Hidden Conformal Symmetry for Dyonic Kerr-Sen Black Hole and Its Gauged Family

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Published in Eur. Phys. J. C 83, 255 (2023).

March 2024

Outline

Background

Background

AdS and CFT?

AdS/CFT correspondence claims

Strongly-coupled 4-dimensional gauge theory = Gravitational theory in 5-dimensional AdS spacetime

or more general

$(N-1)$-dimensional quantum field theory = N-dimensional gravitational theory

AdS/CFT also claims

$$Z_{gauge} = Z_{AdS}$$

Background

Why does 5D gravitational theory correspond to 4D field theory?

Intuitive answer can be seen using a black hole (BH).

- BH $\rightarrow$ thermal system at finite temperature $\rightarrow$ entropy.
- $S_{BH} \sim A/4$ (area), different with statistical entropy $S \sim V$.
- Yet, A in N dimension is V in $(N-1)$ dimension.
- This implies:
  BH lives in 5D (AdS), yet can be portrayed by 4D field theory.

Background

Why do we study BH thermodynamics using AdS/CFT?
We want to relate S_{BH} (gravity) with quantum theory.

- Thermodynamic laws of black hole were derived originally by comparing the quantities in the common thermodynamic laws with BH's properties,

$$\text{(statistical)}\ TdS = dE + PdV \qquad \text{(BH)}\ T_H dS_{BH} = dM + \Omega dJ$$

- The problem of quantum gravity is not completely solved.
- However, Z in quantum field theory is already well-identified.
- AdS/CFT correspondence is used to study the origin S_{BH} of BH using Z from CFT.

Background

Which CFT? 2D CFT.

- Entropy of the black holes satisfies

$$S_{BH} = \frac{A}{4},$$

- S_{CFT} is Cardy formula from 2D CFT, defined by

$$S_{CFT} = \frac{\pi^2}{3}\left(c_L T_L + c_R T_R\right).$$

- $c_{L,R}$ are central charges appearing in Virasoro algebra and $T_{L,R}$ are temperatures.
- In first paper of Kerr/CFT (Guica, Hartman, Song, Strominger, PRD'09), it is shown for extremal Kerr that $S_{CFT} = S_{BH}$.

Background

Absorption cross-section

- In 2D CFT, absorption cross-section is the two-point function.
- Absorption cross-section for scalar field P_{abs} satisfies

$$P_{abs}^{CFT} \sim {T_L}^{2h_L-1} {T_R}^{2h_R-1} \sinh\left(\frac{\omega_L}{2T_L}+\frac{\omega_R}{2T_R}\right)\left|\Gamma\left(h_L+i\frac{\omega_L}{2\pi T_L}\right)\right|^2 \times \left|\Gamma\left(h_R+i\frac{\omega_R}{2\pi T_R}\right)\right|^2 .$$

- It has been shown by Castro, Maloney, Strominger (PRD'10) that

$$P_{abs}^{grav} \sim P_{abs}^{CFT},$$

in low-frequency limit of scalar wave equation.

Background

CFT Dual on Kerr BH

Extremal Kerr

- Conformal symmetry on spacetime metric,
- Spacetime isometry $\rightarrow SL(2,R) \times U(1)$, $SL(2,R) \rightarrow AdS_2$,
- $T_R, c_R = 0$ while T_L, c_L are non-zero $\rightarrow$ CFT_1.

Non-extremal Kerr

- Conformal symmetry on scalar wave equation,
- Isometry of the wave equation $\rightarrow SL(2,R) \times SL(2,R) \rightarrow AdS_3$,
- T_R, T_L, c_L, c_R are non-zero $\rightarrow$ CFT_2.

Objectives

- Finding the hidden conformal symmetry on dyonic Kerr-Sen BH and its gauged family.
- Computation of S_{BH}.
- Computation of P_{abs}.

Hidden Conformal Symmetry of Dyonic Kerr-Sen Black Hole

Metric

Dyonic Kerr-Sen (DKS) black hole's metric (Wu *et al*, PRD'21)

$$ds^2 = -\frac{\Delta}{\varrho^2}X^2 + \frac{\varrho^2}{\Delta}dr^2 + \varrho^2 d\theta^2 + \frac{\sin^2\theta}{\varrho^2}Y^2, \tag{1}$$

where

$$X = dt - a\sin^2\theta d\phi, \quad Y = adt - (r^2 - d^2 - k^2 + a^2)d\phi,$$
$$\varrho^2 = r^2 - d^2 - k^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2mr - d^2 - k^2 + a^2 + p^2 + q^2. \tag{2}$$

m, a, q, p, d, k are mass, spin, electric, magnetic, dilaton charge, and axion charges. q, p, d, k possess the following relation

$$d = \frac{p^2 - q^2}{2m}, \qquad k = \frac{pq}{m}. \tag{3}$$

Lagrangian

DKS solution is the solution to Einstein-Maxwell-Dilaton-Axion (EMDA) theory,

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial\chi)^2 - e^{-\phi}F^2\right] + \frac{\chi}{2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}, \quad (4)$$

One can write the Lagrangian (4) into the effective Lagrangian of the low energy limit of the heterotic string theory,

$$\mathcal{L}_{eff} = \sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - e^{-\phi}F^2 - \frac{1}{12}e^{-2\phi}H^2\right), \quad (5)$$

where $H^2 = H_{\mu\nu\rho}H^{\mu\nu\rho}$ is an antisymmetric tensor where it is defined by $H = d\mathcal{B} - A \wedge F/4 = -e^{2\phi} \star d\chi$.

Fields

The electromagnetic potential, its dual, dilaton, and axion fields related to metric (1) are given by

$$\mathbf{A} = \frac{q(r+d-p^2/m)}{\varrho^2}X - \frac{p\cos\theta}{\varrho^2}Y, \tag{6}$$

$$\mathbf{B} = \frac{p(r+d-p^2/m)}{\varrho^2}X + \frac{q\cos\theta}{\varrho^2}Y, \tag{7}$$

$$e^{\phi} = \frac{(r+d)^2+(k+a\cos\theta)^2}{\varrho^2}, \tag{8}$$

$$\chi = 2\frac{kr-da\cos\theta}{(r+d)^2+(k+a\cos\theta)^2}. \tag{9}$$

The dual gauge potential can be obtained from $-dB = e^{-\phi}\star F + \chi F$.

Thermodynamic Properties

Temperature, entropy, angular velocity, electric potential, and magnetic potential are given by

$$T_H = \frac{r_+ - m}{2\pi(r_+^2 - d^2 - k^2 + a^2)}, \tag{10}$$

$$S_{BH} = \pi(r_+^2 - d^2 - k^2 + a^2), \tag{11}$$

$$\Omega = \frac{a}{r_+^2 - d^2 - k^2 + a^2}, \tag{12}$$

$$\Phi = \frac{q(r_+ + d - p^2/m)}{r_+^2 - d^2 - k^2 + a^2}, \tag{13}$$

$$\Psi = \frac{p(r_+ + d - p^2/m)}{r_+^2 - d^2 - k^2 + a^2}. \tag{14}$$

The position of the inner and outer horizons are as given by

$$r_\pm = m \pm \sqrt{m^2 + d^2 + k^2 - a^2 - p^2 - q^2}. \tag{15}$$

Scalar Wave Equation

Neutral massless scalar field equation

$$\nabla_\alpha \nabla^\alpha \hat{\Phi} = 0. \tag{16}$$

We separate the coordinates in the scalar field

$$\hat{\Phi}(t, r, \theta, \phi) = \mathrm{e}^{-i\omega t + in\phi} R(r) S(\theta). \tag{17}$$

From (17) and (16), we find

$$\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta) - \frac{n^2}{\sin^2\theta} - a^2\omega^2\sin^2\theta\right] S(\theta) = -K_h S(\theta), \tag{18}$$

$$\left[\partial_r(\Delta\partial_r) + \frac{\left[(r^2 - d^2 - k^2 + a^2)\omega - an\right]^2}{\Delta} + 2an\omega\right] R(r) = K_h R(r). \tag{19}$$

where the separation constant K_h is the eigenvalues on a sphere.

Radial Wave Equation

To show the hidden conformal symmetry on radial part, we need to assume the low-frequency limit: $\omega M \ll 1, \omega a \ll 1, \omega q \ll 1, \omega p \ll 1$. Radial wave equation becomes

$$\partial_r \left[(r - r_+)(r - r_-)\partial_r\right] R(r) + \left[\frac{r_+ - r_-}{r - r_+} A + \frac{r_+ - r_-}{r - r_-} B + C\right] R(r) = 0. \quad (20)$$

where

$$A = \frac{\left[(r_+^2 - d^2 - k^2 + a^2)\omega - an\right]^2}{(r_+ - r_-)^2}, \; B = -\frac{\left[(r_-^2 - d^2 - k^2 + a^2)\omega - an\right]^2}{(r_+ - r_-)^2},$$
$$C = -K_h, \quad K_h = h(h+1) \quad (21)$$

Does it have conformal symmetry? We will use coordinate transformations.

Conformal Coordinates

Conformal (locally) coordinate transformations

$$\omega^+ = \sqrt{\frac{r-r_+}{r-r_-}}\mathrm{e}^{2\pi T_R\phi+2n_R t}, \ \omega^- = \sqrt{\frac{r-r_+}{r-r_-}}\mathrm{e}^{2\pi T_L\phi+2n_L t}, \quad (22)$$

$$y = \sqrt{\frac{r_+-r_-}{r-r_-}}\mathrm{e}^{\pi(T_L+T_R)\phi+(n_L+n_R)t}. \quad (23)$$

We can construct operators in terms of conformal coordinates

$$H_1 = i\partial_+, \ H_{-1} = i\left(\omega^{+2}\partial_+ + \omega^+ y\partial_y - y^2\partial_-\right), \ H_0 = i\left(\omega^+\partial_+ + \frac{1}{2}y\partial_y\right), \quad (24)$$

$$\bar{H}_1 = i\partial_-, \ \bar{H}_{-1} = i\left(\omega^{-2}\partial_- + \omega^- y\partial_y - y^2\partial_+\right), \ \bar{H}_0 = i\left(\omega^-\partial_- + \frac{1}{2}y\partial_y\right). \quad (25)$$

Note that $T_L\bar{H}_0 + T_R H_0 = \frac{i}{2\pi}\partial_\phi$.

$SL(2,R) \times SL(2,R)$ Isometry

Each set of conformal operators (24) and (25) satisfies the $SL(2,R)$ algebra

$$[H_0, H_{\pm 1}] = \mp i H_{\pm 1}, \quad [H_{-1}, H_1] = -2iH_0. \tag{26}$$

We find $SL(2,R) \times SL(2,R)$ isometry group $\rightarrow$ isometry of AdS_3 and CFT_2.

Each set of operators satisfies quadratic Casimir operator

$$\mathcal{H}^2 = \bar{\mathcal{H}}^2 = -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) = \frac{1}{4}(y^2 \partial_y^2 - y\partial_y) + y^2 \partial_+ \partial_-. \tag{27}$$

So, radial equation can be written as $\mathcal{H}^2 R(r) = CR(r)$.

Meanwhile, the angular equation possesses $SU(2) \times SU(2)$ isometry group.

Temperature Interpretation

$SL(2,R) \times SL(2,R)$ isometry generates conformal transformation on (ω^+, ω^-). By assuming constant r connected with the (t, ϕ)-plane, we obtain

$$\omega^{\pm} = e^{t^{\pm}} \rightarrow \qquad t^+ = 2\pi T_R \phi + 2n_R t, \quad t^- = 2\pi T_L \phi + 2n_L t. \tag{28}$$

This is precisely the relation between Minkowski $(\omega^{\pm})$ and Rindler $(t^{\pm})$ coords. In the $SL(2,R) \times SL(2,R)$ invariant Minkowski vacuum, observers at fixed position in Rindler coordinates will observe a thermal bath of Unruh radiation. By identifying the rotation on ϕ, $SL(2,R) \times SL(2,R)$ breaks down to $U(1) \times U(1)$, then we find

$$t^+ \sim t^+ + 4\pi^2 T_R, \quad t^- \sim t^- - 4\pi^2 T_L \quad \rightarrow e^{-4\pi^2 i T_R H_0 - 4\pi^2 i T_L \bar{H}_0}. \tag{29}$$

Hence, we get a thermal density matrix at those temperatures. So, this shows that the observer undergoes a thermal radiation with the temperature T_L, T_R. By comparing radial Eq. (20) and Casimir operator (27), we can identify

$$T_L = \frac{r_+^2 + r_-^2 + 2(a^2 - d^2 - k^2)}{4\pi a (r_+ + r_-)}, \quad T_R = \frac{r_+ - r_-}{4\pi a} \sim T_H. \tag{30}$$

Central charges

Central charges for non-extremal BHs can be assumed to connect smoothly with that of the extremal BHs (Castro, Maloney, Strominger, PRD'10),

$$c_L = c_R \sim c_L^{ext} \tag{31}$$

Near-horizon extremal DKS metric is given by

$$ds^2 = \Gamma(\theta)\left(-\hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + \alpha(\theta)d\theta^2\right) + \gamma(\theta)\left(d\hat{\phi} + e\hat{r}d\hat{t}\right)^2, \tag{32}$$

where

$$\Gamma(\theta) = \frac{\varrho_+^2}{\upsilon}, \quad \alpha(\theta) = \frac{\upsilon}{\Delta_\theta}, \quad \gamma(\theta) = \frac{r_0^4 \Delta_\theta \sin^2\theta}{\varrho_+^2 \Xi^2},$$
$$\varrho_+^2 = r_+^2 - d^2 - k^2 + a^2\cos^2\theta, \quad e = \frac{2ar_+\Xi}{r_0^2\upsilon}. \tag{33}$$

Central charge of the CFT related to metric above is given by (Sakti & Burikham, PRD'22)

$$c_L^{ext} = 3e\int_0^\pi d\theta\sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)} = 12ar_+. \tag{34}$$

Cardy entropy

For generic DKS black hole metric, since $r_+ \neq r_-$, we obtain

$$c_L^{ext} \to c_L = c_R = 6a(r_+ + r_-). \tag{35}$$

Another way to compute it is by using the covariant phase space formalism where it is required to include 'Wald-Zoupas' counterterms. (Haco, Hawking, Perry, Strominger, JHEP'18).

Using Cardy entropy formula from 2D CFT,

$$S_{CFT} = \frac{\pi^2}{3}(c_L T_L + c_R T_R), \tag{36}$$

we find

$$S_{CFT} = \pi(r_+^2 - d^2 - k^2 + a^2) = S_{BH}. \tag{37}$$

"Non-extremal DKS BH is holographically dual with 2D CFT"

Hidden Conformal Symmetry of Dyonic Kerr-Sen-AdS Black Hole

Metric

Dyonic Kerr-Sen-AdS (DKSAdS) black hole's metric (Wu *et al*, PRD'21),

$$ds^2 = -\frac{\Delta}{\varrho^2}X^2 + \frac{\varrho^2}{\Delta}dr^2 + \frac{\varrho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\varrho^2}Y^2, \tag{38}$$

where

$$\begin{aligned}
X &= dt - a\sin^2\theta\frac{d\phi}{\Xi}, \quad Y = adt - (r^2 - d^2 - k^2 + a^2)\frac{d\phi}{\Xi}, \\
\Delta &= (r^2 - d^2 - k^2 + a^2)\left(1 + \frac{r^2 - d^2 - k^2}{l^2}\right) - 2mr + p^2 + q^2 \\
\Delta_\theta &= 1 - \frac{a^2}{l^2}\cos^2\theta, \quad \Xi = 1 - \frac{a^2}{l^2}, \quad \varrho^2 = r^2 - d^2 - k^2 + a^2\cos^2\theta.
\end{aligned}$$

DKSAdS solution is the solution to

$$\mathcal{L}_{gauged} = \mathcal{L} + \sqrt{-g}\frac{4 + e^{-\phi} + e^{\phi}(1+\chi^2)}{l^2}. \tag{39}$$

Thermodynamic Quantities

The thermodynamic quantities of DKSAdS BH are given by

$$M = \frac{m}{\Xi}, \quad J = \frac{ma}{\Xi}, \quad Q = \frac{q}{\Xi}, \quad P = \frac{p}{\Xi}, \quad V = \frac{4}{3} r_+ S, \quad \mathcal{P} = \frac{3}{8\pi l^2}, \tag{40}$$

$$T_H = \frac{r_+(2r_+^2 - 2d^2 - 2k^2 + a^2 + l^2) - ml^2}{2\pi(r_+^2 - d^2 - k^2 + a^2)l^2}, \tag{41}$$

$$S_{BH} = \frac{\pi}{\Xi}(r_+^2 - d^2 - k^2 + a^2), \quad \Omega = \frac{a\Xi}{r_+^2 - d^2 - k^2 + a^2}, \tag{42}$$

$$\Phi = \frac{q(r_+ + d - p^2/m)}{r_+^2 - d^2 - k^2 + a^2}, \quad \Psi = \frac{p(r_+ + d - p^2/m)}{r_+^2 - d^2 - k^2 + a^2}, \tag{43}$$

In the rest frame, some quantities change as

$$M \to \frac{m}{\Xi^2}, \quad \Omega \to \Omega + \frac{a}{l^2}, \quad V \to V + \frac{4\pi}{3} aJ, \tag{44}$$

which satisfy

$$dM = T_H dS_{BH} + \Omega dJ + \Phi dQ + \Psi dP + V d\mathcal{P}. \tag{45}$$

Scalar Wave Equation

By using neutral massless scalar field equation $\nabla_\alpha \nabla^\alpha \hat{\Phi} = 0$ and ansatz $\hat{\Phi}(t, r, \theta, \phi) = \mathrm{e}^{-i\omega t + in\phi} R(r) S(\theta)$, we find

$$\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta) - \frac{n^2\Xi^2}{\sin^2\theta} + \frac{2an\omega\Xi - a^2\omega^2\sin^2\theta}{\Delta_\theta}\right] S(\theta) = -K_h S(\theta), \tag{46}$$

$$\left[\partial_r(\Delta\partial_r) + \frac{\left[(r^2 - d^2 - k^2 + a^2)\omega - an\Xi\right]^2}{\Delta} - K_h\right] R(r) = 0, \tag{47}$$

where the separation constant K_h is different with that in ungauged case. To show the conformal symmetry, it is compulsory to approximate $\Delta \simeq \upsilon(r - r_+)(r - r_*)$ in the near-horizon region in addition to low-frequency assumption where

$$\begin{aligned} r_* &= r_+ - \frac{1}{\upsilon r_+}\left[\frac{2r_+^2(2r_+^2 - 2d^2 - 2k^2 + a^2 + l^2)}{l^2} - \frac{(r_+^2 - d^2 - k^2 + a^2)}{l^2}\right. \\ &\times \left.(r_+^2 - d^2 - k^2 + l^2) + q^2 + p^2\right], \quad \upsilon = 1 + \frac{6r_+^2 - 2d^2 - 2k^2 + a^2}{l^2}. \end{aligned}$$

Radial Wave Equation

Radial wave equation

$$\partial_r \left[(r-r_+)(r-r_*)\partial_r\right] R(r) + \left[\frac{r_+-r_*}{r-r_+}A + \frac{r_+-r_*}{r-r_*}B + C\right] R(r) = 0. \qquad (48)$$

where

$$A_s = \frac{\left[(r_+^2 - d^2 - k^2 + a^2)\omega - am\Xi\right]^2}{\upsilon^2 (r_+ - r_*)^2},$$

$$B_s = -\frac{\left[(r_*^2 - d^2 - k^2 + a^2)\omega - am\Xi\right]^2}{\upsilon^2 (r_+ - r_*)^2}, \quad C_s = -\frac{K_h}{\upsilon} \qquad (49)$$

The conformal symmetry can be shown using similar coordinate transformation as ungauged case, yet by changing $r_- \to r_*$. In this case, the radial equation can be shown to have an $SL(2,R) \times SL(2,R)$ isometry group.

CFT Temperatures, Central charges, Entropy

From the conformal coordinate transformation, we can identify the CFT temperatures which are given by

$$T_L = \frac{\upsilon[r_+^2 + r_*^2 + 2(a^2 - d^2 - k^2)]}{4\pi a(r_+ + r_*)\Xi}, \quad T_R = \frac{\upsilon(r_+ - r_*)}{4\pi a\Xi}. \tag{50}$$

The central charges can be computed in the similar way, that results in

$$c_L^{ext} \to c_L = c_R = \frac{6a(r_+ + r_*)}{\upsilon}. \tag{51}$$

Then by using Cardy entropy formula from 2D CFT, we find

$$S_{CFT} = \frac{\pi}{\Xi}(r_+^2 - d^2 - k^2 + a^2) = S_{BH}.$$

"Non-extremal DKSAdS BH is holographically dual with 2D CFT"

Absorption Cross-section Dyonic Kerr-Sen Black Hole

Radial Wave Solution

To further support the dual CFT, we study scattering of non-extremal DKS BH. Firstly, we need to solve radial Eq. (20). We introduce coord. transformation $z = \frac{r-r_+}{r-r_-}$ which implies that when $r_+ \leq r \leq \infty$, we have $0 \leq z \leq 1$.

$$\left[z(1-z)\partial_z^2 + (1-z)\partial_z + \frac{A}{z} + B + \frac{C}{1-z}\right] R(z) = 0, \tag{52}$$

where the ingoing solution to that in the near-region ($r \ll 1/\omega$) is

$$R^{in}(z) = z^{-i\sqrt{A}}(1-z)^{(1+l)}\ {}_2F_1(a_s, b_s; c_s; z), \tag{53}$$

where $a_s = 1 + h - i(\sqrt{A} + \sqrt{-B})$, $b_s = 1 + h - i(\sqrt{A} - \sqrt{-B})$, $c_s = 1 - 2i\sqrt{A}$. In the asymptotic region ($r \gg M$ or $z \rightarrow 1$), above solution will become

$$R^{in}(r \gg M) \sim D_0 r^h + D_1 r^{-1-h}, \tag{54}$$

where

$$D_0 = \frac{\Gamma(c_s)\Gamma(1+2h)}{\Gamma(a_s)\Gamma(b_s)}, \quad D_1 = \frac{\Gamma(c_s)\Gamma(-1-2h)}{\Gamma(c_s - a_s)\Gamma(c_s - b_s)}. \tag{55}$$

Absorption Cross-section

The essential part of the absorption cross-section can be read out directly from the coefficient D_0, namely

$$P_{abs} \sim |D_0|^{-2} \sim \sinh\left(2\pi A^{1/2}\right) |\Gamma(a_s)\Gamma(b_s)|^2. \tag{56}$$

Note that the constant D_1 is suppressed by the constant D_0, so we can ignore D_1. Note that this will agree, up to the undetermined normalization factors, with the CFT result,

$$P_{abs} \sim \sinh\left(\frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R}\right) \left|\Gamma\left(h_L + i\frac{\omega_L}{2\pi T_L}\right)\right|^2 \left|\Gamma\left(h_R + i\frac{\omega_R}{2\pi T_R}\right)\right|^2. \tag{57}$$

In order to match with gravity, we need $\omega_{L,R}, h_{L,R}$. We already have that $h_{L,R} = h + 1$. Then $\omega_{L,R}$ can be computed from equating

$$\delta S_{BH} = \frac{\delta M}{T_H} - \frac{\Omega \delta J}{T_H}, \quad \delta S_{CFT} = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}. \tag{58}$$

CFT Frequencies

From equating the entropies and identifying δM as ω and δJ as n, this yields to identification of $\delta E_{R,L}$ as $\omega_{R,L}$ where

$$\begin{aligned} \omega_L &= \frac{r_+^2 + r_-^2 + 2(a^2 - d^2 - k^2)}{2a}\omega, \\ \omega_R &= \omega_L - n. \end{aligned} \tag{59}$$

Absorption Cross-section Dyonic Kerr-Sen-AdS Black Hole

Radial Wave Equation

To consider the scattering issue of DKSAdS BH, we need to consider near-extremal condition of the radial wave equation because the near-horizon approximation will break down in asymptotic region. In addition, we also consider the scalar field frequency in the near-superradiant bound $\omega = \omega_s + \hat{\omega}\frac{\lambda}{r_0}$ where $\omega_s = n\Omega$. By using near-extremal coord. transformations,

$$r = \frac{r_+ + r_*}{2} + \lambda r_0 y, \quad r_+ - r_* = \mu\lambda r_0, \quad t = \frac{r_0 \Xi}{\lambda}\tau, \quad \phi = \varphi + \frac{\Omega_H r_0 \Xi}{\lambda}\tau, \tag{60}$$

and then followed by $z = \frac{y-\mu/2}{y+\mu/2}$, we can find the follwoing radial wave equation

$$\left[z(1-z)\partial_z^2 + (1-z)\partial_z + \frac{\hat{A}_t}{z} + \hat{B}_t + \frac{C_t}{1-z}\right] R(z) = 0, \tag{61}$$

where

$$\hat{A}_t = \frac{\hat{\omega}^2}{\upsilon^2\mu^2}, \quad \hat{B}_t = -\frac{1}{\upsilon^2}\left(\frac{\hat{\omega}}{\mu} - 2n\Omega_H r_+\right)^2, \quad C_t = C_t(\hat{\omega}). \tag{62}$$

Radial Wave Solution

The ingoing solution to wave equation (61) is

$$R(z) = z^{-i\sqrt{\hat{A}_t}}(1-z)^{1+h}\ {}_2F_1(a_s, b_s; c_s; z), \tag{63}$$

with the parameters
$a_s = 1 + h - i(\sqrt{\hat{A}_t} + \sqrt{-\hat{B}_t}), \ \ b_s = 1 + h - i(\sqrt{\hat{A}_t} - \sqrt{-\hat{B}_t}),$
$c_s = 1 - 2i\sqrt{\hat{A}_t}, \ h = \frac{1}{2}\left(-1 + \sqrt{1 - 4C_t}\right).$
In the asymptotic region ($y \gg \mu/2$ or $z \to 1$), above solution is

$$R(y) \sim D_0 y^h + D_1 y^{-1-h}, \tag{64}$$

where

$$D_0 = \frac{\Gamma(c_s)\Gamma(1+2h)}{\Gamma(a_s)\Gamma(b_s)}, \quad D_1 = \frac{\Gamma(c_s)\Gamma(-1-2h)}{\Gamma(c_s - a_s)\Gamma(c_s - b_s)}. \tag{65}$$

Absorption Cross-section

Similarly with ungaged case, we can find the absorption cross-section as

$$P_{abs} \sim |D_0|^{-2} \sim \sinh\left(2\pi \hat{A}_t^{1/2}\right) |\Gamma(a_s)\Gamma(b_s)|^2. \quad (66)$$

Above P_{abs} will agree with the CFT result (57) with the following quantities

$$\omega_L = n, \quad \omega_R = \frac{r_0}{a\Xi}\left(\hat{\omega} - \mu n \Omega_H r_+\right), \quad (67)$$

while the temperatures and conformal weights are now given by

$$T_L = \frac{\upsilon}{4\pi\Omega_H r_+}, \quad T_R = \frac{\upsilon r_0}{4\pi a\Xi}\lambda\mu, \quad h_L = h_R = 1 + h. \quad (68)$$

Note that for the extremal case, T_R will vanish.

Conclusions

- Neutral scalar wave equation in the low-frequency limit in DKS and DKSAdS BHs's background possesses $SL(2,R) \times SL(2,R)$ isometry $\rightarrow$ isometry of AdS_3 and CFT_2.
- CFT in DKS black hole is represented by

$$T_L = \frac{r_+^2 + r_-^2 + 2(a^2 - d^2 - k^2)}{4\pi a(r_+ + r_-)}, \quad T_R = \frac{r_+ - r_-}{4\pi a}, \quad c_L = c_R = 6a(r_+ + r_-).$$

that will reproduce Bekenstein-Hawking entropy from Cardy formula. While for DKSAdS BH, CFT is represented by

$$T_L = \frac{\upsilon[r_+^2 + r_*^2 + 2(a^2 - d^2 - k^2)]}{4\pi a(r_+ + r_*)\Xi}, \quad T_R = \frac{\upsilon(r_+ - r_*)}{4\pi a\Xi},$$

$$c_L = c_R = \frac{6a(r_+ + r_*)}{\upsilon}.$$

These reduce to those of DKS BH when $1/l^2 = 0$.
- P_{abs} agrees, up to the undetermined normalization factors, with the CFT result by determining the CFT frequencies.

THE END

Thank you for your attention!