{ab}\gamma_{ab}\psi + \frac{1}{4\ell^2}A^{ab}\gamma_{ab}\chi
\end{aligned}
$$

R.h.s. of these supercurvatures to zero (from boundary constraints): Maurer-Cartan eqs. associated with a SUSY extension of the so-called AdS-Lorentz algebra (semi-simple extension of Poincaré algebra)

$\ell \to \infty$ of $\mathcal{L}^{\ell}_{\text{full}}$ is precisely $\mathcal{L}^{\text{flat}}_{\text{full}}$

(Holds also for the supercurvatures and global symmetry at the boundary: Super AdS-Lorentz in the limit $\ell \to \infty$ reduces to super-Maxwell)

APPLICATIONS OF THE FORMALISM TO ASYMPTOTIC BOUNDARIES

☞ AVZ $D = 3$ model (exhibiting "unconventional SUSY") from $\mathcal{N} = 2$, $D = 4$ pure SUGRA with a $3D$ boundary

L. Andrianopoli, B. L. Cerchiai, R. D'Auria, M. Trigiante, 1801.08081

- AVZ model: Based on a $3D$ CS Lagrangian with $\mathrm{OSp}(2|2)$ supergroup, but features a Dirac spinor $\chi^{(\mathrm{AVZ})}$ as the only propagating d.o.f.; Important applications in the description of graphene-like systems near the Dirac points

  P. D. Alvarez, M. Valenzuela, J. Zanelli, 1109.3944

- $\chi^{(\mathrm{AVZ})}$ emerges by imposing the following condition on the spacetime component of the odd CS connection 1-form Ψ:

$$\chi^{(\mathrm{AVZ})}_\alpha = \mathrm{i}\left(\gamma^i\right)_{\alpha\beta}\Psi^\beta_\mu e^\mu_i \qquad (\alpha, \beta = 1, 2\,,\ i = 0, 1, 2\,,\ \mu = 0, 1, 2)$$

- Correspondence with the CS model of AVZ found for specific choice of the $D = 3$ boundary: Local AdS_3 geometry at spatial infinity of the $D = 4$ theory (asymptotically AdS_4 solutions featuring this boundary geometry comprise the "ultraspinning limit" of AdS_4-Kerr black hole)

Applications of the formalism to asymptotic boundaries

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☞ $\mathcal{N}=2$ SUSY extension of EGB term $\rightarrow$ Counterterms for holographic renormalization? Holographic framework for $\mathcal{N}=2$, $D=4$ pure AdS_4 SUGRA, including all the contributions from the fermionic fields

L. Andrianopoli, B. L. Cerchiai, R. Matrecano, O. Miskovic, R. Noris, R. Olea, L. R., M. Trigiante, 2010.02119

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☞ Possible applications to flat SUGRA in a holographic context? $\rightarrow$ A natural boundary dual to flat gravity has been recently identified in the framework of Carrollian fluids ($BMS_4 \cong$ conformal Carroll) $\rightarrow$ At SUSY level?

L. Ciambelli, C. Marteau, A. C. Petkou, P. M. Petropoulos, K. Siampos, 1802.06809

Open directions

Regarding the geometric approach to the boundary problem in SUGRA:

☞ Extension to higher-dimensional, as well as to $\mathcal{N}$-extended, pure or matter coupled, SUGRA models (including fields with spin < 1)

- The SUSY extension of the EGB term is unique for a given theory with $\mathcal{N} \geq 2$ SUSY; it is total derivative, corresponding to a boundary term in superspace
- Topological index in superspace associated with this invariant?
- Could be investigated using the formalism of integral forms in superspace

L. Castellani, R. Catenacci, P. A. Grassi, 1409.0192, 1503.07886

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☞ Holographic contact with the AVZ model and "SCFT side" $\rightarrow$ Dual field theory of which the AVZ model provides an effective description? Still dual SCFT?

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- Role of the "topological auxiliary fields" A^{ab} and χ?
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Thank you!