Quantum Improved Regular Kerr Black Holes

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15 February 2024 @ Proca Seminars Series

CMC, Y. Chen, A. Ishibashi, N. Ohta and D. Yamaguchi, PRD 105 (2022) 106026 [arXiv:2204.09892 [hep-th]],
CMC, Y. Chen, A. Ishibashi and N. Ohta, CQG 40 (2023) 215007 [arXiv:2303.04304 [hep-th]]
CMC, Y. Chen, A. Ishibashi and N. Ohta, [arXiv:2308.16356 [hep-th]]

Outline

- Asymptotically safe gravity
- Running couplings: Newton coupling and cosmological constant
- Identification: Consistency with thermodynamics
- Phase structure of quantum improved Schwarzschild-(A)dS black holes
- Quantum improved regular Kerr black holes
- Discussion

Asymptotically Safe Gravity

- Black holes, cosmological models et al. have various kinds of singularity. Hawking, Penrose
- Einstein equations (classical) are not valid near the curvature divergent points.
- Can/How quantum effects resolve singularity?
- Asymptotically safe gravity: quantum gravity by functional renormalization group
  - Physical couplings depend on the energy scale.
  - All dimensionless couplings go to finite fixed point at UV (a finite theory).

Reuter [hep-th/9605030]

Niedermaier, Reuter (2006) Living Rev. Rel. **9**, 5

Percacci (2017); Platania (2018); Ohta (2021, Japanese)

Asymptotically Safe Gravity

- Quantum improved black holes/cosmology:
  - Action level: (very difficult) Reuter, Weyer [hep-th/0311196]
  - Equation level: (not easy) Platania [2302.04272 [gr-qc]]
  - Solution level: energy scale of prober/observer
- Quantum effects: replacing couplings (const.) in classical "solutions" with running couplings (dep. on energy).
- It still needs a suitable choice of the identification of the energy scale with some length scale in the solution.

Running Couplings

- Einstein theory with a cosmological constant

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g}\,(R - 2\Lambda)$$

- Running Newton coupling and cosmological constant: dep. on energy scale k (dominated effects with small Λ/k^2)

$$G(k) = \frac{G_0}{1 + \omega G_0 k^2}, \quad \Lambda(k) = \Lambda_0 + \lambda k^2$$

Bonanno, Reuter [hep-th/0002196]

Pawlowski, Stock [1807.10512 [hep-th]]

- Scale identification: position-dependent energy scale

$$k = \xi / d(P)$$

$d(P)$: distance scale, ξ: dimensionless constant

- How the energy scale is related to the distance scale?

Running Newton Coupling: Schwarzschild

- Scale identification: for Schwarzschild black holes
- geodesic distance: Bonanno, Reuter [hep-th/0002196]

$$d(r) = \begin{cases} r, & r \to \infty \\ \frac{2r^{3/2}}{3\sqrt{2G_0 M}}, & r \to 0 \end{cases} \quad \Rightarrow \quad d(r) = \left(\frac{r^3}{r + \gamma G_0 M} \right)^{1/2}$$

  - Newton coupling: $\gamma = 9/2$

$$G(r) = \frac{G_0 r^3}{r^3 + \omega \xi^2 G_0 (r + \gamma G_0 M)}$$

  - for small distance: The singularity at origin is resolved.

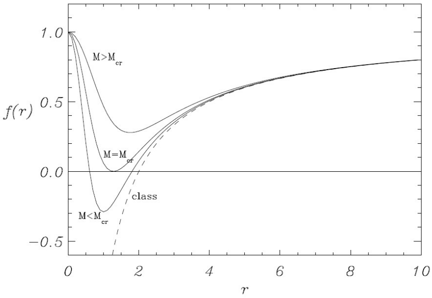
$$G(r) \approx \frac{r^3}{\omega \xi^2 \gamma G_0 M} \qquad \text{(repulsive force, } \Phi \sim -r^2\text{)}$$

Running Newton Coupling: Schwarzschild

- laps function:

$$f(r) = 1 - \frac{2G(r)M}{r}$$

- Identification depends on choice of coordinates/boundary conditions.



- Kretschmann invariant: Pawlowski, Stock [1807.10512 [hep-th]]

$$k = \xi \left(R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}\right)^{1/4} = \xi \left(48M^2G_0^2/r^6\right)^{1/4}$$

  - Newton coupling

$$G(r) = \frac{G_0 r^3}{r^3 + 4\sqrt{3}\omega\xi^2 G_0^2 M}$$

Running Newton Coupling: Kerr

- Scale identification: for Kerr black holes

  Pawlowski, Stock [1807.10512 [hep-th]]

- Both above identifications lead to an angle dependent Newton coupling $G = G(r, \theta)$
  - Horizon is not a round sphere. $\Delta = r^2 - 2G(r,\theta)Mr + a^2 = 0$
  - Surface gravity may not be a constant. (not thermal equilibrium)
  - singularity at “horizon”: $\Sigma = r^2 + a^2 \cos^2\theta$

$$R = \frac{2M}{\Sigma}\left[r\partial_r^2 G(r,\theta) + 2\partial_r G(r,\theta) - \frac{Mr^2}{\Delta^2}\Big(\partial_\theta G(r,\theta)\Big)^2\right]$$

Running Newton Coupling: Kerr

- Identification in Eddington-Finkelstein coordinates
  - This gives a different solution.
  - The scalar curvature does not diverge.

    Held [2105.11458 [gr-qc]]
  - This solution encounters another problem of “parallelly propagated curvature singularity.”

    CMC, Chen, Ishibashi, Ohta [2308.16356 [hep-th]]
- The angular dependence should be excluded.
- angle-independent $G(r)$:
  - Consistent BH thermodynamics (system with multiple variables)

$$S(M,a) \quad \Rightarrow \quad \partial_a \partial_M S = \partial_M \partial_a S$$

  - This is a fundamental requirement which is independent of theory.

Running Newton Coupling: Kerr

- First law of thermodynamics

$$dM = TdS + \Omega dJ \quad \Rightarrow \quad dS = (dM - \Omega dJ)/T$$

  - mass: $\Delta(r_+) = 0$ (assuming $\partial_M G(r) = 0$)

$$M = (r_+^2 + a^2)/2G(r_+)r_+ \qquad \boxed{\partial_a \partial_+ S = \partial_+ \partial_a S}$$

  - angular momentum and angular velocity

$$J = Ma, \qquad \Omega = a/(r_+^2 + a^2)$$

  - temperature: κ surface gravity

$$T = \frac{\kappa}{2\pi} = \frac{(r_+^2 - a^2)G(r_+) - r_+(r_+^2 + a^2)\partial_+ G(r_+)}{4\pi r_+(r_+^2 + a^2)G(r_+)}$$

  - entropy: $S = S(r_+; a) \rightarrow dS = \partial_+ S\, dr_+ + \partial_a S\, da$

$$\partial_+ S = \frac{\partial_+ M - \Omega\, \partial_+ J}{T}, \qquad \partial_a S = \frac{\partial_a M - \Omega\, \partial_a J}{T}$$

Consistency of Thermodynamics

- Newton coupling is also independent of a:
  - consistency check:

$$\begin{aligned} & \partial_a\left(\partial_+ S\right) = \partial_+\left(\partial_a S\right) \\ \Rightarrow \quad & r_+(r_+^2 - a^2)\partial_+^2 G(r_+) - 2a^2\partial_+ G(r_+) = 0 \\ \Rightarrow \quad & \partial_+^2 G(r_+) = 0, \quad \partial_+ G(r_+) = 0 \end{aligned}$$

  - The coupling does not run in order to ensure the first law.
- There has not been any physical principle to determine the scale identification.
- Guiding principle: Identification, at least near horizon, should be consistent with the first law of thermodynamics.

Consistency of Thermodynamics: Mass Independent

- Mass independent Newton coupling: $G = G(r; a)$

  CMC, Chen, Ishibashi, Ohta, Yamaguchi, PRD (2022) [2204.09892]

  - The derivatives of entropy from first law

$$
\begin{aligned}
\partial_+ S &= \frac{2\pi r_+}{G(r_+, a)} \\
\partial_a S &= \frac{2\pi \left[r_+^2 (r_+^2 + a^2) \partial_a G(r_+, a) - a (r_+^2 - a^2) G(r_+, a) \right]}{G(r_+, a) \left[r_+ (r_+^2 + a^2) \partial_+ G(r_+, a) - (r_+^2 - a^2) G(r_+, a) \right]}
\end{aligned}
$$

  - consistency

$$
\partial_+ \frac{r_+^2 (r_+ \partial_a G - a \partial_+ G)}{r_+ (r_+^2 + a^2) \partial_+ G - (r_+^2 - a^2) G} = 0
$$

  - simple solution: $A_h = 4\pi (r_+^2 + a^2)$

$$
r_+ \partial_a G - a \partial_+ G = 0 \quad \Rightarrow \quad G(r_+; a) = G(r_+^2 + a^2) = G(A_h)
$$

Consistency of Thermodynamics: Mass Independent

- derivatives of entropy $A_h = 4\pi(r_+^2 + a^2)$

$$\partial_+ S = 2\pi r_+/G(A_h),\ \partial_a S = 2\pi a/G(A_h) \ \Rightarrow\ dS = dA_h/4G(A_h)$$

- Universal formula for quantum entropy

$$S(A_h) = \int \frac{dA_h}{4G(A_h)}$$

- This formula is valid also for
  - 5D Myers-Perry black holes (2 angular momenta)
  - Kaluza-Klein black strings
  - Kerr-(A)dS black holes (with a constant Λ)
- The same consequence was discussed/obtained by "assuming" to preserve the relation of entropy variation $\delta S = \delta A_h/4G$. Falls, Litim, PRD (2014) [1212.1821]

Consistency of Thermodynamics: Mass Independent

- suggested identification: simple dimension analysis

$$k = \frac{\xi}{\sqrt{A}} = \frac{\tilde{\xi}}{(r_+^2 + a^2)^{1/2}} \Rightarrow G(r_+; a) = \frac{G_0(r_+^2 + a^2)}{r_+^2 + a^2 + \tilde{\omega} G_0}$$

- quantum improved entropy

$$S = \frac{\pi(r_+^2 + a^2)}{G_0} + \pi\tilde{\omega} \ln(r_+^2 + a^2)$$

  - A typical logarithmic correction to the Bekenstein-Hawking formula.

- Natural idea: extending the identification away from horizon

$$k = k(r_+^2 + a^2) \to k(r^2 + a^2), \quad G = G(r^2 + a^2)$$

  - It is "impossible" to resolve singularity of rotating BHs.

$$\lim_{r \to 0} k(r^2 + a^2) \xrightarrow{?} \infty \quad \text{or} \quad \lim_{r \to 0} G(r^2 + a^2) \xrightarrow{?} 0$$

Phase Structure of Quantum Improved Schwarzschild

- Phase structure of quantum improved Sch.-AdS BHs

  CMC, Chen, Ishibashi, Ohta, CQG (2023) [2303.04304]

  - Quantum effect provides a repulsive force in the core region near singularity.
  - It stabilizes the thermodynamically unstable small black holes, and also creates a zero temperature state with finite size (candidate for dark matter).
  - We find a new second order phase transition between small and large black holes for quantum improved Schwarzschild-Anti de Sitter black holes.
  - We also discuss the black holes with different spatial topologies and find a notable duality.

Consistency of Thermodynamics: Mass Dependent

- Is it possible to revolve the ring singularity of Kerr BHs?

  CMC, Y. Chen, A. Ishibashi and N. Ohta, [arXiv:2308.16356 [hep-th]]

  - necessary condition: $\lim_{r\to 0} G \to 0$
    but $\lim_{r\to 0} G(r^2+a^2) \nrightarrow 0$

- An interesting observation

$$\Delta(r_+) = r_+^2 - 2G(r_+^2+a^2)Mr_+ + a^2 = 0$$

$$\Rightarrow \qquad Mr_+ = \frac{r_+^2+a^2}{2G(r_+^2+a^2)}$$

  - A function of Mr_+ can be reexpressed as a function of $r_+^2 + a^2 = A_h/4\pi$.
  - Another possible consistent identification: $G = G(Mr_+)$

Consistency of Thermodynamics: Mass Dependent

- General identification: mass dependent

$$G = G(r_+; a, M)$$

- The consistency gives the following condition:

$$Ma\partial_M G + r_+^2 \partial_a G - ar_+ \partial_+ G = 0$$

- consistent identification

$$G(r_+; M, a) = G(Mr_+, r_+^2 + a^2) \quad \xrightarrow{\Delta(r_+)=0} \quad G(A_h)$$

- Generalized formula for the entropy: $M\partial_M G$ is still a function of area.

$$S = \int \frac{dA_h}{4(G + M\partial_M G)}$$

Quantum Improved Regular Kerr

- It is possible to resolve ring singularity by a natural extension of the identification. (Mr: dimensionless)

$$k^2 = \frac{\xi^2}{G_0 M^3 r^3}$$

- Newton coupling

$$G(r) = \frac{G_0 M^3 r^3}{M^3 r^3 + \tilde{\omega}}$$

  - asymptotic value:

$$G(\infty) = G_0$$

  - properties at $r = 0$

$$G(0) = 0, \quad G'(0) = 0, \quad G''(0) = 0$$

  - The quantum improved Kerr black holes are regular.

Torres, [arXiv:2208.12713 [gr-qc]]

Quantum Improved Regular Kerr

- Quantum improved Schwarzschild: “Hayward black holes”

$$f(r) = 1 - \frac{2G_0 M^4 r^2}{M^3 r^3 + \omega} = 1 - \frac{2G_0 M r^2}{r^3 + \omega/M^3}$$

  - The blue part of Hayward black holes is a parameter which is independent of mass.

- We can construct quantum improved “likely” regular Kerr-(A)dS as a generalization of Hayward black holes.

- Peculiar property?

$$\lim_{r\to 0} k = \lim_{M\to 0} k \to \infty, \qquad \lim_{r\to 0} G = \lim_{M\to 0} G \to 0$$

Quantum Improved Regular Kerr

- Is it really a regular black hole?
- Kretschmann scalar on the disk $r \to 0$
  - classical Kerr: ring singularity at $\theta = \pi/2$ and discontinuous

$$\lim_{r\to 0} K = \frac{48M^2G^2}{r^6} \to \infty \quad (\theta = \pi/2) \quad \text{or} \quad -\frac{48M^2G^2}{a^6\cos^6\theta} \quad (\theta \neq \pi/2)$$

  - quantum improved Kerr with $G''(0) = 0$: no divergent at $\theta = \pi/2$ but still discontinuous

$$\lim_{r\to 0} K = \frac{96G_0^2M^8}{\omega^2} \quad (\theta = \pi/2) \quad \text{or} \quad 0 \quad (\theta \neq \pi/2)$$

- We may need more a strong condition, such as $G'''(0) = 0$

Quantum Improved Regular Kerr

- a simple identification

$$k^2 = \frac{1}{G_0 M^p r^p}, \qquad G(r) = \frac{G_0 M^p r^p}{M^p r^p + \omega}$$

- Curvature and Kretschmann scalar on the disk $r \to 0$

$$R \sim \begin{cases} r^{p-1}, & \theta \neq \pi/2 \\ r^{p-3}, & \theta = \pi/2 \end{cases}, \quad K \sim \begin{cases} r^{2(p-1)}, & \theta \neq \pi/2 \\ r^{2(p-3)}, & \theta = \pi/2 \end{cases}$$

- Similarly to the Hayward black holes, the geodesics are not smooth at $r = 0$ for any value of p.
- The extension to negative value of r is inevitable, as far as the unique continuation is required.

Quantum Improved Regular Kerr

- "new" singularity at $M^p r^p = -\omega$ for odd p

$$R = -\frac{2\omega G_0 p M^{p+1} r^{p-1}\left[(p-1)M^p r^p - (p+1)\omega\right]}{(r^2 + a^2\cos^2\theta)(M^p r^p + \omega)^3}$$

- A physically desirable choice $p = 4$: minimal value that
  - No discontinuity at $r = 0, \theta = \pi/2$.
  - No singularity at $r < 0$.
- Closed Timelike Curves

$$g_{\varphi\varphi} = \sin^2\theta\left(r^2 + a^2 + \frac{2G_0 M^{p+1} r^{p+1} a^2 \sin^2\theta}{(M^p r^p + \omega)\Sigma}\right)$$

- Necessary conditions for $g_{\varphi\varphi}|_{\theta=\pi/2} > 0$

$$1 < \frac{\tilde{\omega}^{1/p}}{G_0 M^2}, \qquad \frac{|a|}{G_0 M} < \frac{\tilde{\omega}^{1/p}}{G_0 M^2}, \qquad \tilde{\omega} = (p-1)\omega$$

Discussion

- The consistency of the thermodynamics gives a physical principle to determine the identification.
- The Newton coupling at the horizon should be a function of the horizon area.
- Black holes have extremal limit (remnants).
- A more interesting phase structure appears in quantum improved BHs.
- Regular Kerr black hole:
  - admitting a consistent BH thermodynamics at the horizon,
  - resolving the ring singularity,
  - partially eliminating closed time-like curves present in the classical Kerr black holes.